Towards a Categorical Treatment of Economics

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What is Economics?

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- A more general definition is that Economics studies the *interaction among intentional entities* (no wonder that other social scientists think that Economics is "imperialistic"!).

Why a categorical treatment?

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This presentation

- The main problem we want to address is the impossibility of separating different "local" interactions as if all the others remained fixed.
- That is, we want to get rid of the *cæteris paribus* clause frequently applied by economists in their analyses.
- One instance of the problem arises when we try to see whether *agenthood* is well-defined.
- Another instance appears when we want to see whether the intended solutions to interaction problems scale up with their aggregation.
- Both instances of the original problem reveals the need for a *level-agnostic* (or *continuous with respect to subagents*) Economic Theory.

Single agents and different economic problems

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- In applications of this model it is customary to reduce the analysis to a subspace of the space of alternatives, simplifying the problem of making a decision.
- This requires to assume the independence of the preferences over the subspace from the preferences over the rest of the larger space of alternatives.

Local problems

- Consider a family of local *problems* each with its own domain, say D_i and each with a problem-specific function u_i.
- Hypothesis: there exist a global function U over D ($D_i \subseteq D$ for all i).
- Then, U must be such that $u_i = U_{|D_i}$ for each i.
- To recover the hypothetical U, we must be able to patch together the local restrictions in a consistent way.

Decision-making: local vs. global

- Let $\mathcal{L}$ be a space of possible **options** that an agent may select.
- Each $x \in \mathcal{L}$ is evaluated by means of a *utility* function, $U : \mathcal{L} \rightarrow \mathbb{R}$.
- Given a family of constraints limiting the set of options for the agent to $\hat{L} \subseteq \mathcal{L}$, the goal of the agent is to find some $\mathbf{x}^*$ that yield the highest value of U over $\hat{L}$.

Decision-making: local vs. global

- Consider a family $\{L^k\}_{k=0}^{\kappa}$ of closed linear subspaces of $\mathcal{L}$.
- Let us define
$$\text{Proj}_k : \mathcal{L} \to L^k$$
such that $\text{Proj}_k(x) = x^k$, is the *projection* of x on L^k.
- The projection of a global solution $\mathbf{x}^*$ onto L^k will return the point in L^k that is closer to $\mathbf{x}^*$.

Decision-making: local vs. global

- In case the projection does not return a local solution, however, we can still define an operator, which we call $\Gamma_k(x)$ that formalizes the idea of "best choice" within a local problem.

- Let us define a new correspondence, $\Gamma_k : \hat{L} \to \hat{L}^k$:

$$\Gamma_k(x) = \{x^k \in \hat{\mathbf{X}}^k : x^k \in \text{argmin}_{y \in \hat{\mathbf{X}}^k} |y - \text{Proj}_k(x)|\}.$$

The category of local problems

Definition

A *local problem* is $s^k = \langle L^k, \hat{L}^k, u^k, \hat{\mathbf{X}}^k \rangle$, where $\hat{\mathbf{X}}^k$ is the class of "highest values" of a utility function u^k over a compact set $\hat{L}^k \subseteq L^k$.

The category of local problems

Definition

Let $\mathcal{PR}$ be the category of local problems, where

- $\mathrm{Obj}(\mathcal{PR})$ is the class of objects. Each one is a problem s^k.

- a morphism $\rho_{kj} : s^k \to s^j$ exists if two conditions are fulfilled:
  - $\hat{L}^k \subseteq \hat{L}^j$, $u^k = u^j|_{L^k}$ and
  - $\dim(L^k) \leq \dim(L^j)$.

- Given two morphisms $\rho_{kj} : s^k \to s^j$ and $\rho_{jl} : s^j \to s^l$ there exists their composition $\rho_{jl} \circ \rho_{kl} = \rho_{kl}$.

A sheaf of local problems

- We can also define $\mathcal{P}(\mathcal{L})$ as the category in which the objects are subsets of $\mathcal{L}$ and morphisms are composable correspondences (multivalued functions).

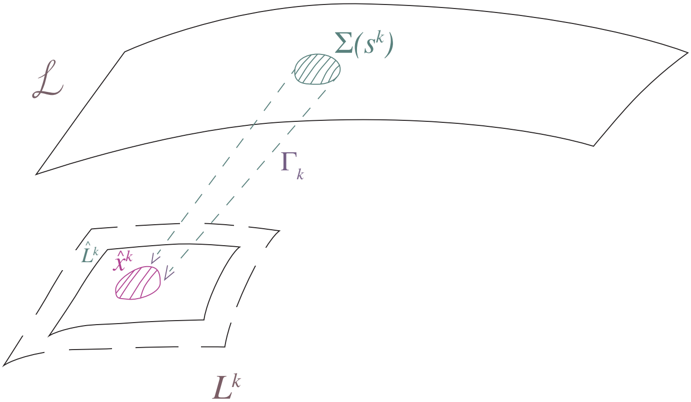
- We can now define a functor

$$\Sigma : \mathcal{PR} \longrightarrow \mathcal{P}(\mathcal{L})$$

which assigns to a problem s^k the subset $\Sigma(s^k) \subseteq \mathcal{L}$:

$$\Sigma(s^k) = \{y \in \mathcal{L} \mid \Gamma_k(y) \in \hat{\mathbf{X}}^k\}$$

A sheaf of local problems



The sheaf of local problems

- A *section* σ_k over s^k assigns the elements of $\Sigma(s^k)$ to s^k:

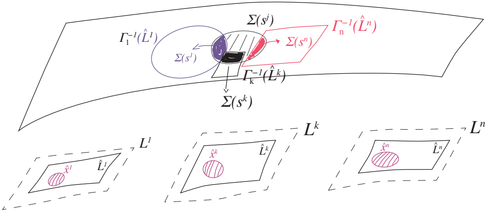
$$\sigma_k : s^k \mapsto \Sigma(s^k).$$

- Given two problems, s^k and s^j, let us write $s^k \lhd s^j$ iff there exists a morphism $s^k \rightarrow s^j$ in $\mathcal{PR}$ ($s^k \lhd s^j$ indicates that s^k is a restriction of s^j).
- Given $\rho_{kj} : s^k \rightarrow s^j$, the correspondence r_k^j is such that

$$r_k^j = \Sigma(\rho_{kj}) : \Sigma(s^j) \rightarrow \Sigma(s^k)$$

such that $r_k^j(\Sigma(s^j) = \Gamma_k^{-1}[proj_k(\Sigma(s^j))] = \Sigma(s^k)$.

A sheaf of local problems



Example

Consider $\mathcal{L}$ *to be* $\mathbb{R}^3$ *(the three-dimensional real Euclidean space) and the utility function:*

$$U(x, y, z) = 3 - 2x^2 - y^2 - 3z^2$$

It has a single global solution $\hat{\mathbf{X}} = \{(0, 0, 0)\}$.

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- s^1: $L^1 = \{(x, y, z) : z = 0\}$, *with* $u^1(x, y, z) = U_{|L^1} = 3 - 2x^2 - y^2$ *over* $\hat{L}^1 = \{(x, y, 0) \in L^1 : x^2 + y^2 = 1\}$. *The solutions are* $\hat{\mathbf{X}}^1 = \{(0, 1, 0), (0, -1, 0)\}$.

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- s^2: $L^2 = \{(x,y,z) : (x,y,z)\cdot(1,-1,1) = 0\}$, *with* $u^2(x,y,z) = 3 - 3x^2 - 4z^2 - 2xz$ *over* $\hat{L}^2 = \{(x,y,z) : 2x^2 + 2z^2 + 2xz = 1\}$. *The solution set is:* $\hat{\mathbf{X}}^2 = \{(-\sqrt{\frac{1}{3}}, -\frac{1}{2\sqrt{3}} - \frac{1}{2}, \frac{1}{2\sqrt{3}} - \frac{1}{2}), (\sqrt{\frac{1}{3}}, \frac{1}{2\sqrt{3}} + \frac{1}{2}, \frac{1}{2} - \frac{1}{2\sqrt{3}})\}$.

Example

Then, $\Gamma_1(0,0,0) = \hat{\mathbf{X}}^1$ *and* $\Gamma_2(0,0,0) = \hat{\mathbf{X}}^2$.
Consider a new problem s^0*, the optimization of* U *over the surface of the three-dimensional sphere*
$\hat{L}^0 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ *and thus,*
$\hat{\mathbf{X}}^0 = \{(0, 1, 0), (0, -1, 0)\}$.

Example

We define $\Sigma : \mathcal{PR} \to \mathcal{P}(\mathcal{L})$*, summarized by the following table (each row being a section* σ_i*,* $i = 0, 1, 2$*):*

Problems	a_1	b_1	a_2	b_2
s^1	X	–	X	–
s^2	–	X	–	X
s^0	X	–	X	–

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Problems	a_1	b_1	a_2	b_2
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s^0	X	$-$	X	$-$

The range of Σ *is based only of four elements in* $\mathcal{L}$:

$$a_1 = (0, 1, 0) \quad a_2 = (0, -1, 0)$$

and

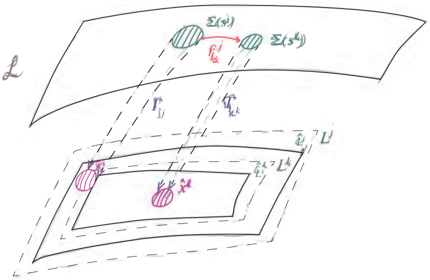
$$b_1 = \left(-\sqrt{\frac{1}{3}}, -\frac{1}{2\sqrt{3}} - \frac{1}{2}, \frac{1}{2\sqrt{3}} - \frac{1}{2}\right) \quad b_2 = \left(\sqrt{\frac{1}{3}}, \frac{1}{2\sqrt{3}} + \frac{1}{2}, \frac{1}{2} - \frac{1}{2\sqrt{3}}\right)$$

Example

It is easy to check that $s^i \triangleleft s^0$ *for* $i = 1, 2$*, since on one hand each problem* s^i *can be seen as the maximization of* U *restricted to subsets of the domain of problem* s^0*. On the other hand,* $f_i^0(\Sigma(s^0)) = \Sigma(s^i)$*.*

- *For* s^1 *it is clear that this is the case.*
- *For* s^2*, let us note that* b_1, b_2 *are the solutions of the problem* s^0 *restricted to* $\hat{L}^2$*, seen as the inverse projection over the surface* $\hat{L}^0$*.*

A sheaf of local problems



A sheaf of local problems

- Σ is a presheaf (i.e. a *contravariant* functor between $\mathcal{PR}$ and $\mathcal{P}(\mathcal{L})$).

- A family $\{s^k\}_{k\in K} \subseteq \mathrm{Obj}(\mathcal{PR})$ is said to be a *cover* of problem s^j if $s^k \lhd s^j$ for each $k \in K$ and $\hat{L}^j \subseteq \cup_{k\in K} \hat{L}^k$.

- The family of sections $\{\sigma_k\}_{k\in K}$ is said to be *compatible* if for any pair $k, l \in K$,

$$\Gamma_k(\Sigma(s^k)) \cap \Gamma_l(\Sigma(s^k)) = \Gamma_k(\Sigma(s^l)) \cap \Gamma_l(\Sigma(s^l))$$

A sheaf of local problems

- Given a cover $\{s^k\}_{k \in K}$ of a problem s^j with compatible sections, Σ is then *sheaf* if there exists a unique $\sigma_j = \Sigma(s^j)$ such that for each $k \in K$,

$$\sigma_k = \sigma_j \cap \Gamma_k^{-1}(\hat{\mathbf{X}}^k)$$

- Intuitively, Σ is a sheaf if σ_j in fact "glues" together all the assignments σ_k in $\mathcal{P}(\mathcal{L})$.

Example

We can check that in our example $\{\sigma^1, \sigma^2\}$ *is a compatible family of sections. Notice that* $\hat{L}^1 \cap \hat{L}^2$ *does not include the solutions to either problem. Then, the sections satisfy, trivially, the compatibility condition.*
Then, Σ *satisfies the sheaf condition.*

Always a sheaf?

- Given $\Sigma : \mathcal{PR} \to \mathcal{P}(\mathcal{L})$, is the sheaf condition always satisfied?.
- Given a problem s, the sheaf condition implies that its solution remains independent of other solutions and thus it disregards their *contextual* relevance.
- If we consider two sequences $s^1, \ldots, s^n$ and $s^{1'}, \ldots, s^{n'}$ in $\mathrm{Obj}(\mathcal{PR})$, such that $s^n{=}s{=}\ s^{n'}$, understood as two different paths (of problems previously solved), the sheaf condition implies that the solution to s is independent of the path followed. That is, the solution is purely *local*.

Always a sheaf?

Proposition

If for every s^k *in* $\mathcal{PR}$

- *The elements in* $\tilde{\mathbf{X}}^k$ *are the maximizers of* u^k *and*
- u^k *is the constraint of a single function* (U) *over* L^k*.*

Then $\Sigma : \mathcal{PR} \to \mathcal{P}(\mathcal{L})$ *is a sheaf.*

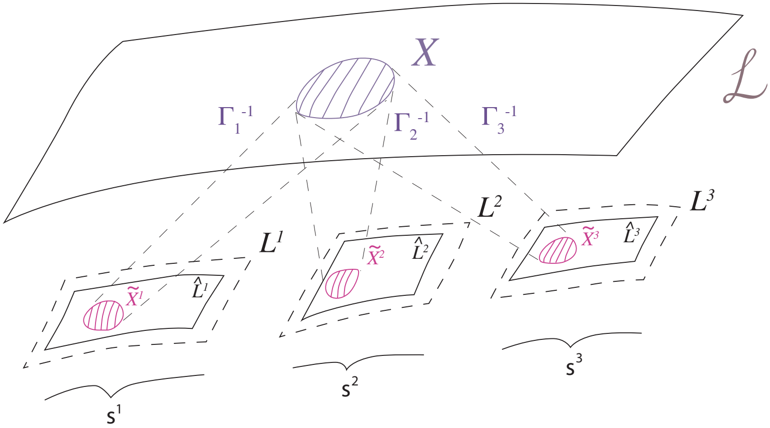
Always a sheaf?

- To establish this claim we start by defining a functor $\Lambda : \mathcal{P}(\mathcal{L}) \to \mathcal{PR}$.
- For any $X \in \mathcal{P}(\mathcal{L})$:

$$\Lambda(X) = \{s^k = \langle L^k, \hat{L}^k, u^k, \tilde{\mathbf{X}^k} \rangle \in \mathrm{Obj}(\mathcal{PR}) : X = \Gamma_k^{-1}(\tilde{\mathbf{X}}^k)\}$$

- That is, given $X \subseteq \mathcal{L}$, Λ yields the problems that have as solutions the projections of X.

Always a sheaf?



$$\Lambda(x) = \{s^1, s^2, s^3\}$$

Always a sheaf?

Proposition

For any $s \in Obj(\mathcal{PR})$, $s \in \Lambda(\Sigma(s))$.

and

Proposition

If $\bigcup_{k \in K} \Gamma_k^{-1}(\tilde{\mathbf{X}}^k) = \tilde{\mathbf{X}}$ *then* $\Lambda(\Sigma(s)) \subseteq \{s\}$.

Always a sheaf?

- Under the conditions of the last Proposition, Λ can be seen as defining a *fiber bundle*.
- Moreover, it is a *trivial bundle* with fiber $\Lambda(\Sigma(\mathbf{s}))$, where **s** is the global problem.

Proposition

If for every $s^k = \langle L^k, \hat{L}^k, u^k, \tilde{\mathbf{X}}^k \rangle$ *in* $\mathcal{PR}$, $\Lambda(\Sigma(s^k)) = \{s^k\}$ *then* Λ *is trivial iff there exists* U, *such that* u^j *has the same optimal points as* $U_{|L^j}$.

Examples

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- *Case-Based Decision Theory* (Itzhak Gilboa and David Schmeidler) assumes that the similarity to previous problems, stored in memory, is used to obtain solutions to decision problems. But then, if a sequence of previous cases is different, even if the final class of problems is the same, the decisions may be different. That is, the non-locality of solutions makes Λ non-trivial.

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- Interactions in fixed frameworks are formalized in Economics as *games*.
- But an open problem is whether games can be "connected", in ways that lead players to become subject to different rules and even change the way in which outcomes are evaluated.
- *Open Games* (Jules Hedges, Neil Ghani, etc.) address this issue, using the concept of *lenses*. We will present an alternative categorical view of games.

A category of games

Consider a category $\mathcal{G}$ of *games*. Each object G is a game $G = \langle (I_G, S_G, \mathbf{O}_G, \rho_G), \pi_G \rangle$, such that $(I_G, S_G, \mathbf{O}_G, \rho_G)$ is a *game form*:

- I_G is the set of players.
- $S_G = \prod_{i \in I_G} S_i^G$ is the class of strategy profiles, where $S_i^G \subseteq S_i$ is the set of strategies of player $i \in I_G$.
- $\mathbf{O}_G$ represents the possible *outcomes* of the game while $\rho_G : S_G \to \mathbf{O}_G$ is a bijection.
- $\pi_G = (\pi_i^G)_{i \in I_G}$, is *a vector of payoffs*, where $\pi_i^G : \mathbf{O}_G \to \mathbb{R}^+$ is the payoff function of $i \in I_G$.

A category of games

Given

$$G = \langle (I_G, S_G, \mathbf{O}_G, \rho_G), \pi_G \rangle \quad \text{and} \quad G' = \langle (I_{G'}, S_{G'}, \mathbf{O}_{G'}, \rho_{G'}), \pi_{G'} \rangle,$$

a morphism

$$G \rightarrow G'$$

is such that:

- $I_G \subseteq I_{G'}$.
- $S_i^G \subseteq S_i^{G'}$ for each $i \in I_G$.
- There exist two functions:
  - An *inclusion* $p_{\mathbf{O}_G}^{\mathbf{O}_{G'}} : \mathbf{O}_{G'} \hookrightarrow \mathbf{O}_G$.
  - A *projection* $p_{S_G}^{S_{G'}} : S_{G'} \rightarrow S_G$.

Properties of $\mathcal{G}$

Proposition

$\mathcal{G}$ is a category with finite colimits.

- We can define cospans in $\mathcal{G}$. Consider three objects G, G' and G'' with morphisms $G \xrightarrow{f} G'' \xleftarrow{g} G'$. This means that G and G' are subgame forms of the same game (G'').
- We can take as monoidal product the coproduct $G + G'$.

A derived category

- Consider the monoidal symmetric category of cospans in $\mathcal{G}$, $\mathbf{W}_{\mathcal{G}} = \text{cospan}_{\mathcal{G}}$.
- Let us define $\psi : G_1, G_2, \ldots, G_n \to \bar{G}$ as the "wiring" $\phi : G_1 + G_2 + \ldots + G_n \to \bar{G}$. Then

$$G_1 + G_2 + \ldots + G_n \xrightarrow{f} C \xleftarrow{\bar{f}} \bar{G}$$

means that, when f and $\bar{f}$ isomorphims:

Proposition

$\bar{G}$ is the minimal game of which $G_1, \ldots, G_n$ are subgame forms.

A hypergraph category of games

- Consider the *hypergraph category* $\langle \mathcal{G}, \mathrm{Eq} \rangle$ where $\mathrm{Eq} : \mathbf{W}_{\mathcal{G}} \rightarrow \prod_i S_i$, is such that for each G in $\mathbf{W}_{\mathcal{G}}$, $\mathrm{Eq}(G)$ is a subset of the class of strategies of G, $\prod_{i \in I} S_i^G$.
- We say that $\mathrm{Eq}(G)$ is a class of *equilibria* of G.
- Consider the operation $\hat{\cup}$ that, given two equilibria $s \in \mathrm{Eq}(G)$ and $s' \in \mathrm{Eq}(G')$, yields $s - s' \in \mathrm{Eq}(G) \hat{\cup} \mathrm{Eq}(G')$.
- This operation is such that $i \in I_G \cap I_{G'}$ obtains a new strategy that combines s_i and s_i', while the other individual strategies in G and G' remain the same. That is, $\pi_i^{G \hat{\cup} G'}(s - s') = \pi_i^G(s) \times \pi_i^{G'}(s^{'})$ for $i \in I_G \cap I_{G'}$.

Hypergraph category of games

Proposition

Given two games, G and G', $Eq(G) \hat{\cup} Eq(G') = Eq(G + G')$.

Then:

Proposition

Eq is a lax monoidal functor.

Example

Let us consider two *games*, the *Battle of the Sexes* and the *Prisoner's Dilemma*.

Player 2 (columns) / Player 1 (rows)

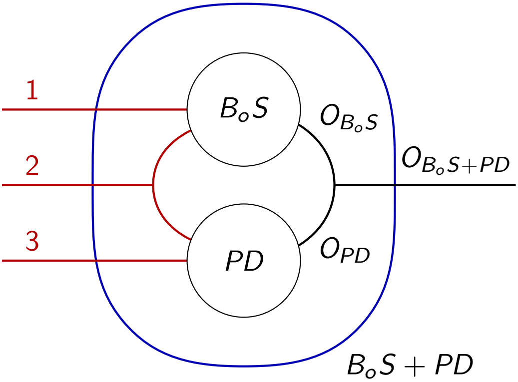
	Bx	BII
Bx	1, 1	0, 0
BII	0, 0	1, 2

Player 3 (columns) / Player 2 (rows)

	C	S
C	2, 2	0, 3
D	3, 0	1,1

Example

The wiring diagram of **BoS** and **PD** is:



Example

$G + G'$ can be described by two matrices. One corresponds to 3 choosing C:

Player 2

		Bx/C	Bx/D	BII/C	BII/D
Player 1	Bx	$2,1 \times 2,2$	$2,1 \times 3,0$	$0,0 \times 2,2$	$0,0 \times 3,0$
	BII	$0,0 \times 2,2$	$0,0 \times 3,0$	$1,2 \times 2,2$	$1,2 \times 3,0$

Example

The other matrix corresponds to 3 choosing D:

Player 2

		Bx/C	Bx/D	BII/C	BII/D
Player 1	Bx	$2,1 \times 0,3$	$2,1 \times 1,1$	$0,0 \times 0,3$	$0,0 \times 1,1$
	BII	$0,0 \times 0,3$	$0,0 \times 1,1$	$1,2 \times 0,3$	$1,2 \times 1,1$

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- $\langle \mathcal{G}, \mathsf{Eq} \rangle$ is too rigid to capture the dynamics of economic interactions.
- We need a more flexible structure.
- A possibility is to consider a $\mathbb{O}\mathbf{rg}$-enriched dynamic category (David and Brandon).

Economies as dynamic monoidal category

- Let us recall that $\mathbb{O}\mathbf{rg}$ is a bicategory, where Ob($\mathbb{O}\mathbf{rg}$) = Ob(**Poly**) and Morph($\mathbb{O}\mathbf{rg}$) consists of the categories $[p, q] - \mathbf{Coalg}$.
- $[p, q]$ is an internal hom in **Poly** that can be seen as a process that takes as inputs both "flows" from outputs of p to outputs of q and from inputs of q to inputs of p and yields as output morphisms $\phi : p \to q$.
- A $[p, q] - \mathbf{Coalg}$ is a category in which each object is a *state* with a rule that assigns both a corresponding *interaction pattern* (an output of $[p, q]$) and an update of the state in response to that pattern.

Economies as dynamic monoidal category

- Then, an $\mathbb{O}$**rg**-enriched dynamic multicategory is such that, briefly:
  - for each object a it corresponds a p_a in **Poly**,
  - for objects $a_1, \ldots, a_n, b$ there corresponds a $[p_{a_1} \oplus \ldots \oplus p_{a_n}, p_b] - \mathbf{Coalg}$ of *states* $\mathbf{S}_{a_1,\ldots,a_n,b}$,
  - Morphisms are such that each object a satisfies an "identitor" condition and pairs of morphisms can satisfy a "compositor" condition. Both indicate, roughly, that morphisms inherit identity and compositionality properties from $\mathbb{O}$**rg**.

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Economies as dynamic monoidal category

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- Perhaps $\mathcal{G}$ is the best candidate. Notice that $\text{Ob}(\mathcal{PR}) \subseteq \text{Ob}(\mathcal{G})$ since individual decision-making settings can be seen as single-player games.
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- Each state in the morphism between games indicates a different way of connecting them dynamically.

Conclusion

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Thanks!!