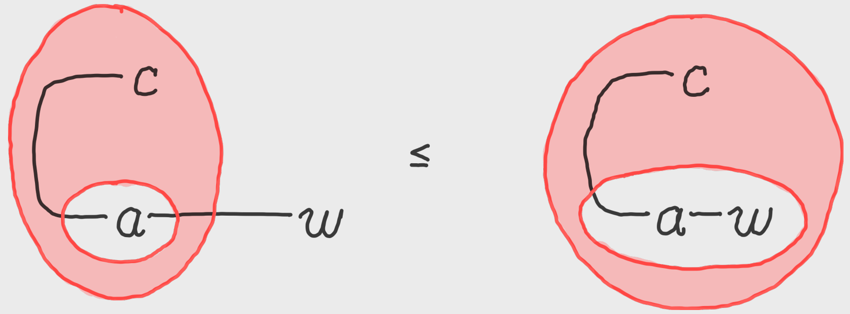
The Second (Graphical) Calculus of Relations: Peirce’s Existential Graphs

Nathan Haydon
University of Waterloo



$$[\ (c \multimap a) \otimes w \leq \ c \multimap (a \otimes w)\ ]$$

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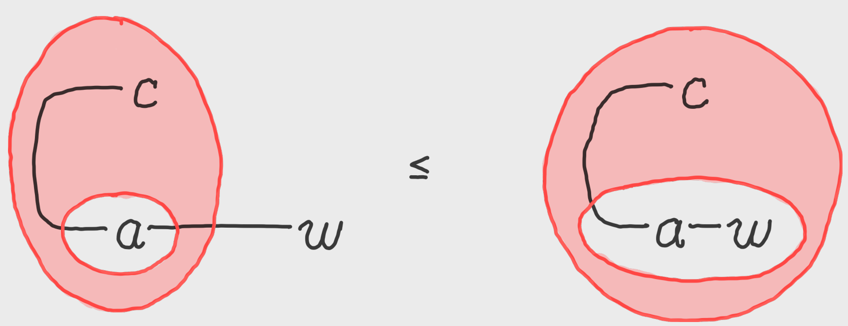
Vaughan Pratt's 'The Second Coming of Binary Relations'



$$[\ (c \multimap a) \otimes w \leq c \multimap (a \otimes w)\ ]$$

Aim

Introduce Peirce's *Existential Graphs*…

… as a precursor to string diagrams …

... and as the inspiration for recent
developments in categorical logic.

Outline

- introduce Peirce's Existential Graphs (à la regular logic and cartesian bicategories)
- move to the Neo-Peircean Calculus of Relations (à la residuation and cyclic bilinear logic)
- demonstrate topological advantages

The *Existential Graphs* are a graphical calculus for the logic of relations developed by Charles S. Peirce from the 1880s-1914…

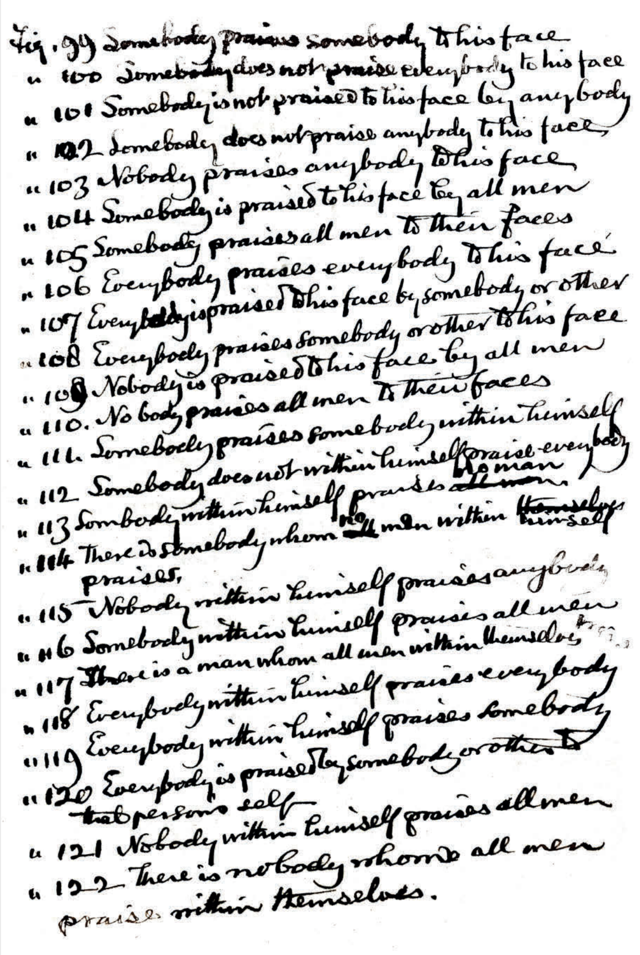
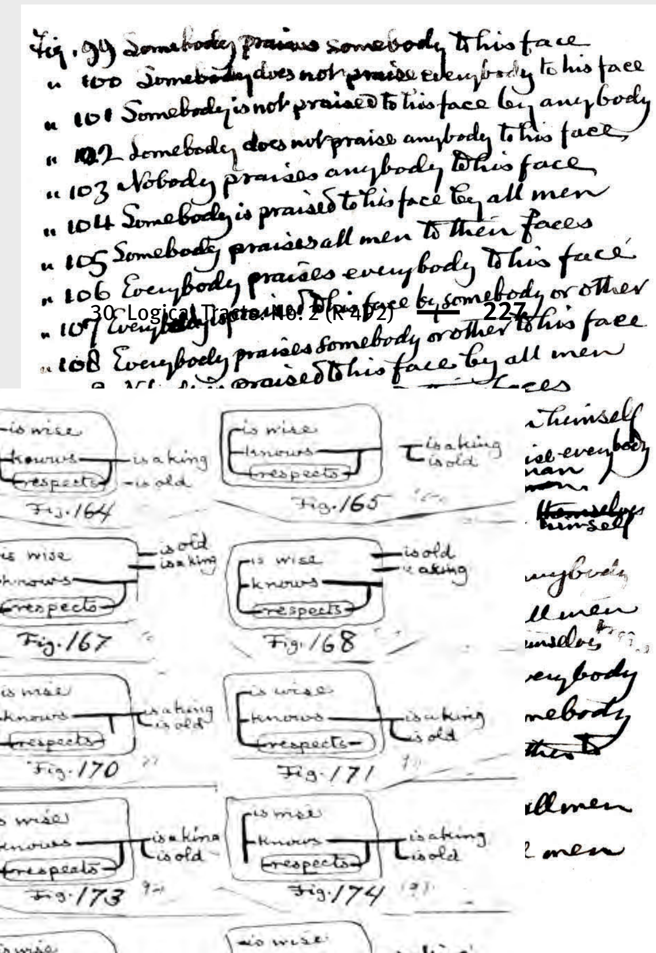


Fig. 99 Somebody praises somebody to his face
" 100 Somebody does not praise everybody to his face
" 101 Somebody is not praised to his face by anybody
" 102 Somebody does not praise anybody to his face
" 103 Nobody praises anybody to his face
" 104 Somebody is praised to his face by all men
" 105 Somebody praises all men to their faces
" 106 Everybody praises everybody to his face
" 107 Everybody is praised to his face by somebody or other
" 108 Everybody praises somebody or other to his face
" 109 Nobody is praised to his face by all men
" 110. No body praises all men to their faces
" 111. Somebody praises somebody within himself
" 112 Somebody does not within himself praise everybody
" 113 Somebody within himself praises ~~all men~~ no man
" 114 There is somebody whom ~~all~~ no men within ~~themselves~~ himself praises.
" 115 Nobody within himself praises anybody
" 116 Somebody within himself praises all men
" 117 There is a man whom all men within themselves praise
" 118 Everybody within himself praises everybody
" 119 Everybody within himself praises somebody
" 120 Everybody is praised by somebody or other ~~to that person's self~~
" 121 Nobody within himself praises all men
" 122 There is nobody whom all men praise within themselves.

Two holograph pages (Harvard Peirce Papers, R 493).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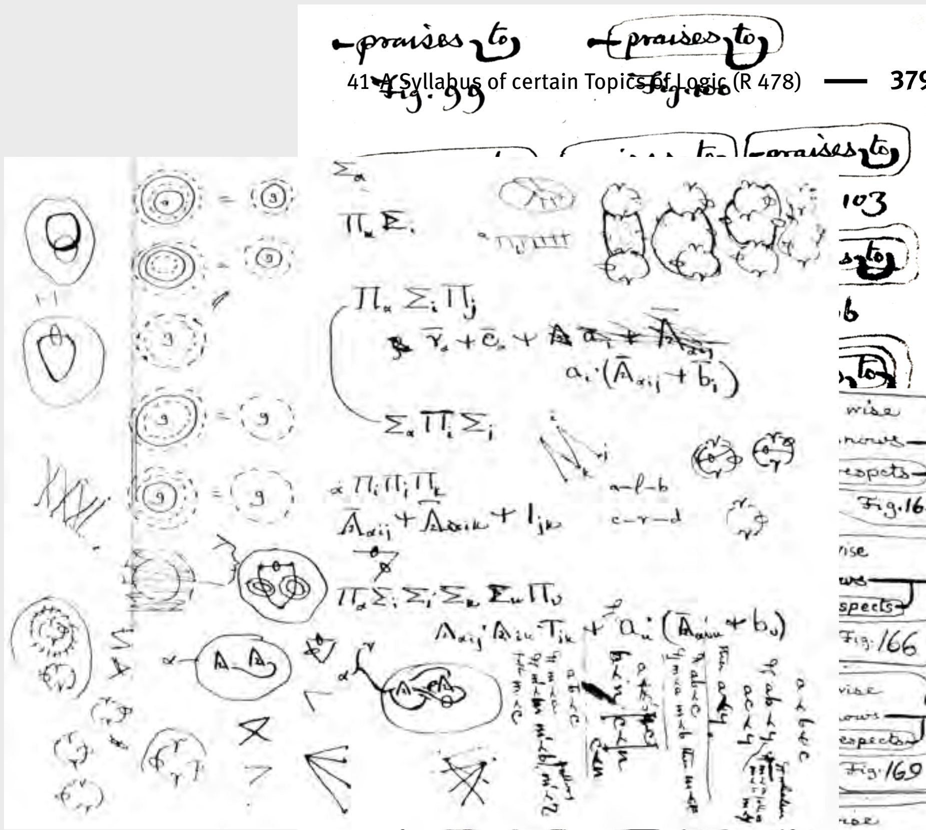
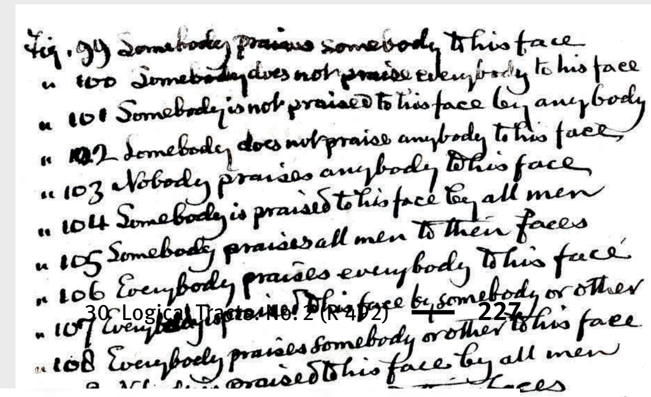
Two holograph pages (Harvard Peirce Papers, R 493).

30 Logical Tracts. No. 2 (R 492) 227

[A mutilated page of R 492, showing abundant editorial marks and cut-and-paste clippings by the editors of the *Collected Papers* (Harvard Peirce Papers).]



e of R 492, showing abundant editorial marks and cut-and-paste clippings by
e *Collected Papers* (Harvard Peirce Papers).]



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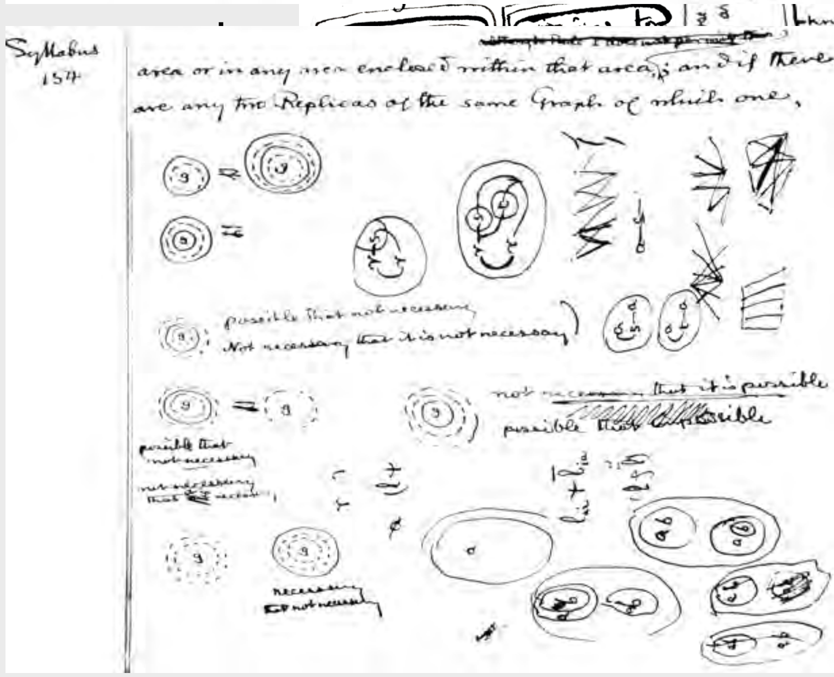

[Two holograph images (Harvard Peirce Papers, R 478(s)): a reversed verso of an abandoned ms draft page 137 (above) and an abandoned ms draft page 154 (below).]

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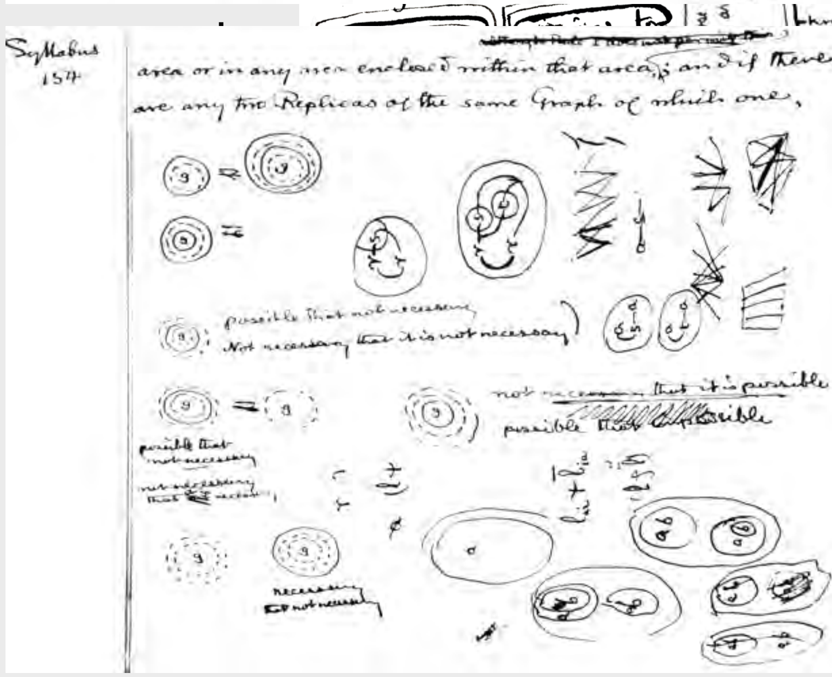
…e *Collected Papers* (Harvard Peirce Pap…



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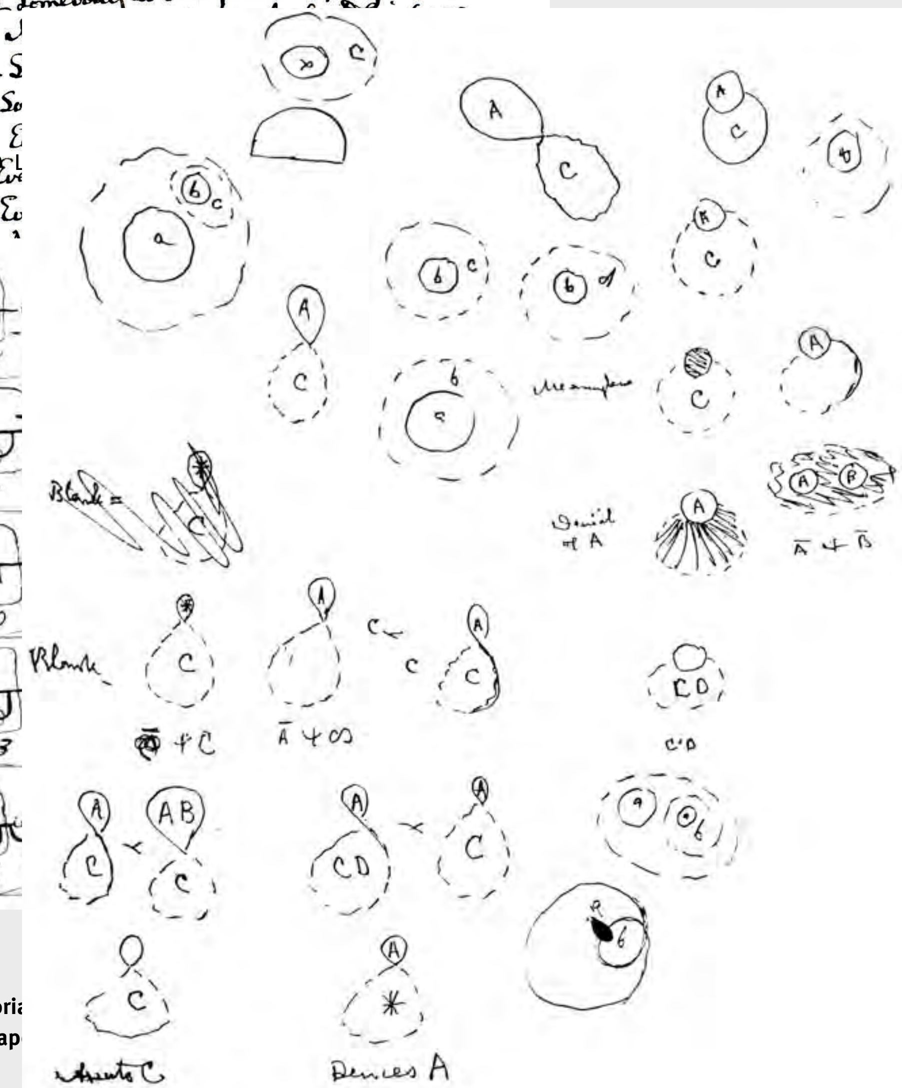
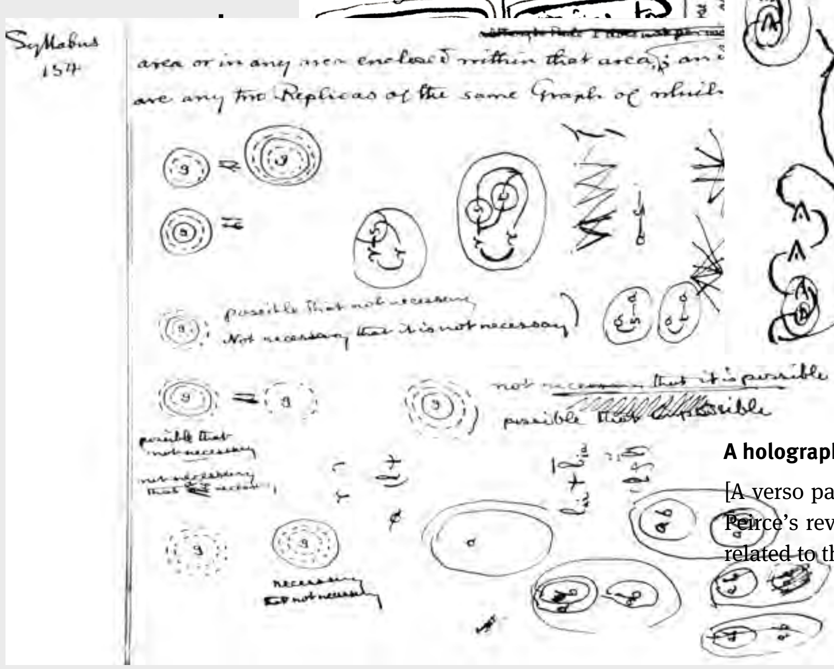


A holograph page (Harvard Peirce Papers, R 1070).

[An undated proposal representing variants of logical graphs that have a rather special look. This one-page study is inscribed on the back page of a leaflet, "Inventory of Instruments", which Peirce had retained from the Coastal and Geodetic Survey perhaps since late 1880s. Three pages of that leaflet have been preserved in R 1070; perhaps Peirce kept them as scrap paper which he was constantly in shortage of. The graphs are likely to have been scribed much later, however, and possibly date from the post-1903 era. Non-standard notations are created especially for the blot, thus effecting contradictions ("meaningless") and denials of an assertion, as well as for the scroll, in which the inloops are pulled out from the outloop to form these 8-shapes distinguished from the latter by dashed boundaries. Rules for inserting on the antecedent and erasing from the consequent appear near the bottom left of the page.]

41 A Syllabus of certain Topics of Logic



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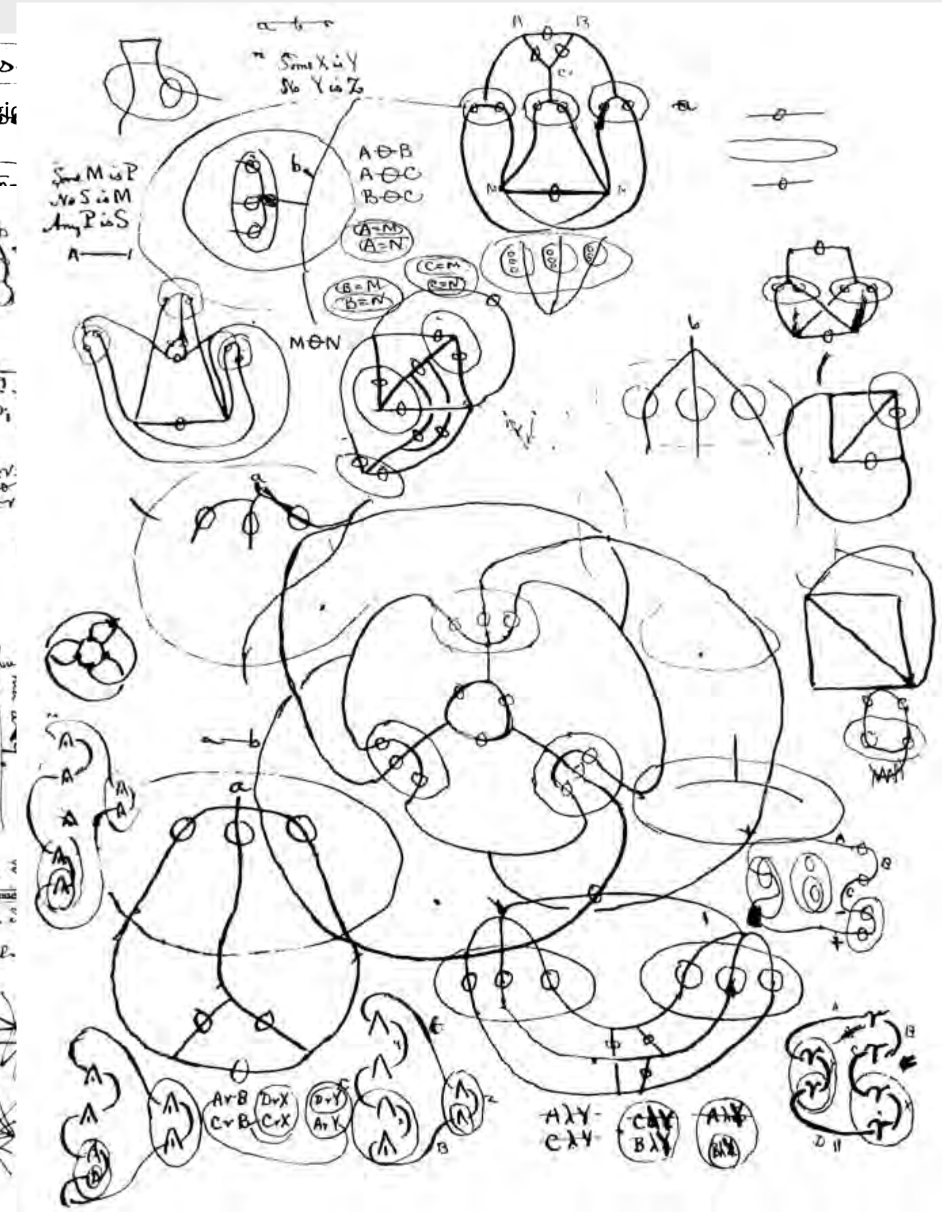
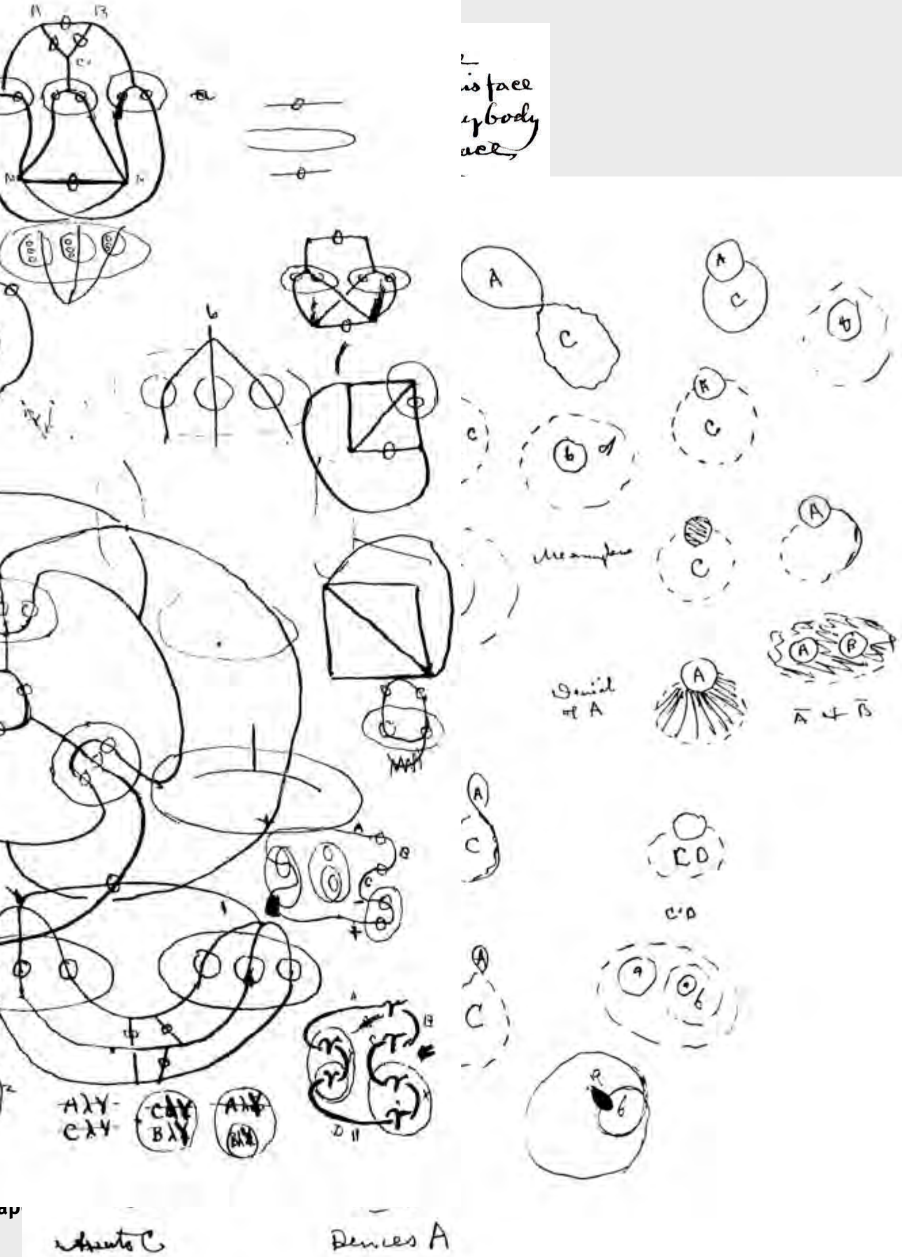


Collected Papers (Harvard Peirce Pap

A holograph page (Harvard Peirce Papers, R 1333)

[A verso page filled with studies on graphs. The r[illegible] Peirce's review on English scientists, published in [illegible] related to the topics taken up in the pre-drafts of h[illegible]



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The *Existential Graphs* are a graphical calculus for the logic of relations developed by Charles S. Peirce from the 1880s-1914…

…and had almost no uptake within the broader logic community.

Why return to the Existential Graphs?…

Peirce presented some of the earliest instances of string diagrams as we might recognize them today…

… and stated some essential features and laws.

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Suppose we wish to express the algebraic principle that $(x+y)+z=(z+y)+x$. We will write the letter s with three hooks, as in Fig. 33; to express that w is equal to a result of adding something equal to u to something equal to v.

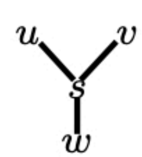

Fig. 33

Fig. 34

Fig. 35

Fig. 36

Then Fig. 34 expresses that, in a universe of values, whatever be the values of x, y, and z, there is a value, m, of $x+y$ such that a value, t, of $m+z$ is a value of $n+x$ where n is a value of $z+y$. Of course, a system of representation designed to express all propositions as analytically as possible cannot, from the nature of things, express the mathematical relation with the same elegance as a system designed only to express the special kind of

(1903)

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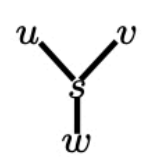

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Fig. 35

Fig. 36

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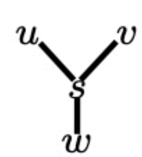

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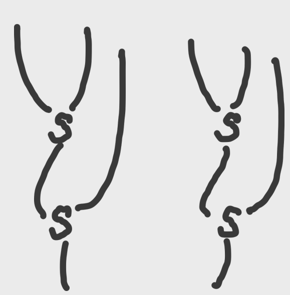

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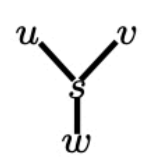

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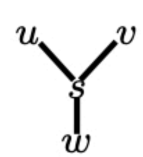

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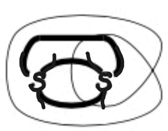

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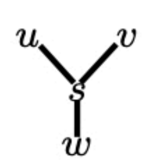

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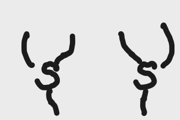

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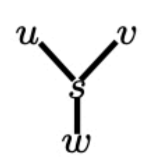

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What are the insights that led to the development of EGs?

&

How exactly do they compare to string diagrams as we know them?

Developing Alpha
(propositional logic)

[‘Sheet of Assertion’]

Convention: Universe of discourse…

[‘Sheet of Assertion’]

Convention: Universe of discourse…
… what we assert to be true.

['Sheet of Assertion']

Convention: Universe of discourse…

… what we assert to be true.

The lilacs are in bloom.

[‘Sheet of Assertion’]

Convention: Universe of discourse…

… what we assert to be true.

['Sheet of Assertion']

Convention: Universe of discourse…
… what we assert to be true.

A pear is ripe.

['Sheet of Assertion']

It hails.

It hails.

It is cold.

Convention: Juxtaposition as conjunction…

It hails.

It is cold.

Convention: Juxtaposition as conjunction…
… they are asserted together.

It hails.

It is cold.

Convention: Juxtaposition as conjunction…
…they are asserted together.

It hails.

It is cold.

[It hails
…and…
It is cold.]

It hails.

It is cold.

Inference Rule: Erasure…

It hails.

It is cold.

Inference Rule: Erasure…

It hails.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

[‘then it is true that’]

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

[‘then it is true that’] It hails.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

It is cold.

['then it is true that']

It is cold.

Inference Rule: Erasure…

…once said, can be unsaid.

It hails.

['then it is true that']

It is cold.

It hails.

['then it is true that']

It is cold.

It hails.

['then it is true that']

It hails.

['then it is true that']

It hails. It hails.

['then it is true that']

It hails. ['then it is true that'] It hails.

It hails.

Inference Rule: Iteration…

…once asserted, can be asserted once more.

It hails.

[‘then it is true that’]

It hails.

It hails.

It hails.

It is cold.

It hails.

It is cold.

['Cut']

Convention: ‘Cut’ as negation…

It hails.

It is cold.

[‘Cut’]

Convention: 'Cut' as negation…

It hails.

It is cold.

[It hails
…and…
It is not cold.]

['Cut']

Convention: 'Cut' as negation…

… as separation from sheet.

It hails.

It is cold.

['Cut']

[It hails

…and…

It is not cold.]

It hails.

It is cold.

It hails.

It is cold.

It hails.

It is cold.

It hails.

It is cold.

It hails.

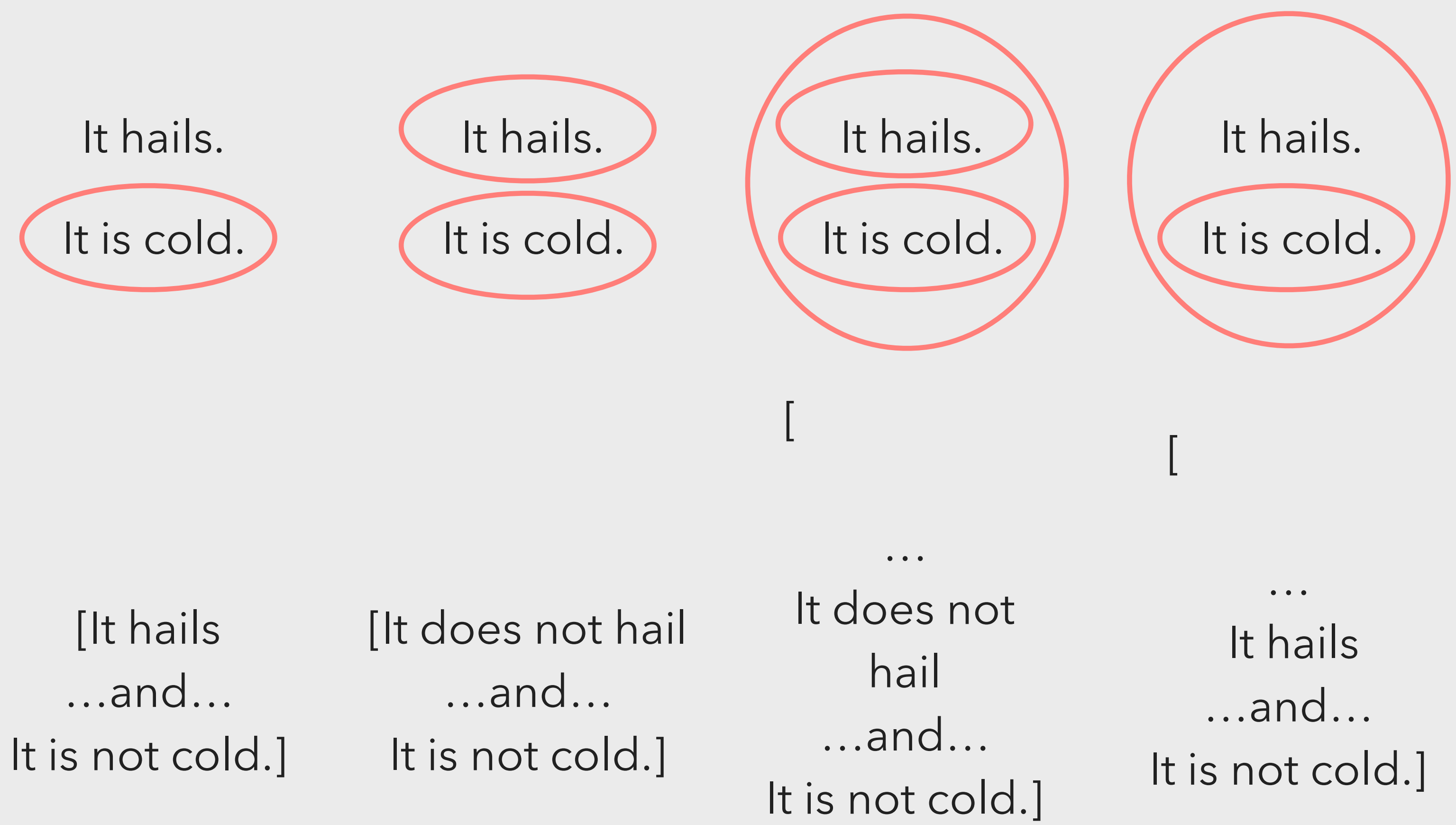
It is cold.

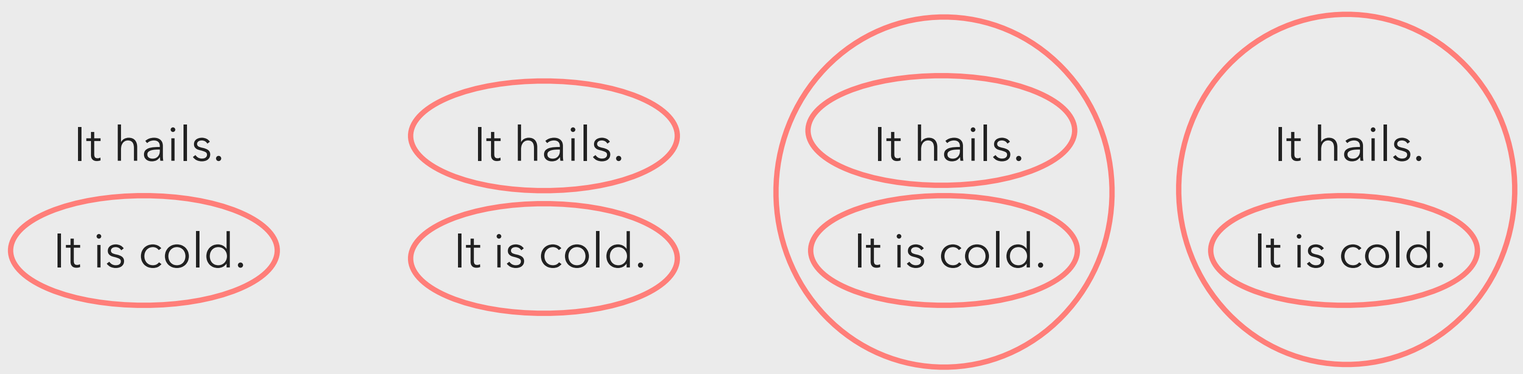
It hails.

It is cold.

It hails. It hails. It hails. It hails.

It is cold. It is cold. It is cold. It is cold.


It hails.
It is cold.
[It hails
...and...
It is not cold.]
It hails.
It is cold.
[It does not hail
...and...
It is not cold.]
It hails.
It is cold.
[
...
It does not
hail
...and...
It is not cold.]
It hails.
It is cold.
[
...
It hails
...and...
It is not cold.]



[It is not the case that…

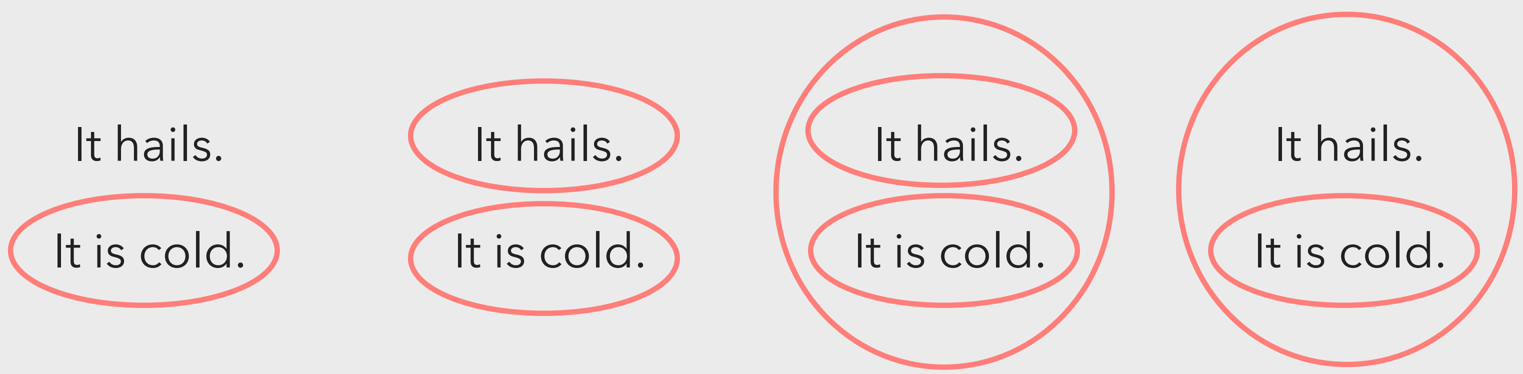
[It is not the case that…

[It hails …and… It is not cold.]

[It does not hail …and… It is not cold.]

It does not hail …and… It is not cold.]

It hails …and… It is not cold.]



[It hails
…and…
It is not cold.]

[It does not hail
…and…
It is not cold.]

[It hails
…or…
It is cold.]

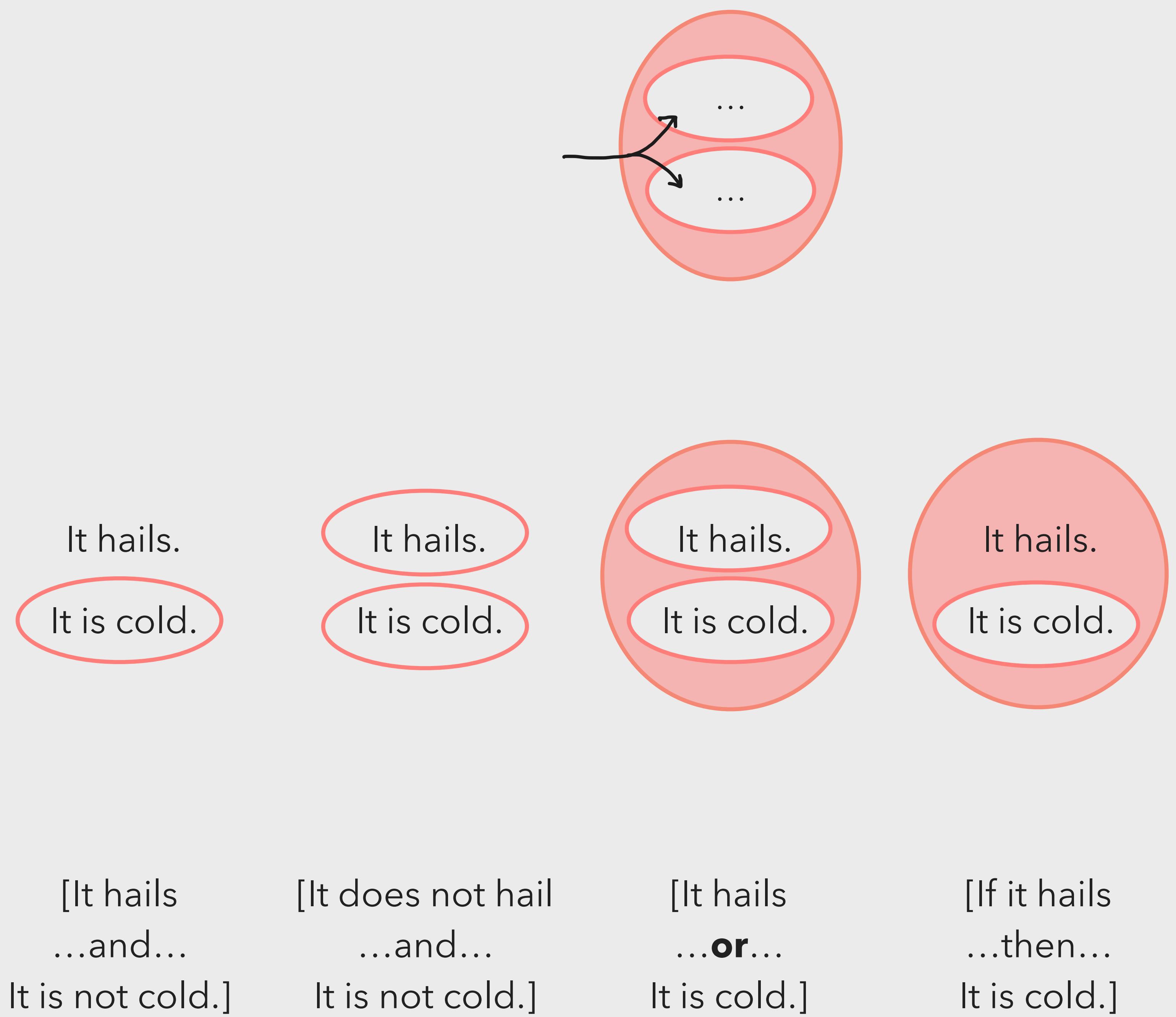
[If it hails
…then…
It is cold.]

[It hails
…and…
It is not cold.]

[It does not hail
…and…
It is not cold.]

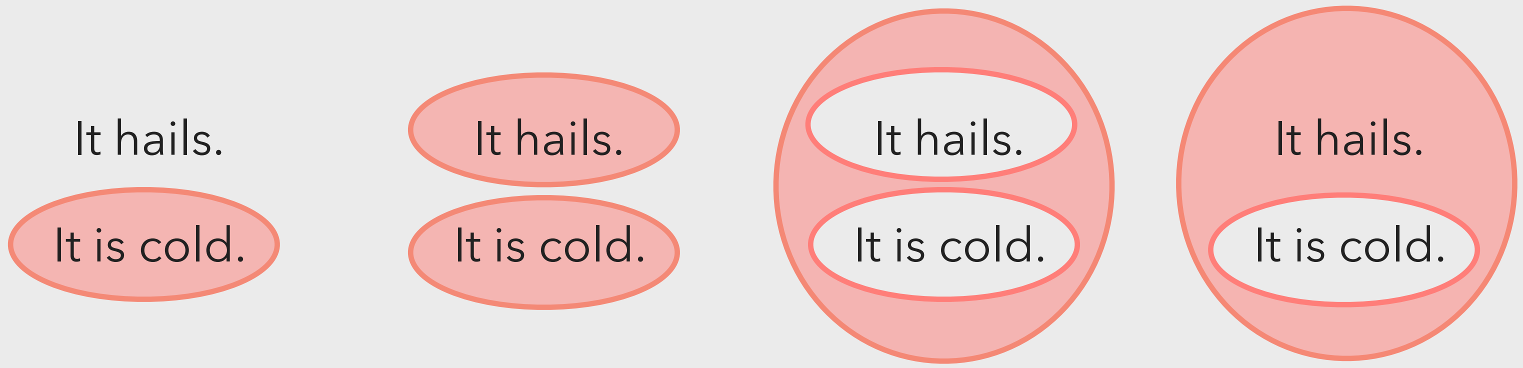
[It hails
…or…
It is cold.]

[**If** it hails
…**then**…
It is cold.]


...
...
It hails.
It is cold.
It hails.
It is cold.
It hails.
It is cold.
It hails.
It is cold.
[It hails
...and...
It is not cold.]
[It does not hail
...and...
It is not cold.]
[It hails
...**or**...
It is cold.]
[If it hails
...then...
It is cold.]

Takeaway: Nested ‘cuts’ capture logical connectives…

…one sign is sufficient..



[It hails
…and…
It is not cold.]

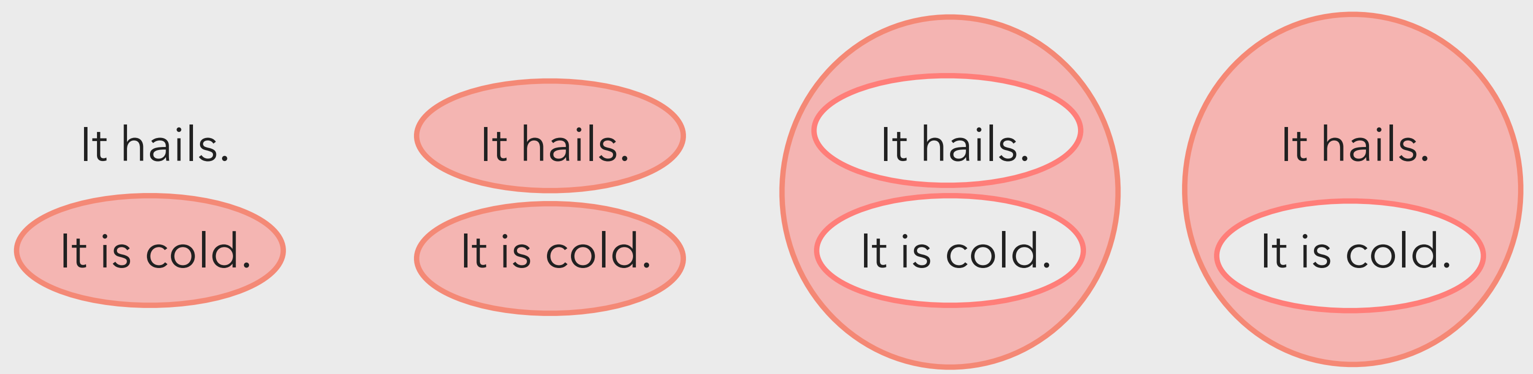
[It does not hail
…and…
It is not cold.]

[It hails
…or…
It is cold.]

[If it hails
…then…
It is cold.]

Takeaway: Nested ‘cuts’ capture logical connectives…

… … with reading rules allowing different interpretations to be read from the same graph.



[It hails …and… It is not cold.]

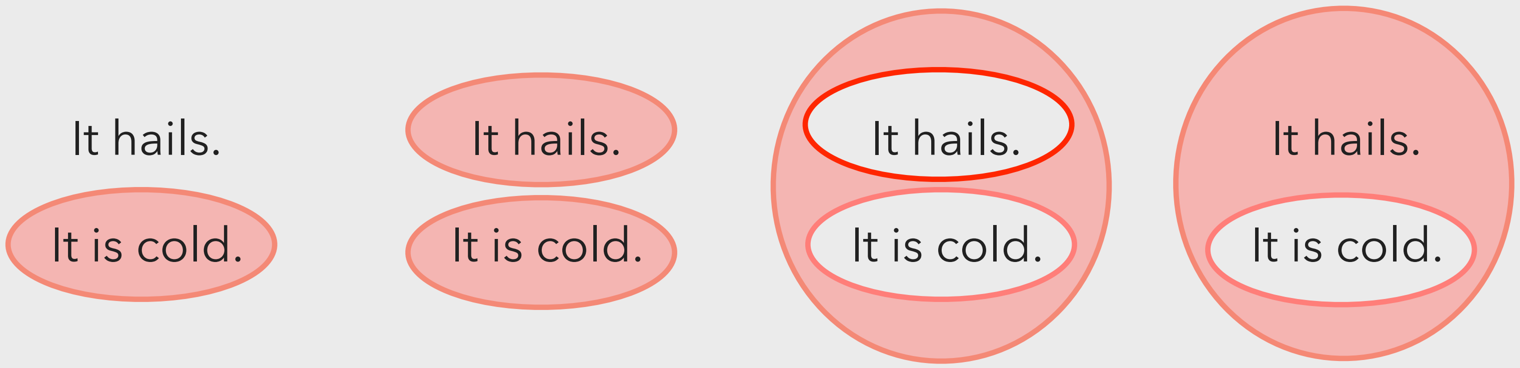
[It does not hail …and… It is not cold.]

[It hails …or… It is cold.]

[If it hails …then… It is cold.]

Takeaway: Nested 'cuts' capture logical connectives…

… … with reading rules allowing different interpretations to be read from the same graph.



[If it does not hail
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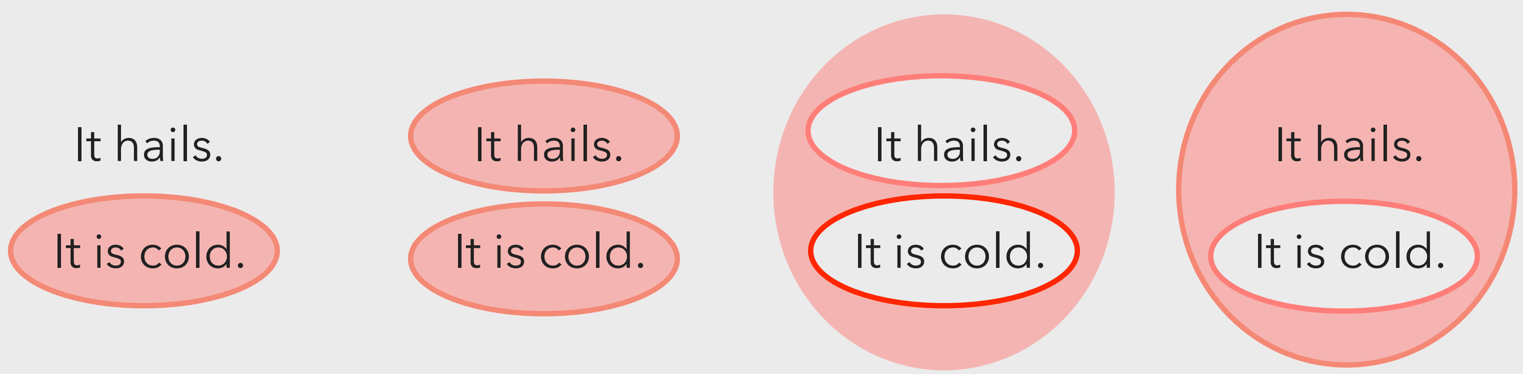
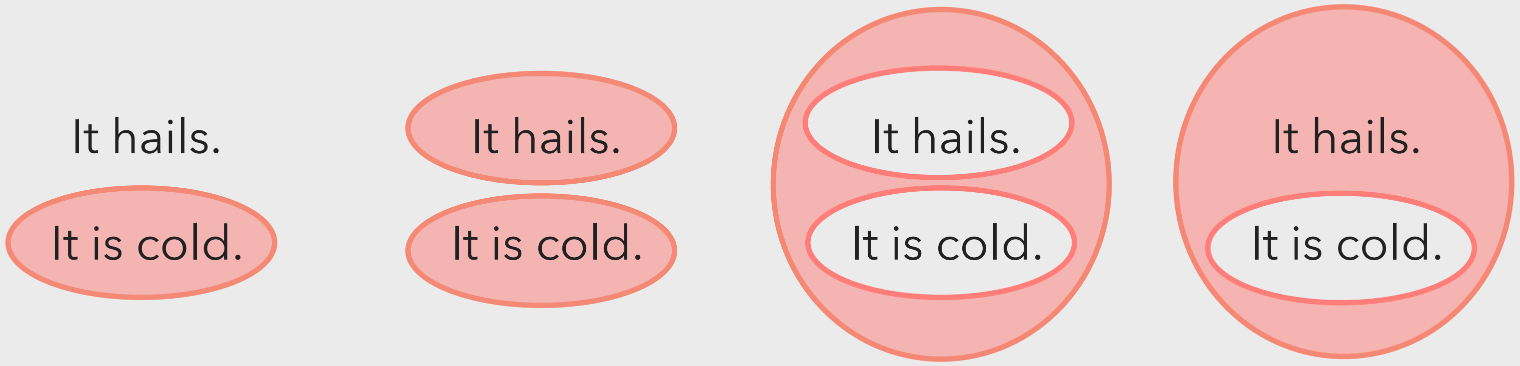


[If it is not cold
…then…
It hails.]


It hails.
It is cold.
It hails.
It is cold.
It hails.
It is cold.
It hails.
It is cold.

It hails.

It hails.

It hails.

It hails.

It hails.

Inference Rule: modus ponens…

It hails.

Inference Rule: modus ponens…

… deiteration

It hails.

Inference Rule: modus ponens…

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It hails.

Inference Rule: modus ponens…

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It hails.

Inference Rule: modus ponens…

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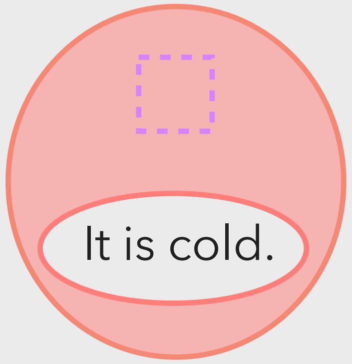
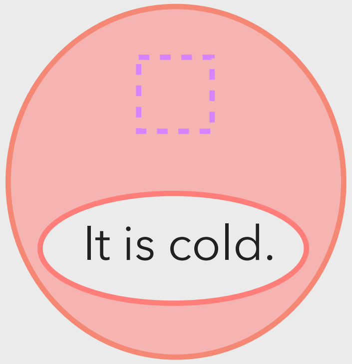
It hails.



[If SA
…then…
It is cold.]

Inference Rule: modus ponens…
… deiteration
… 'double-cut' elimination

It hails.



[If SA
…then…
It is cold.]

Inference Rule: modus ponens…

… deiteration

It hails.

Inference Rule: modus ponens…

… deiteration

… ‘double-cut’ elimination

It hails.

It is cold.

Inference Rule: modus ponens…

… deiteration

… 'double-cut' elimination

It hails.

It is cold.

Inference Rule: modus ponens…

… deiteration

… 'double-cut' elimination

… erasure

It hails.

It is cold.

Inference Rule: modus ponens…

… deiteration

… ‘double-cut’ elimination

… erasure

It is cold.

Inference Rule: modus ponens…

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… ‘double-cut’ elimination

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It is cold.

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Inference Rule: modus ponens…

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… 'double-cut' elimination

… erasure

It is cold.

Inference Rule: modus ponens…
… deiteration
… 'double-cut' elimination
… erasure

It hails.

['then it is true that']

It is cold.

Developing Beta
(predicate logic)

A leaf rustles over the ground.

A leaf rustles over the ground.

______ rustles over the ground
A leaf rustles over _________
______ rustles over _________
______ over ______

Convention: ‘–‘ as ‘something’…

A leaf rustles over the ground.

______ rustles over the ground
A leaf rustles over _________
______ rustles over _________
______ over ______

A leaf rustles over the ground.

______ rustles over the ground
A leaf rustles over _________
______ rustles over _________
______ over ______

God gives some good to every person.

God gives some good to every person.

____ gives ____ to ___
____ gives some good to ___
____ gives ____ to every person
God gives ____ to ___
God gives some good to ___
God gives ____ to every person
____ gives some good to every person

_____ gives _____ to ____

_____ gives _____ to ____

_____ gives  to _____

[Think of valency in chemistry:
H-O-H]

_____ gives _____ to _____

_____ gives  to _____

_____ gives to ___

_______ rustles over the ground

_____ gives to ____

_______ rustles over the ground

Convention: ‘–‘ as ‘something’…

____ gives ____ to ____

________ rustles over the ground

Convention: '—' as 'something'…

… branch as 'same as'.

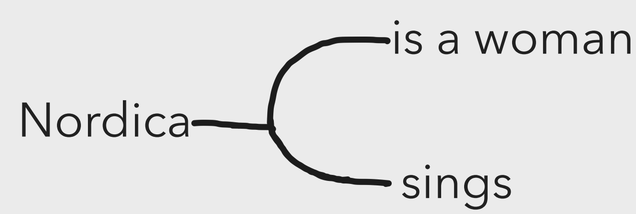
____ gives ____ to ____

________ rustles over the ground

Convention: '–' as 'something'…

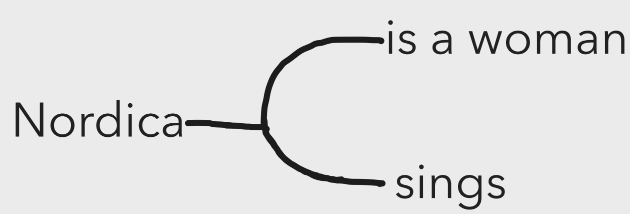
… branch as 'same as'.

['lines of identity']


Nordica
is a woman
sings

Inference Rule: Erasure…

…line can be broken.



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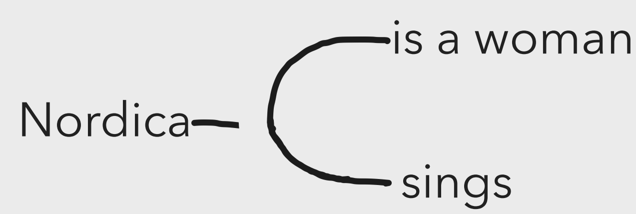

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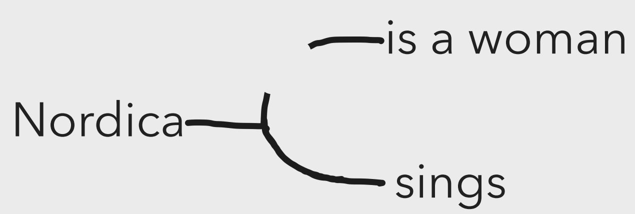

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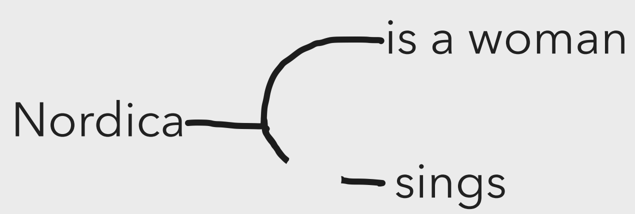

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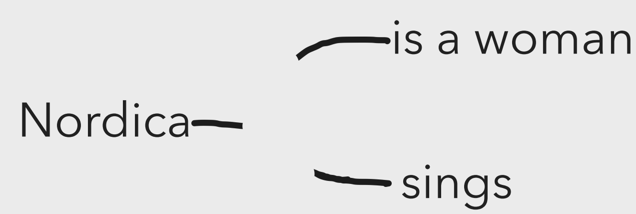


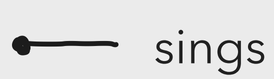
—— sings ○—— sings ●—— sings

[P.H.]

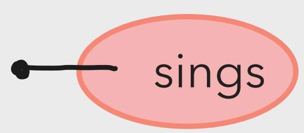
sings

sings sings sings sings

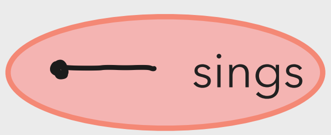

sings
sings
sings
sings



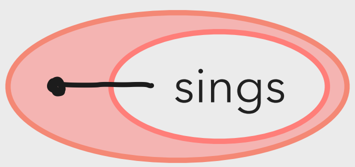
['Somebody sings']



['Somebody does not sing']



['Nobody sings']



['Everybody sings']

[Standard EGs]

Syntax	Conventions	Inference Rules
Asserted Relations (P , Q , R…)	‘sheet of assertion’	Erasure/Insertion
‘Cut’	Juxtaposition as ‘conjunction’	Iteration/Deiteration
‘Line of Identity'	‘Cut’ as negation	Add/Remove ‘Double-Cut’
	‘Line of Identity’ as something exists	(Principle of Contraposition)

[Standard EGs]

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See Roberts’ ‘The Existential Graphs of C.S. Peirce’ (1973).

Outline

- introduce Peirce’s Existential Graphs (à la regular logic and cartesian bicategories)
- move to the Neo-Peircean Calculus of Relations (à la residuation and cyclic bilinear logic)
- demonstrate topological advantages

String diagrams for regular logic….

Cartesian Bicategories of Relations
(Carboni and Walters, 1987)

Graphical Conjunctive Queries
(Bonchi, Seeber, Sobocinski, 2018)

and

Regular Logic
$\exists, \wedge, \top$

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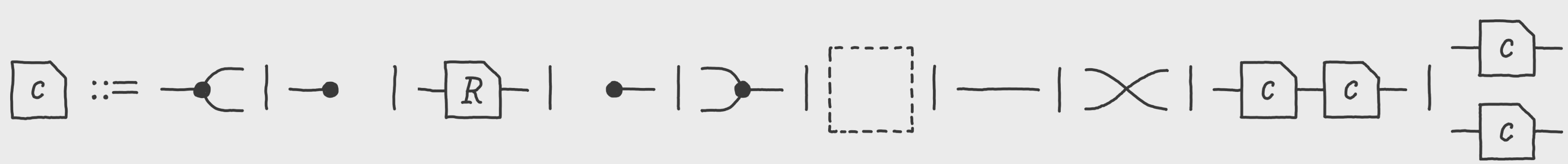
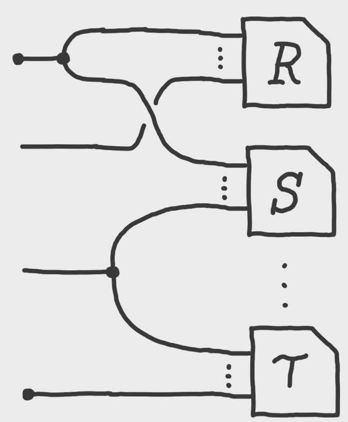
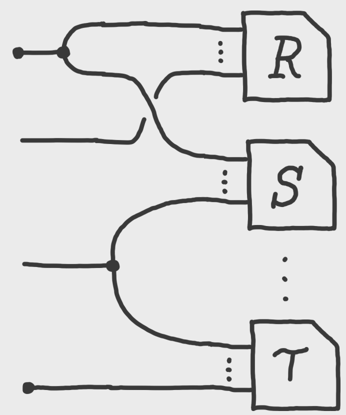


'One-sided' Presentation:



‘One-sided’ Presentation -> ‘Two-sided’ Presentation:



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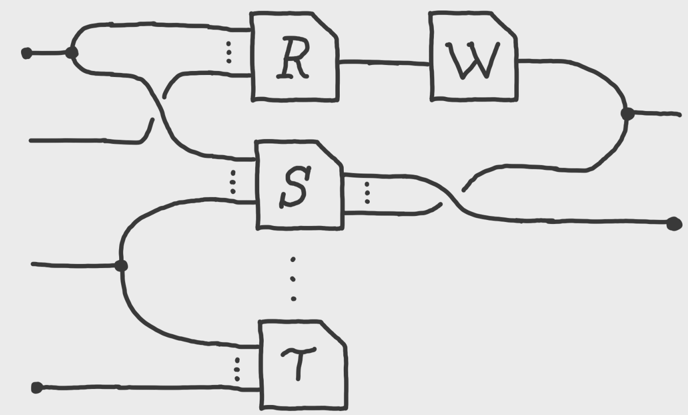
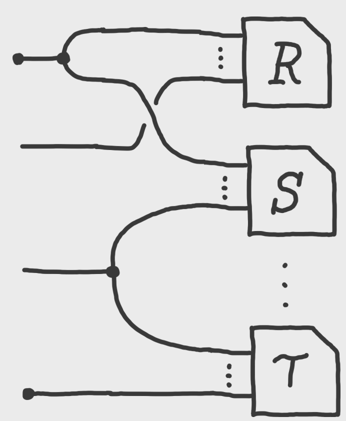


‘One-sided’ Presentation -> ‘Two-sided’ Presentation:



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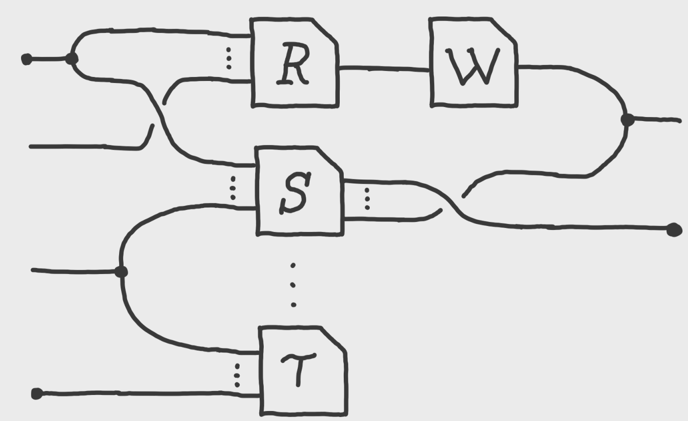
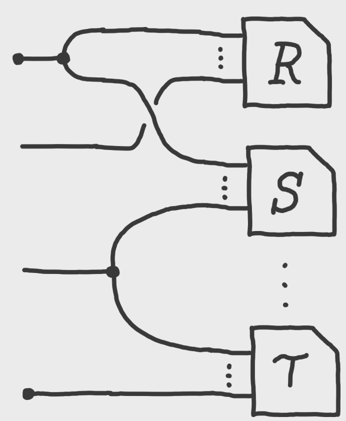


This is sufficient for regular logic, i.e. the fragment of first-order logic with ∃, ∧ , ⊤ .

Tarski’s Relation Algebra

(really following Boole, De Morgan, Schröder, and Peirce…)

‘One-sided’ Presentation -> ‘Two-sided’ Presentation:



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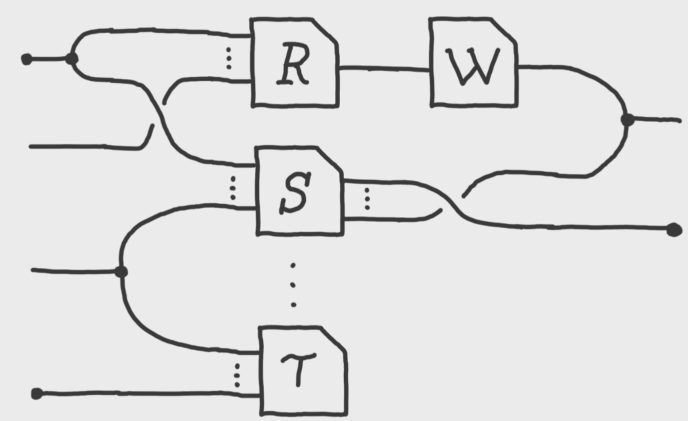
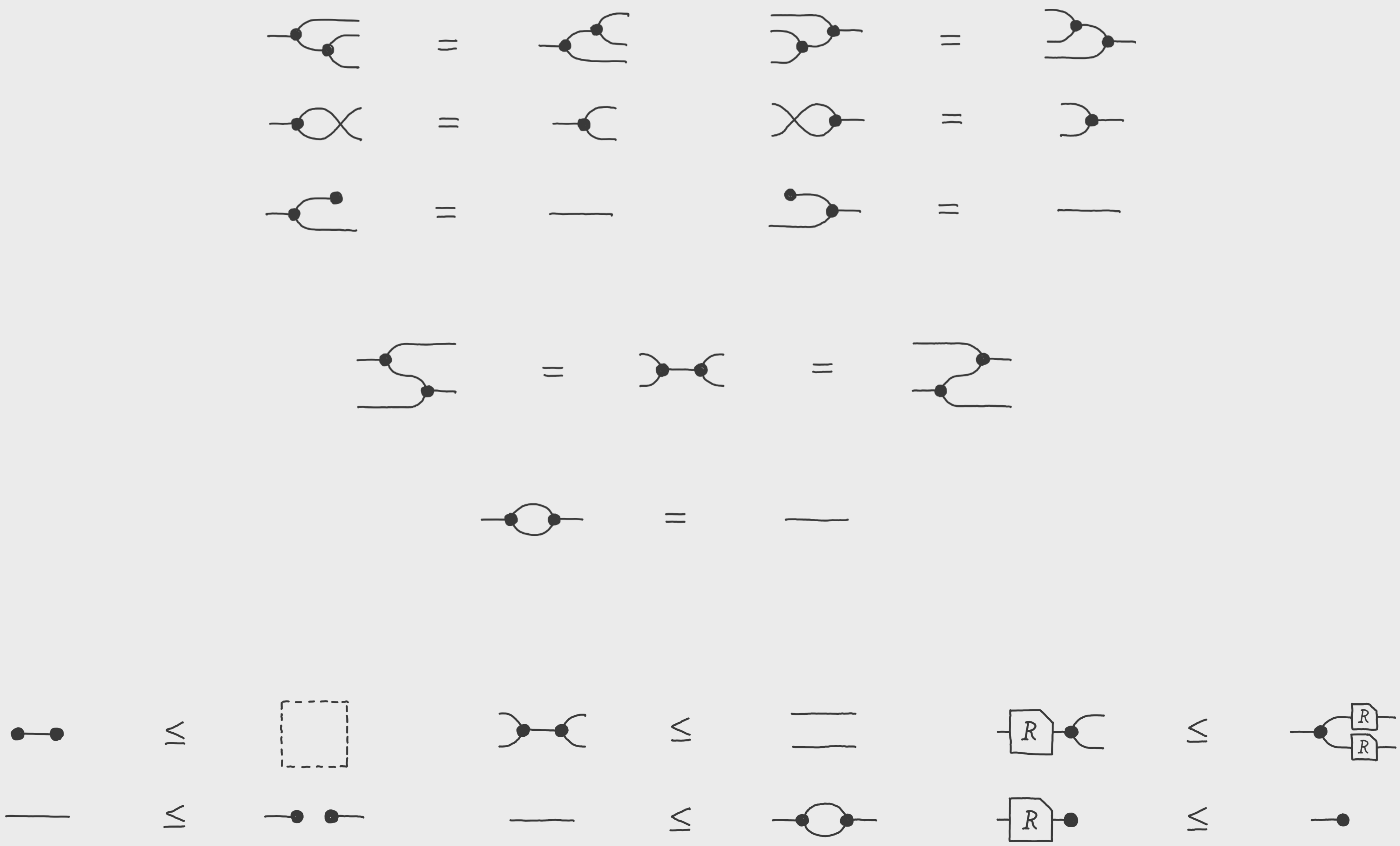


This is sufficient for regular logic, i.e. the fragment of first-order logic with $\exists$, $\wedge$, $\top$.

What are the graphical rewrites, i.e. the inference rules?

String Diagrams for Cartesian Bicategories



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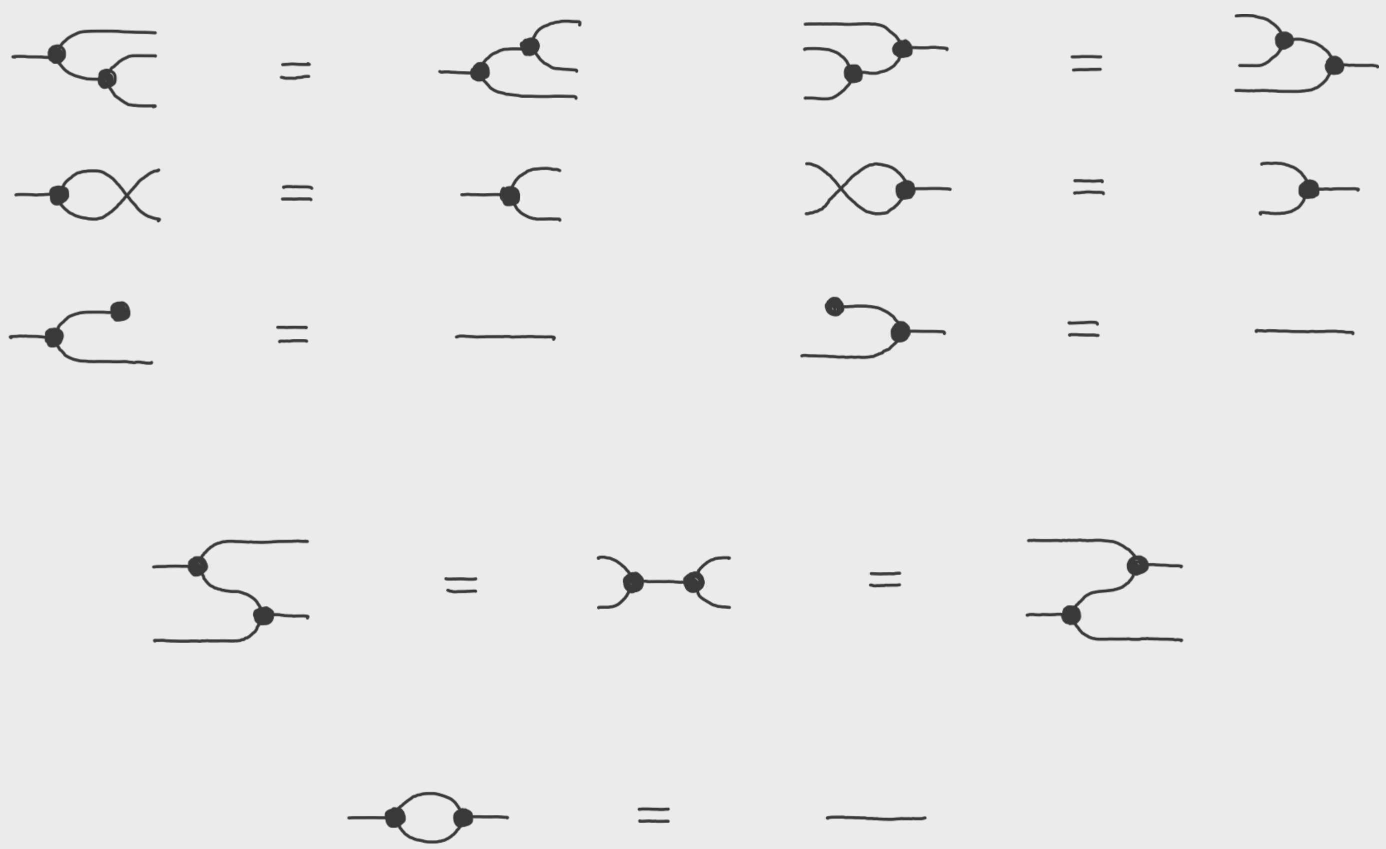
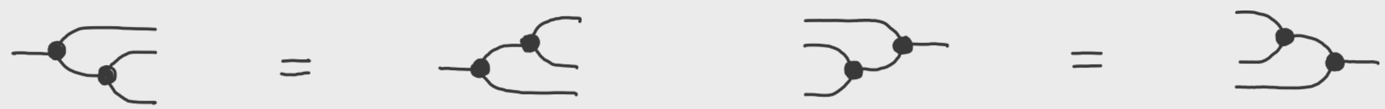
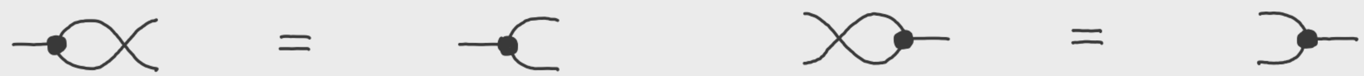


String Diagrams for Cartesian Bicategories

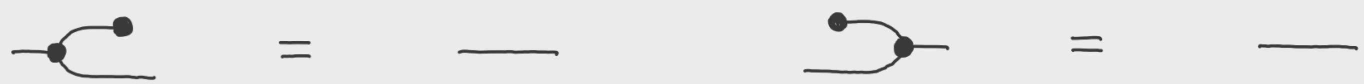
Associativity

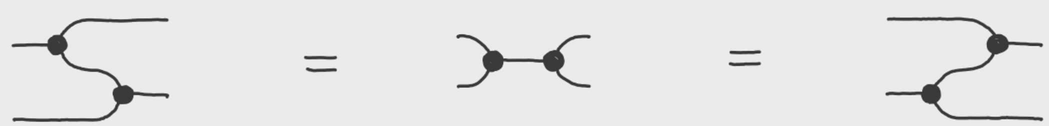


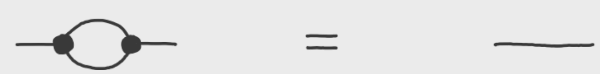
Commutativity



Unit







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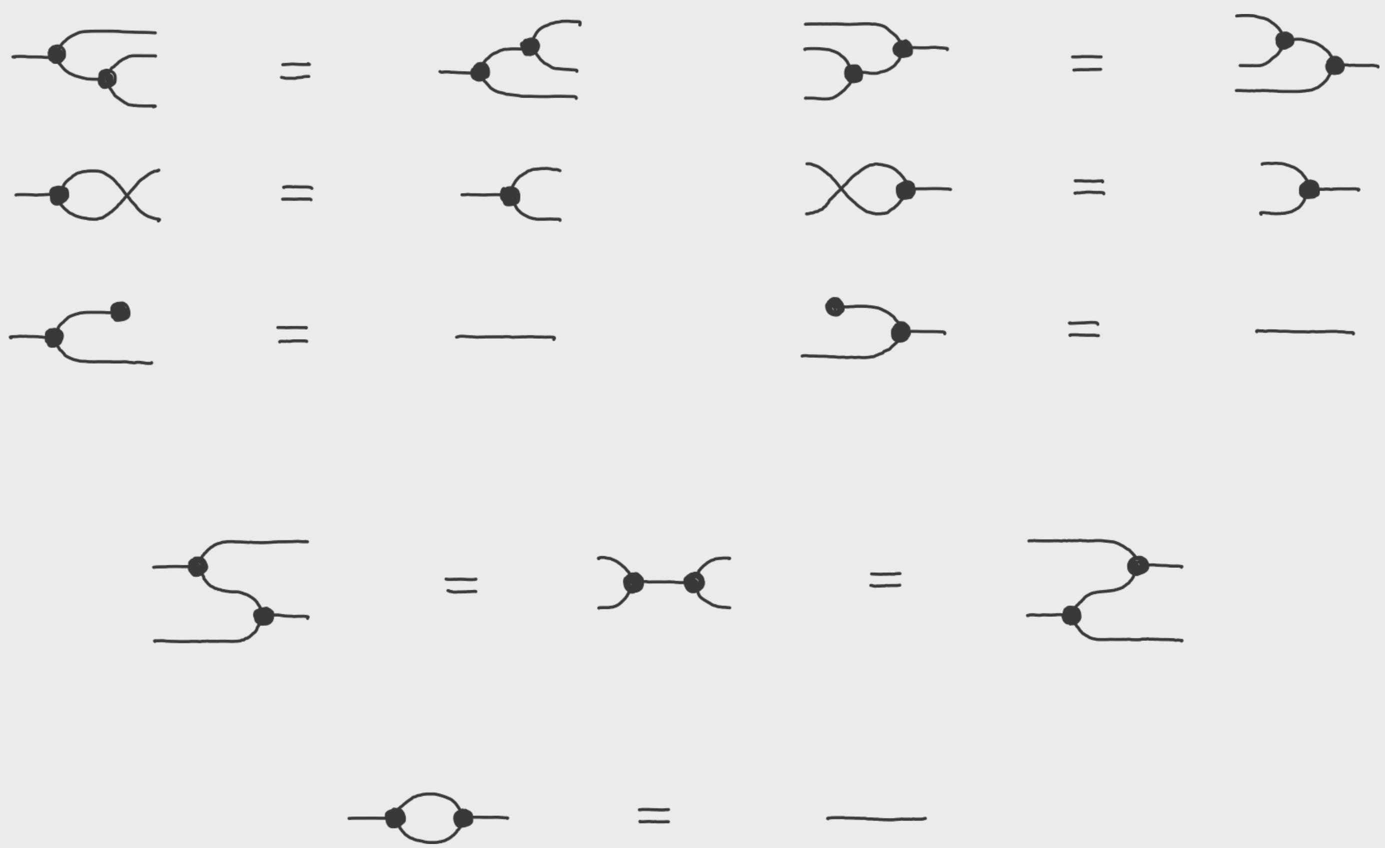
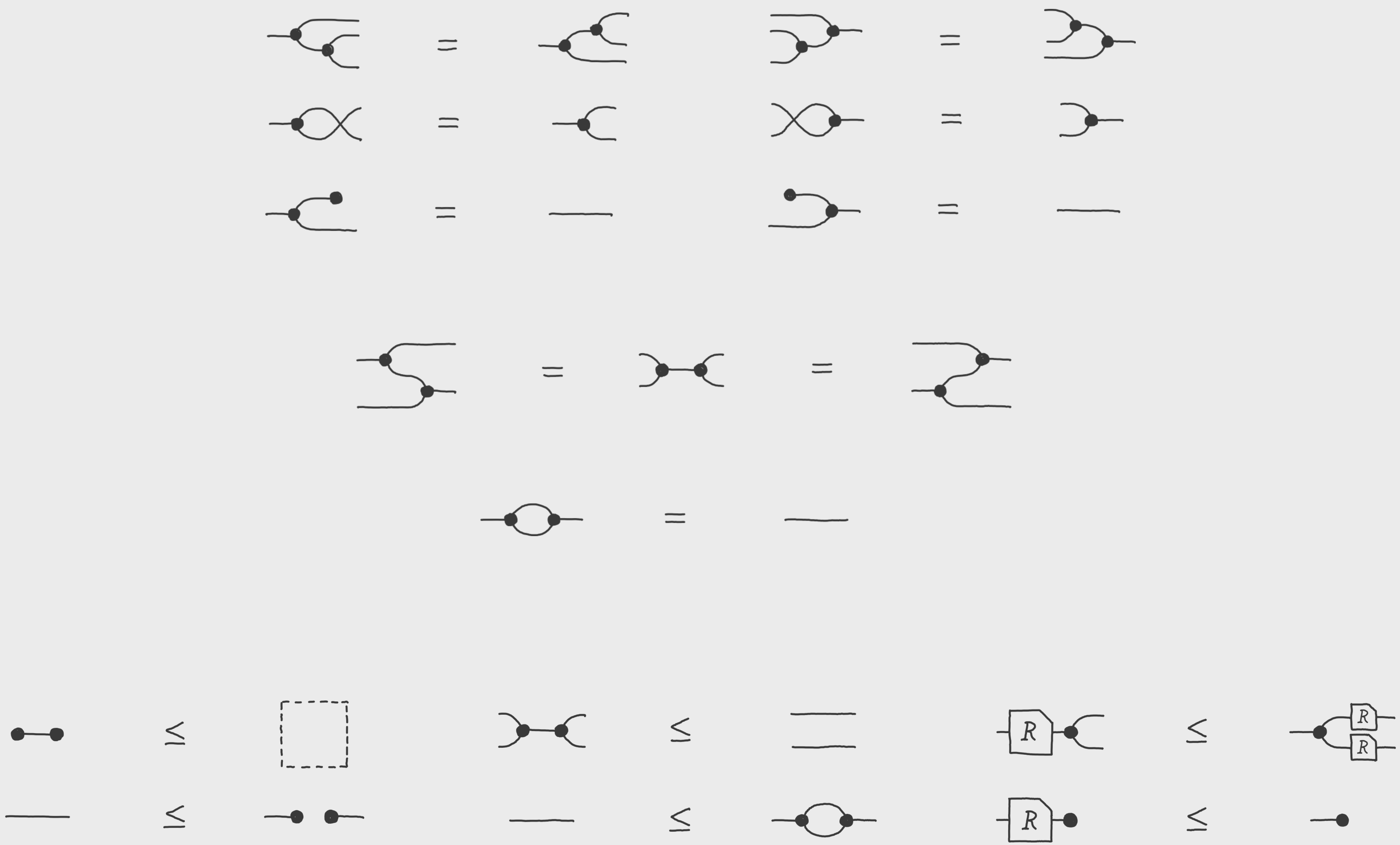


"only connectivity matters…"

String Diagrams for Cartesian Bicategories



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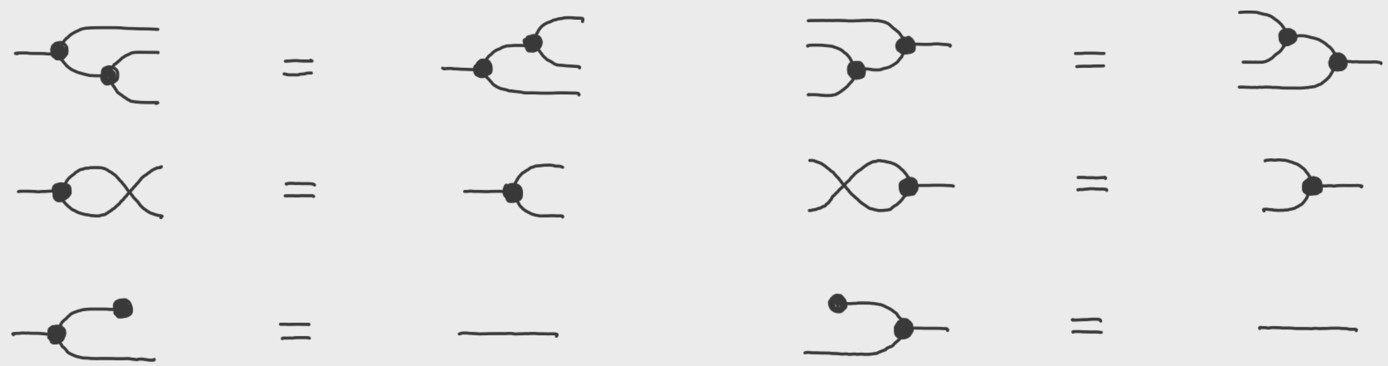
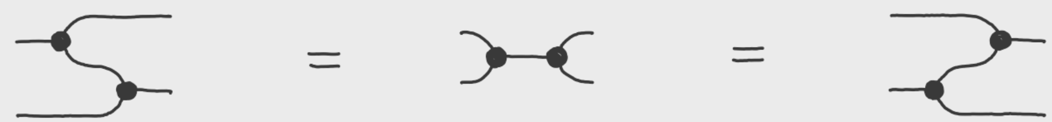
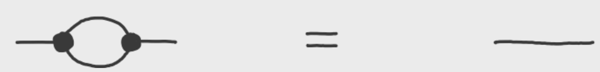


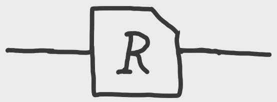




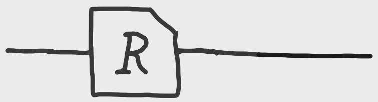
Primitive inference rules…

String Diagrams for Cartesian Bicategories

String Diagrams for Cartesian Bicategories


R

String Diagrams for Cartesian Bicategories



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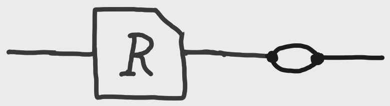

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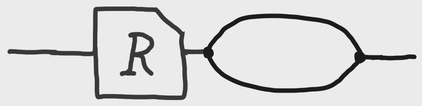

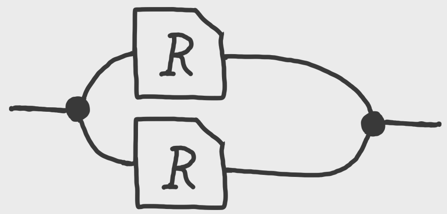
R
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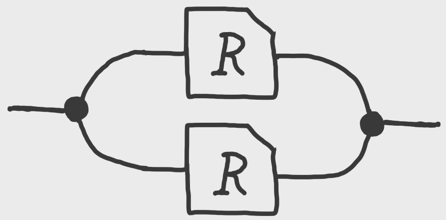

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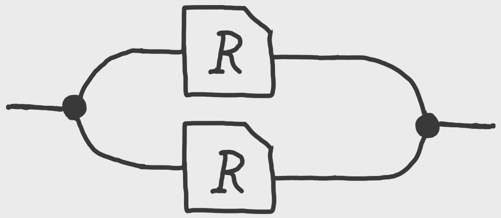

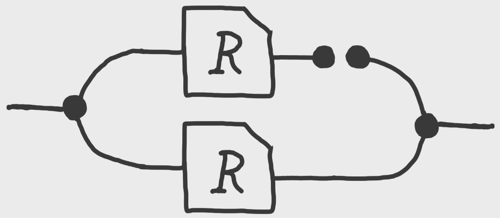
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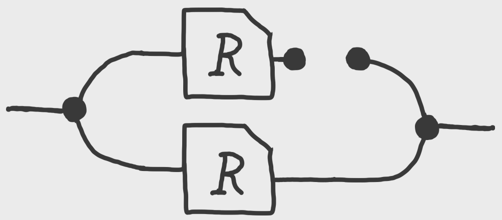
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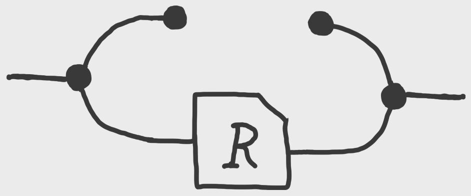
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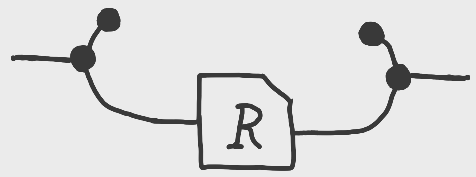
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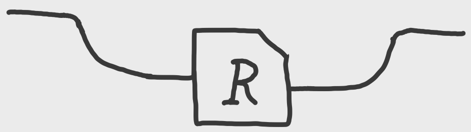
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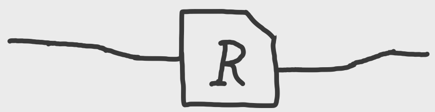
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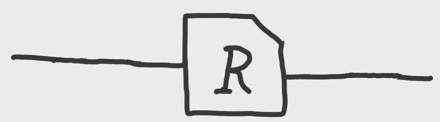
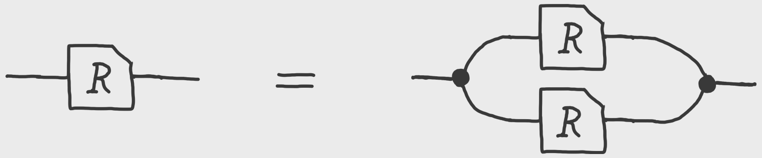
String Diagrams for Cartesian Bicategories



String Diagrams for Cartesian Bicategories



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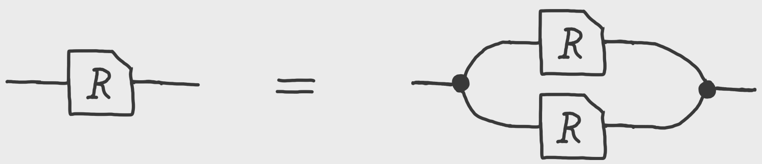

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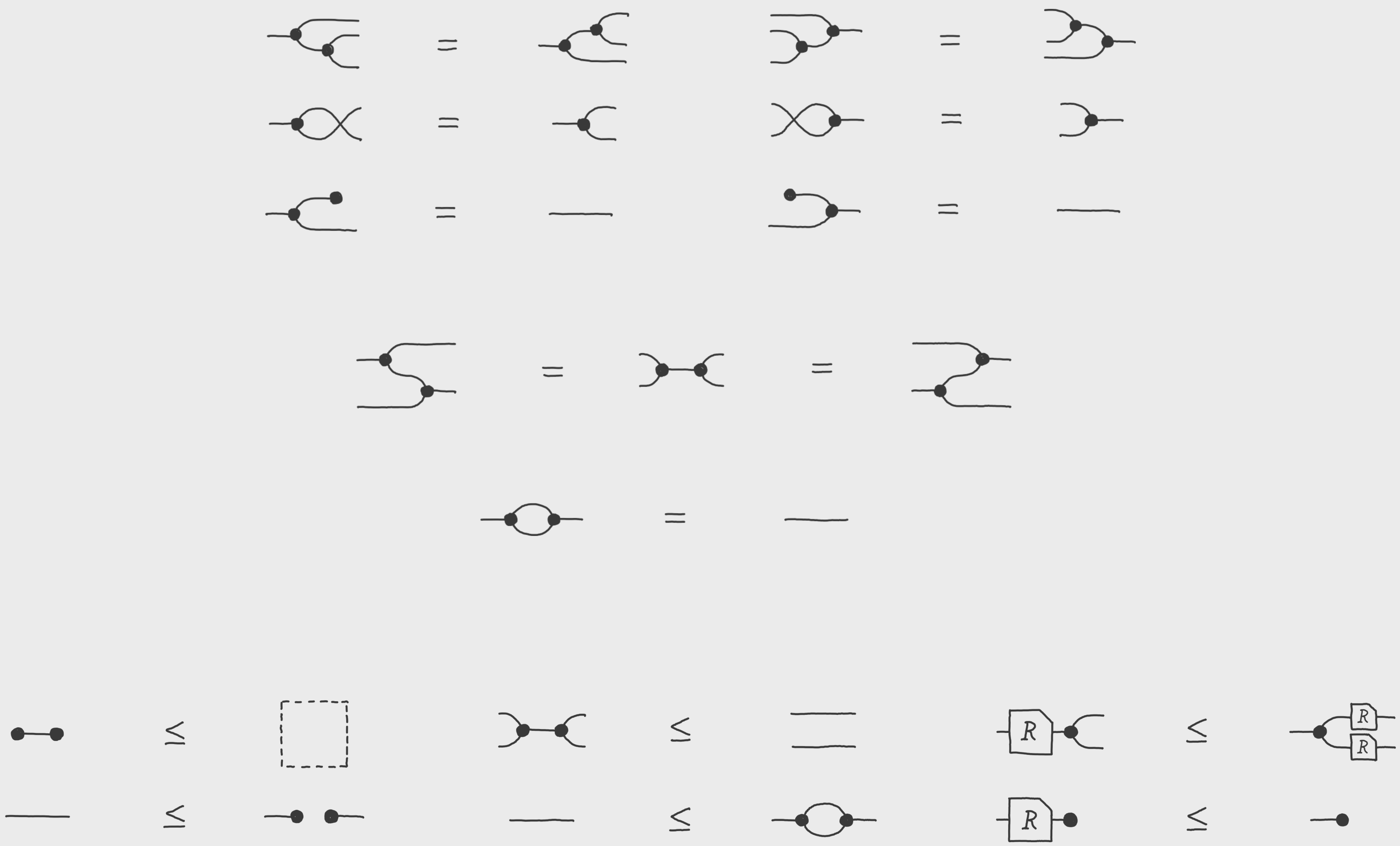

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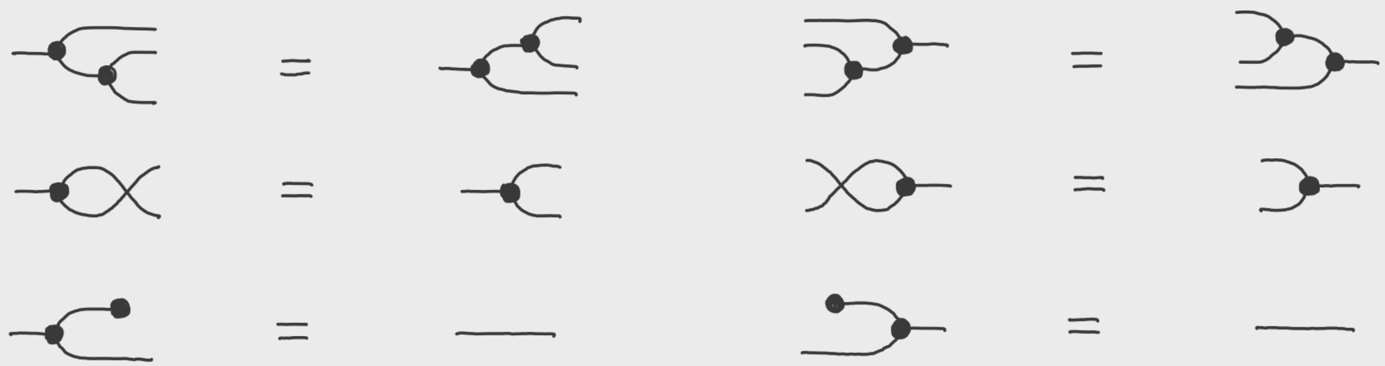
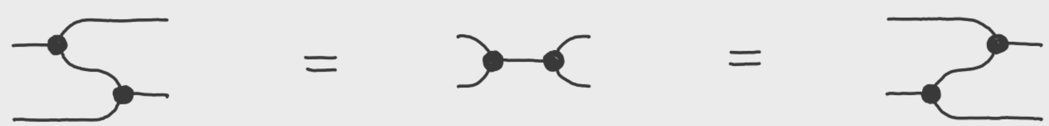
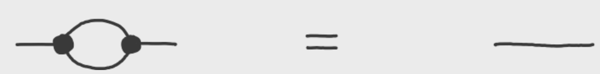
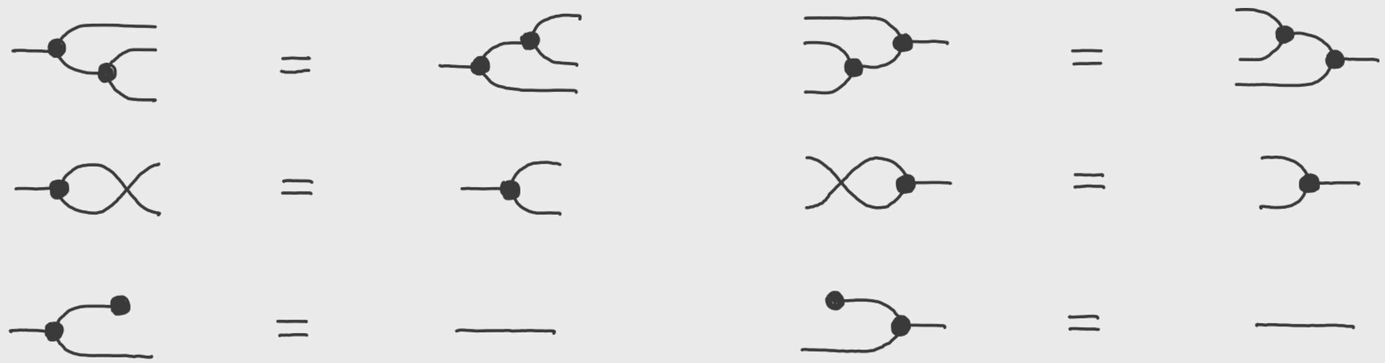


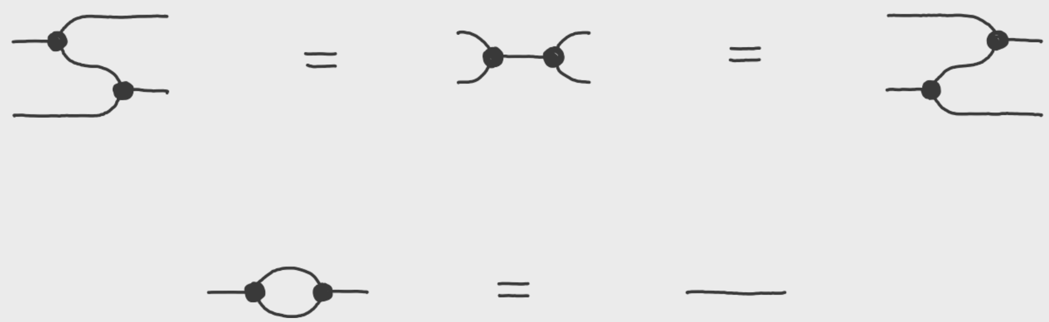
Peirce's
'lines of identity'





String Diagrams for Cartesian Bicategories

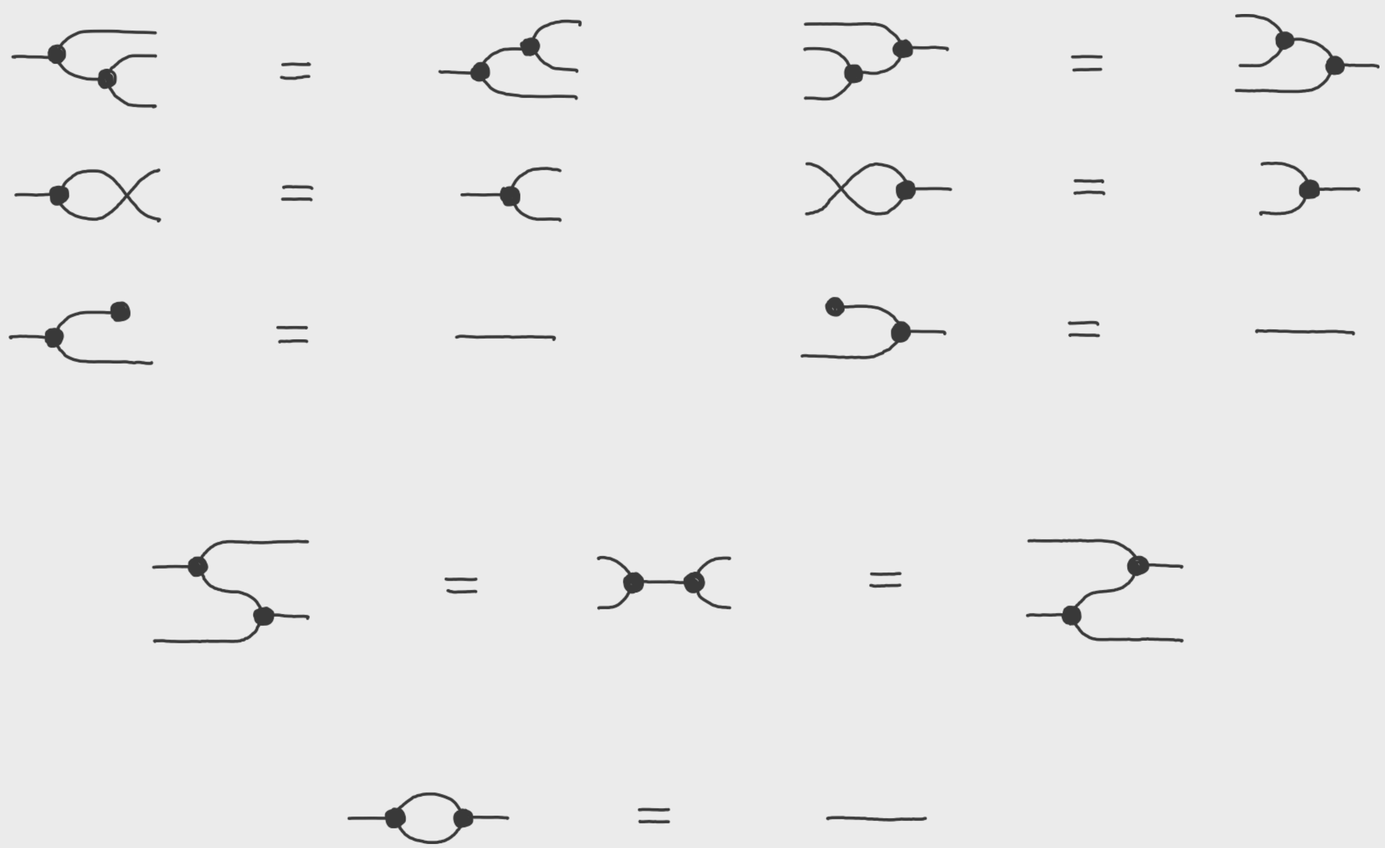




'Teridentity'

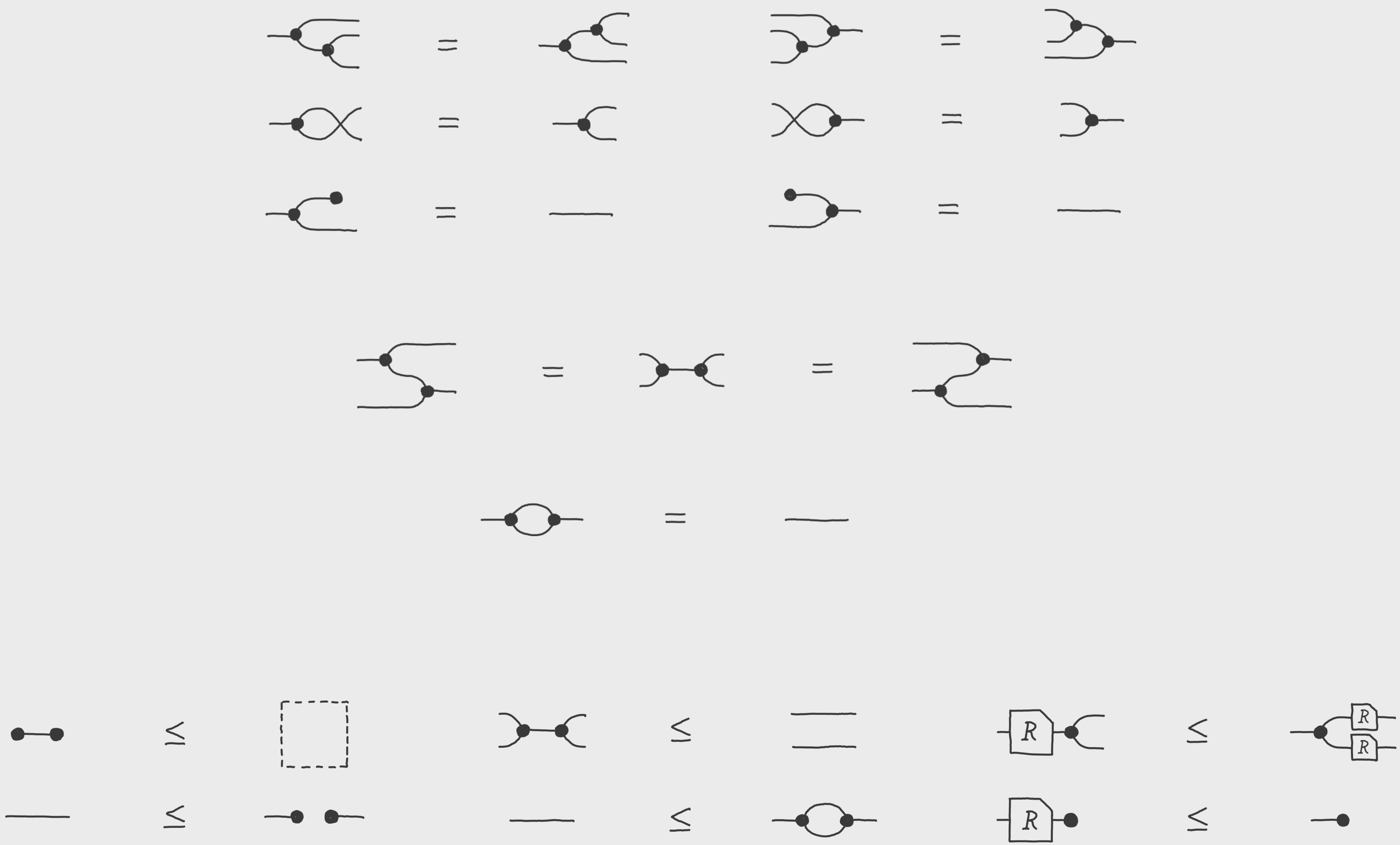
'Tri-identity'

String Diagrams for Cartesian Bicategories



Graphical rules for (positive fragment of) Existential Graphs

String Diagrams for Cartesian Bicategories



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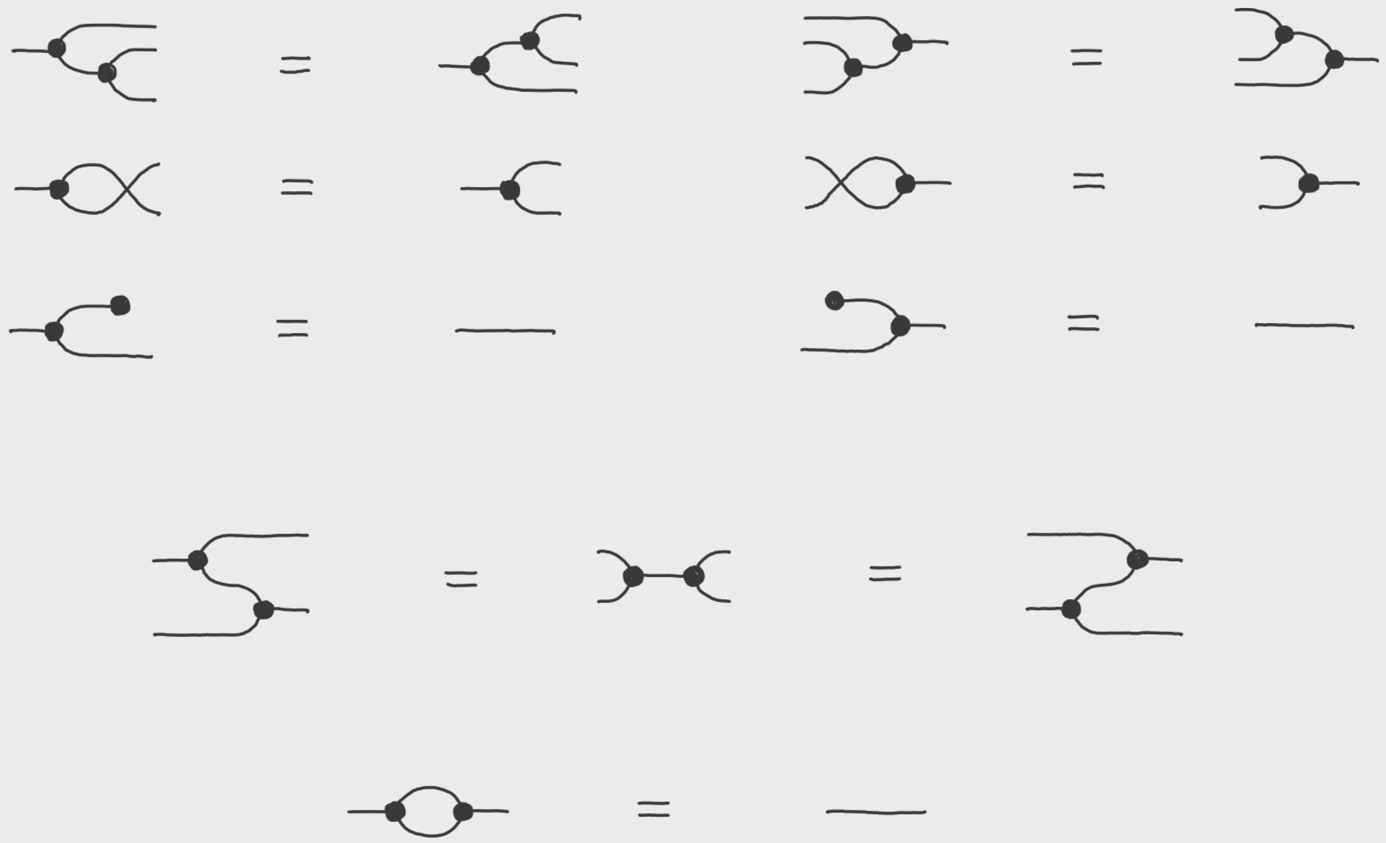

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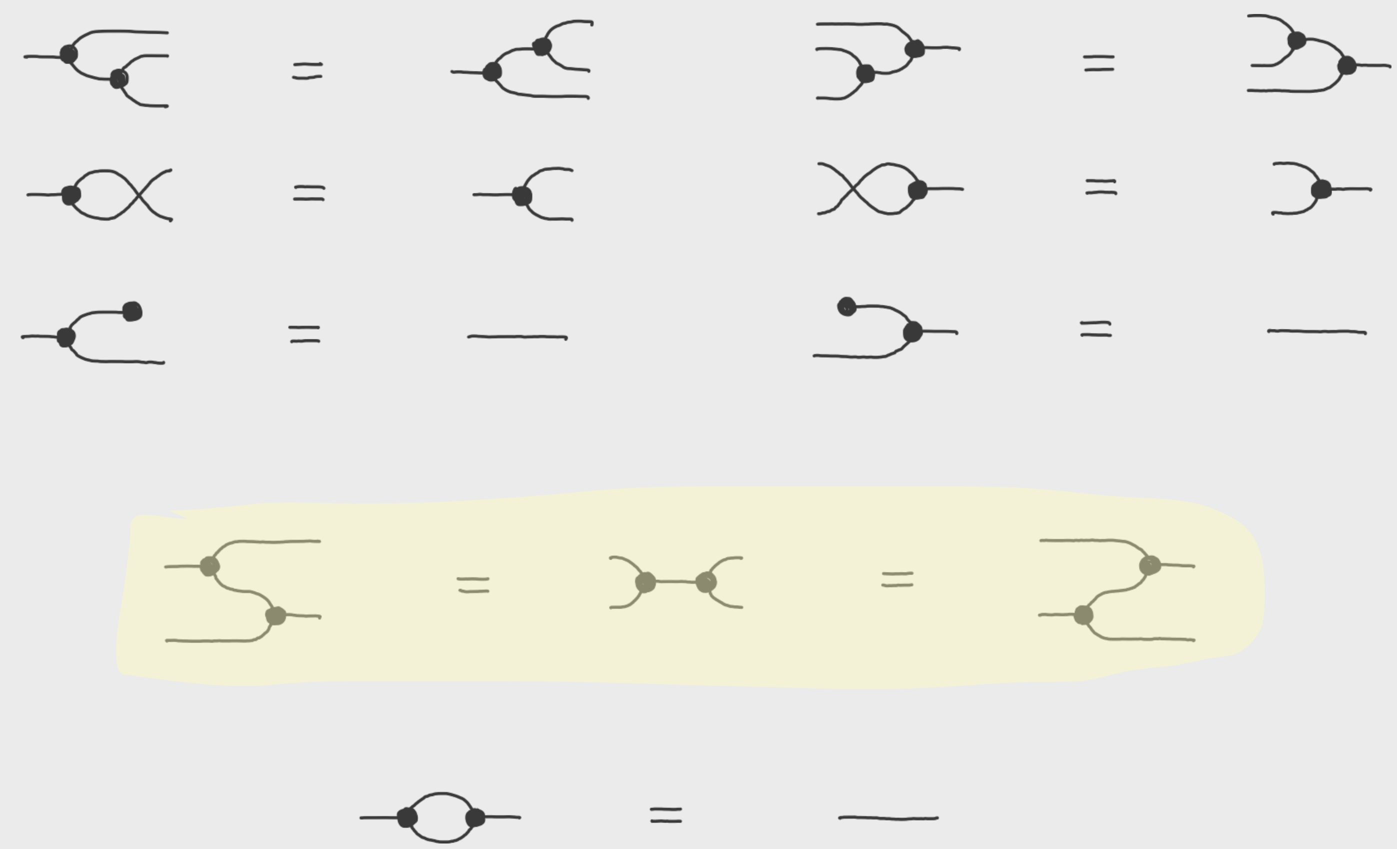
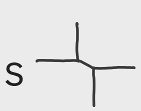
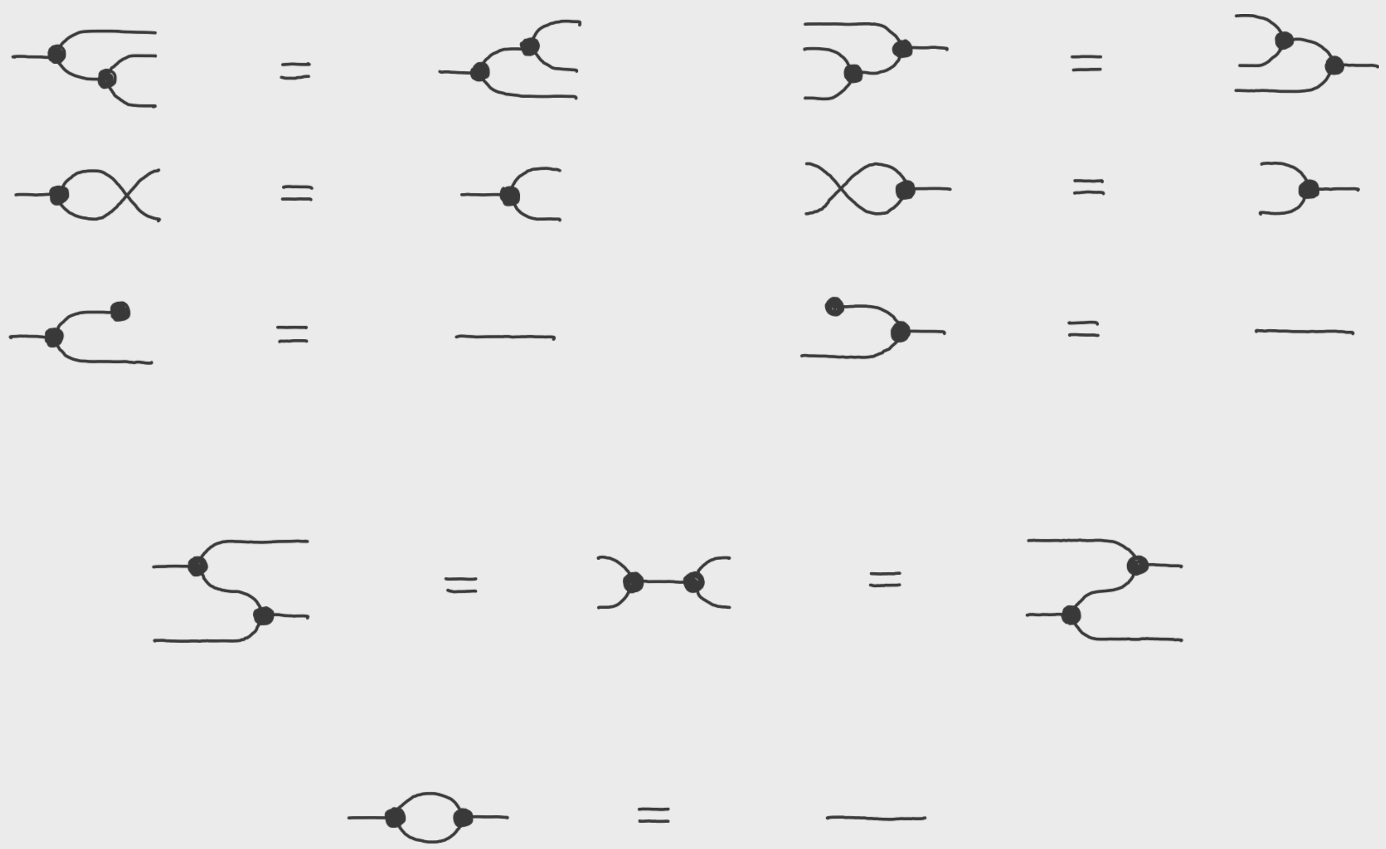


“Quateridentity is obviously composed of two teridentities; i.e. This  is  or  or .”

String Diagrams for Cartesian Bicategories



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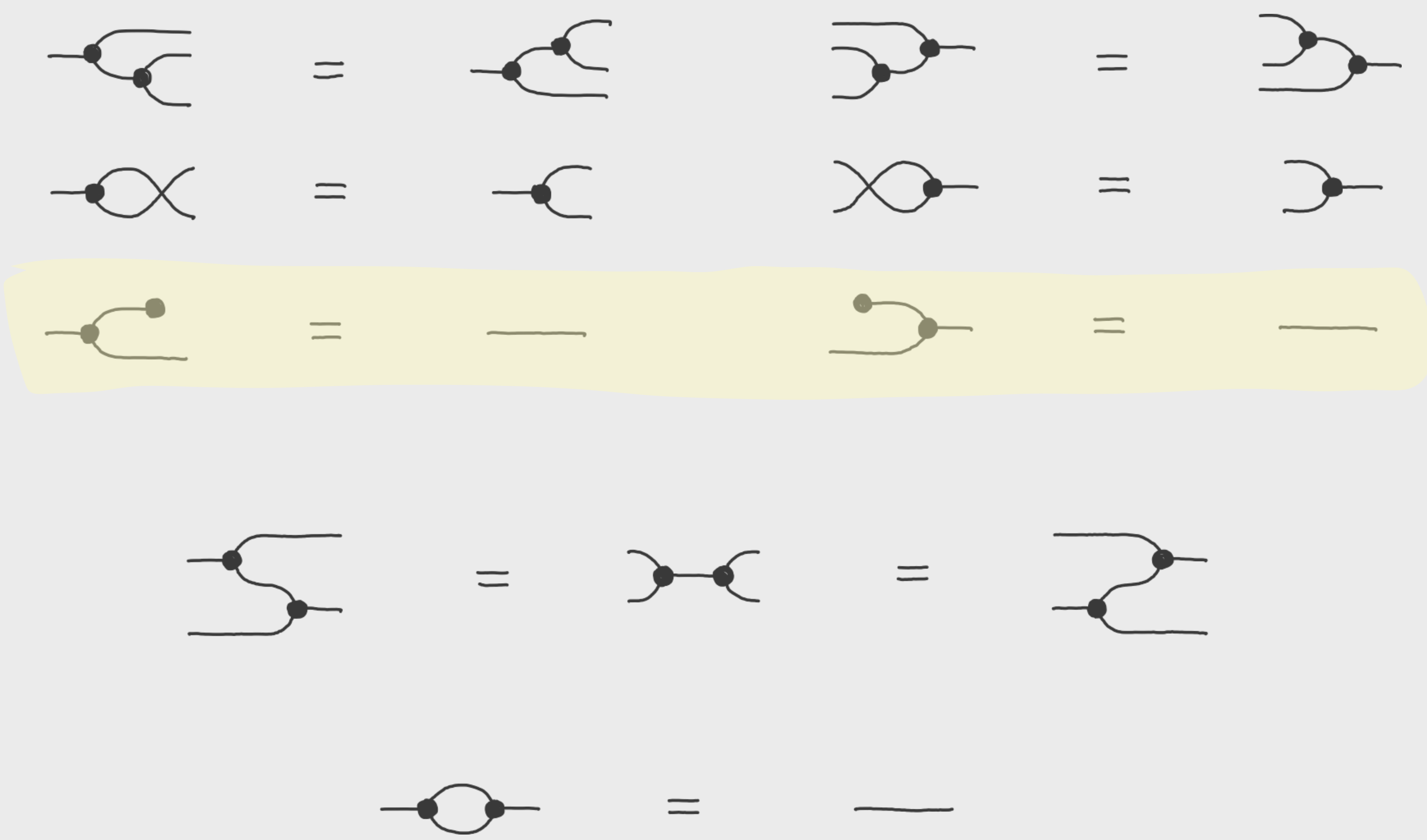
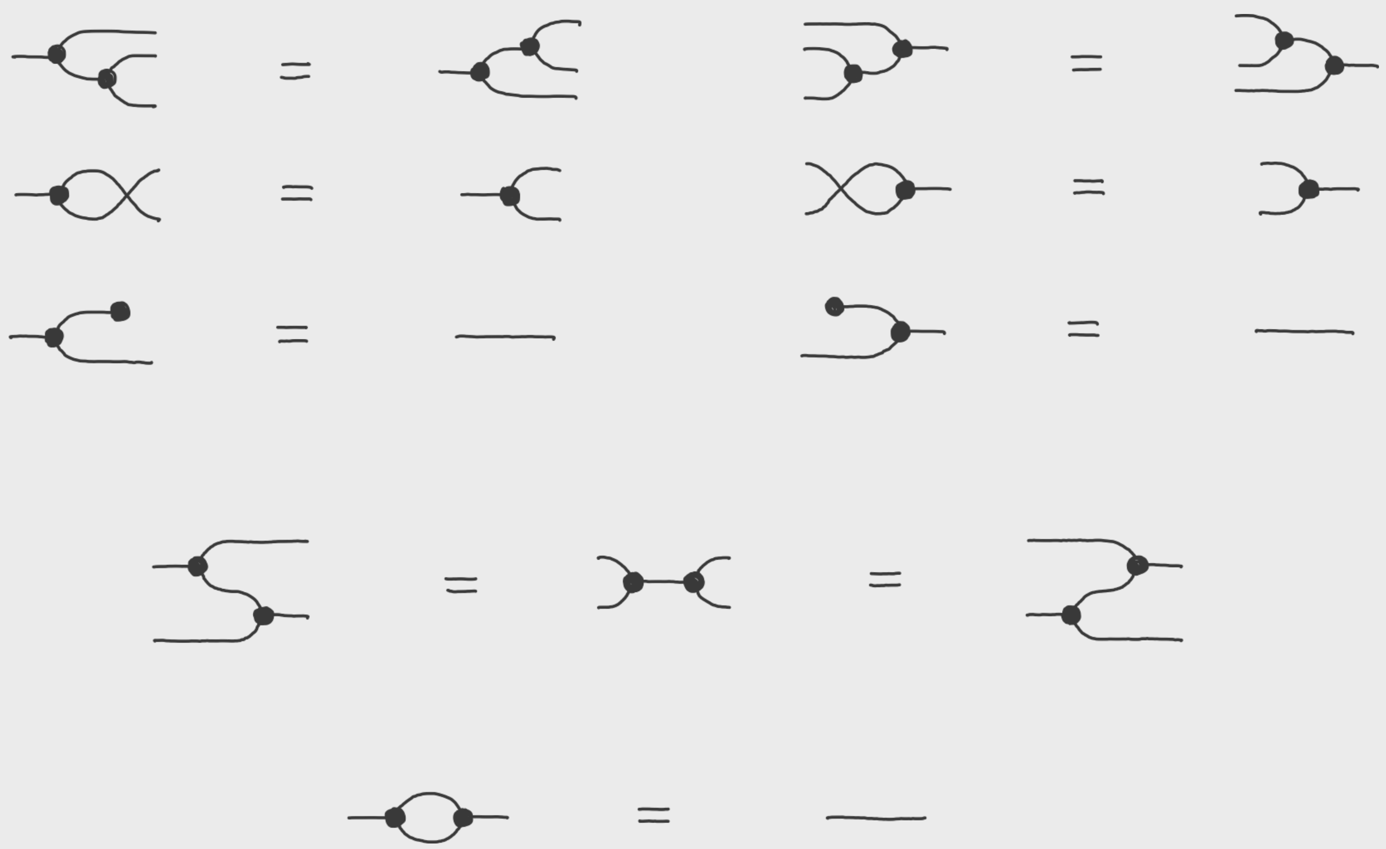


“the line of identity…must be understood quite differently. We must hereafter understand it to be potentially the graph of teridentity by which means there will virtually be at least one loose end in every graph”

String Diagrams for Cartesian Bicategories



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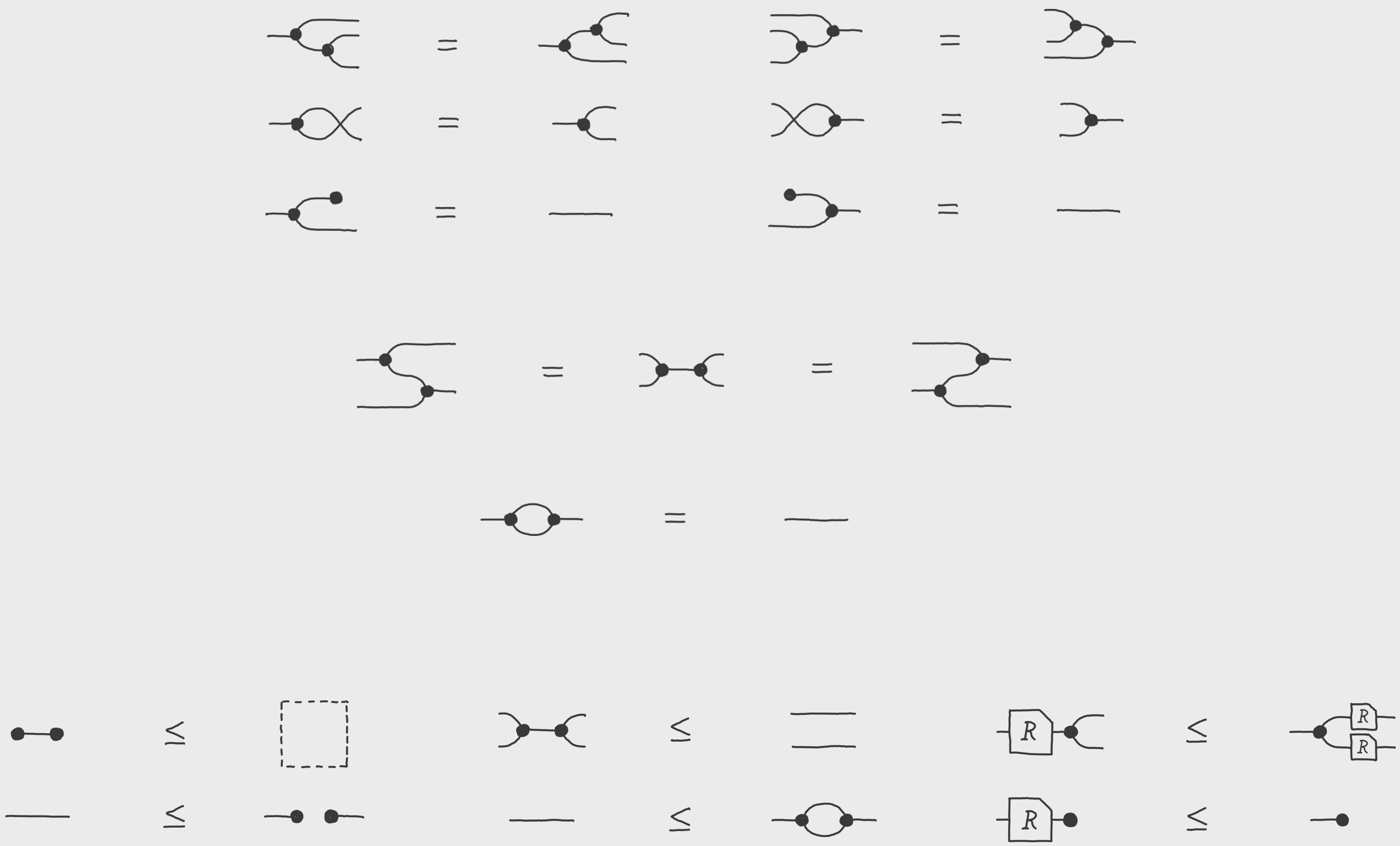

String Diagrams for Cartesian Bicategories

String Diagrams for Cartesian Bicategories

“*Rule of Erasure: A*ny partial or total graph can be erased.

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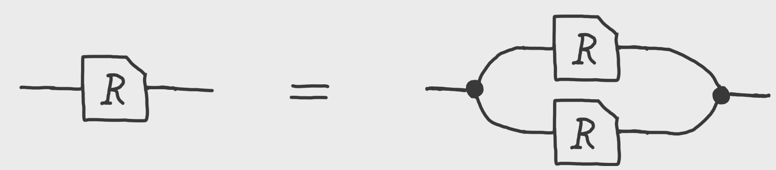
String Diagrams for Cartesian Bicategories

"*Rule of Iteration:* Any partial or total graph may be iterated within the same [area],

String Diagrams for Cartesian Bicategories

“*Rule of Iteration:* Any partial or total graph may be iterated within the same [area], the new replica having *junctures* connecting each hook with the corresponding hook of the original graph.



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The (positive fragment) of Peirce’s Existential Graphs corresponds to…

… the string diagrammatic presentation of cartesian bicategories of relations (CBR)…

…How to most directly extend regular logic and CBR to first-order logic?

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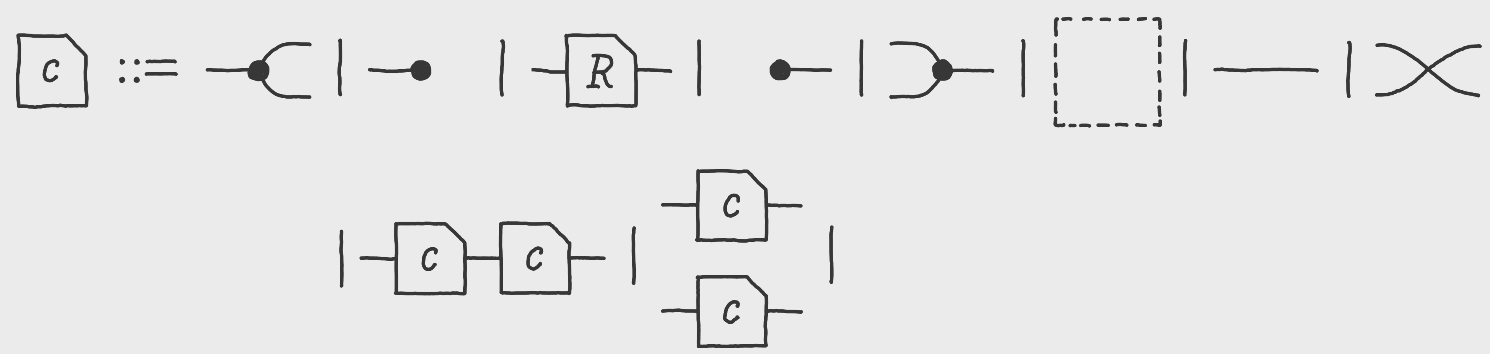

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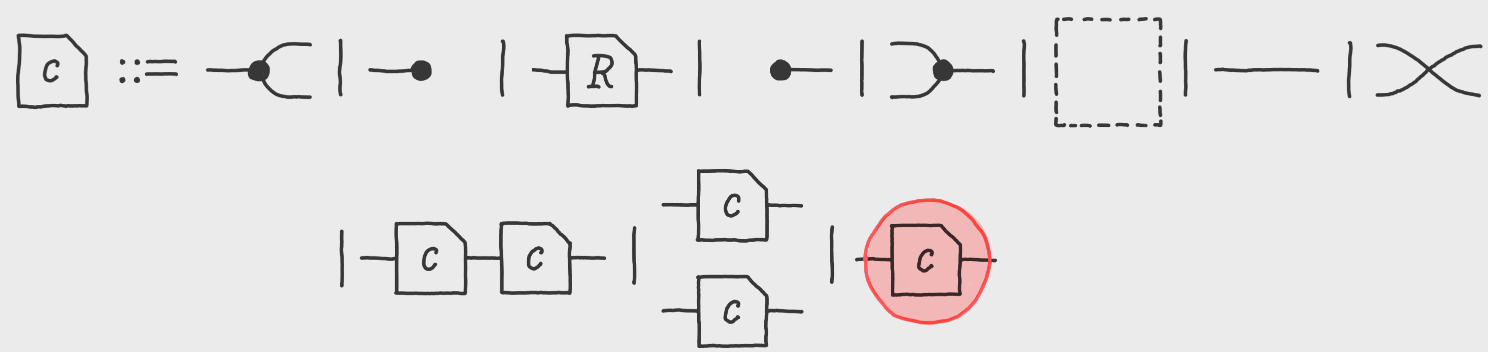

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‘Principle of Contraposition’

If $P \leq Q$ then $\neg Q \leq \neg P$

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'Principle of Contraposition'

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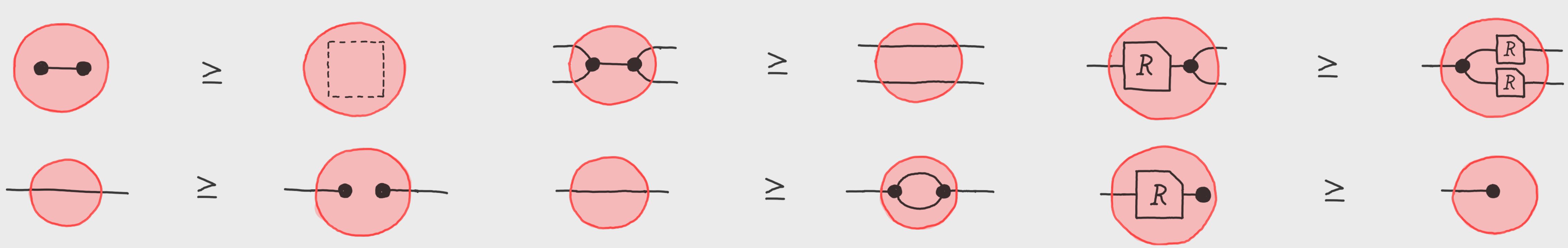

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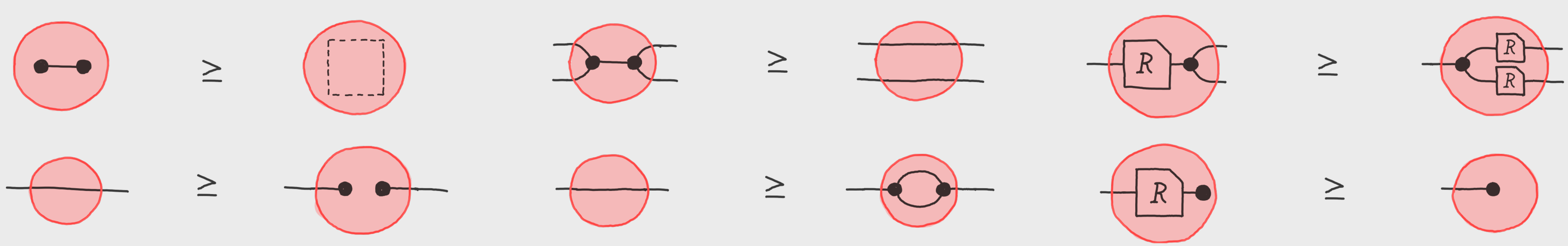
Erasure ↦ Insertion

Iteration ↦ Delteration



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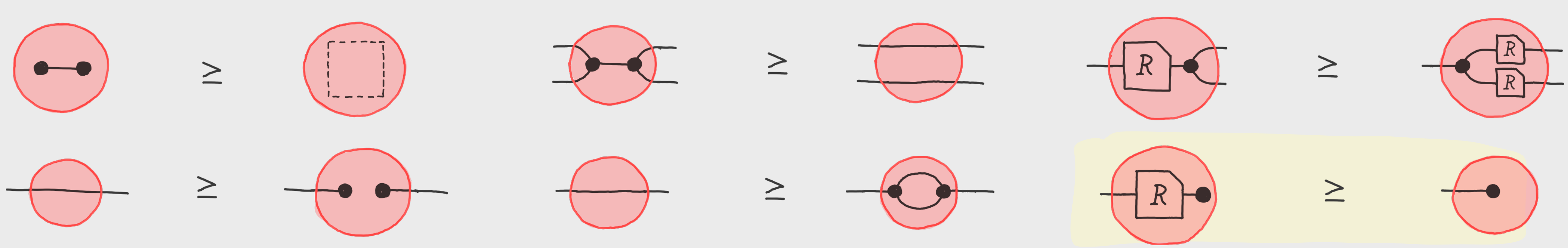

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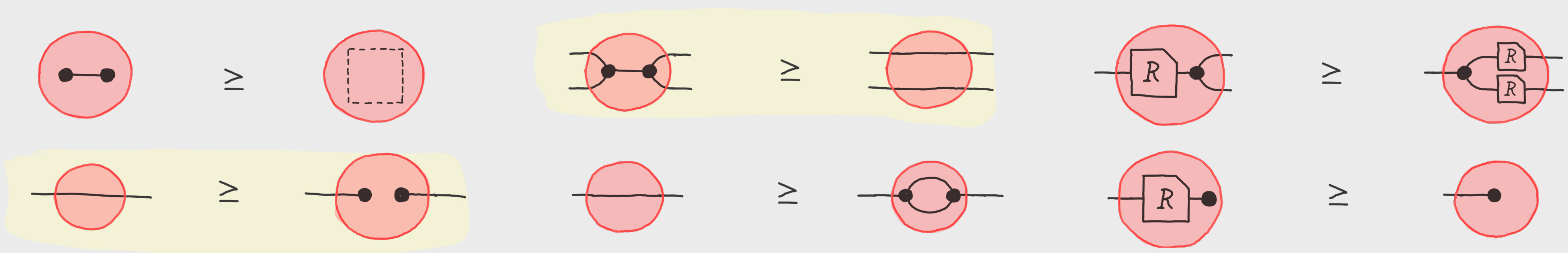

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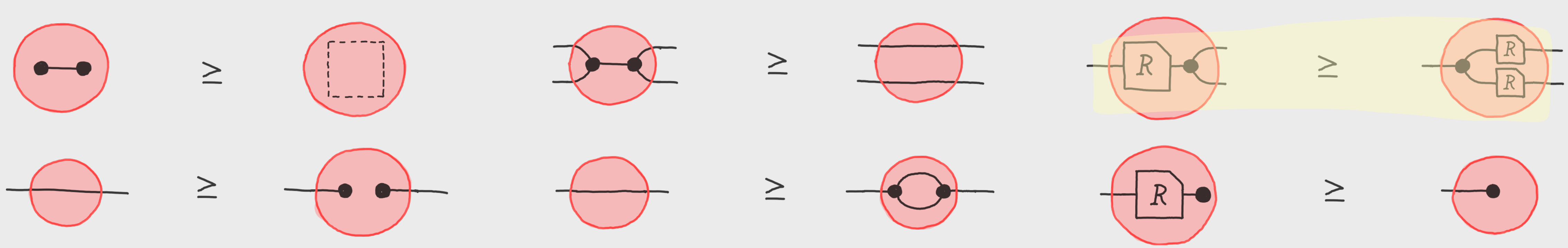

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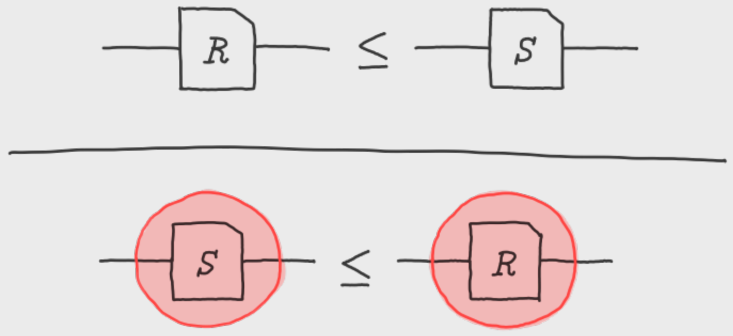

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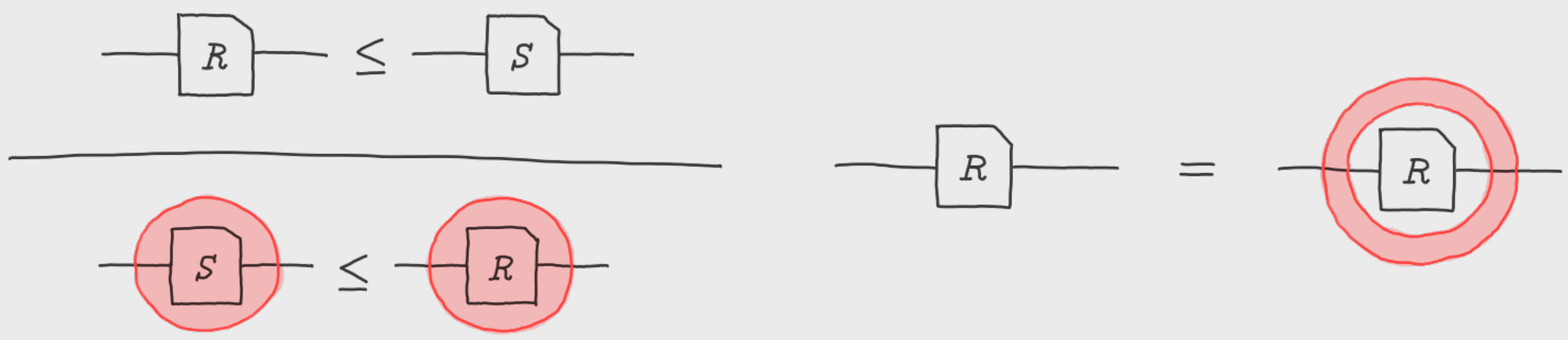

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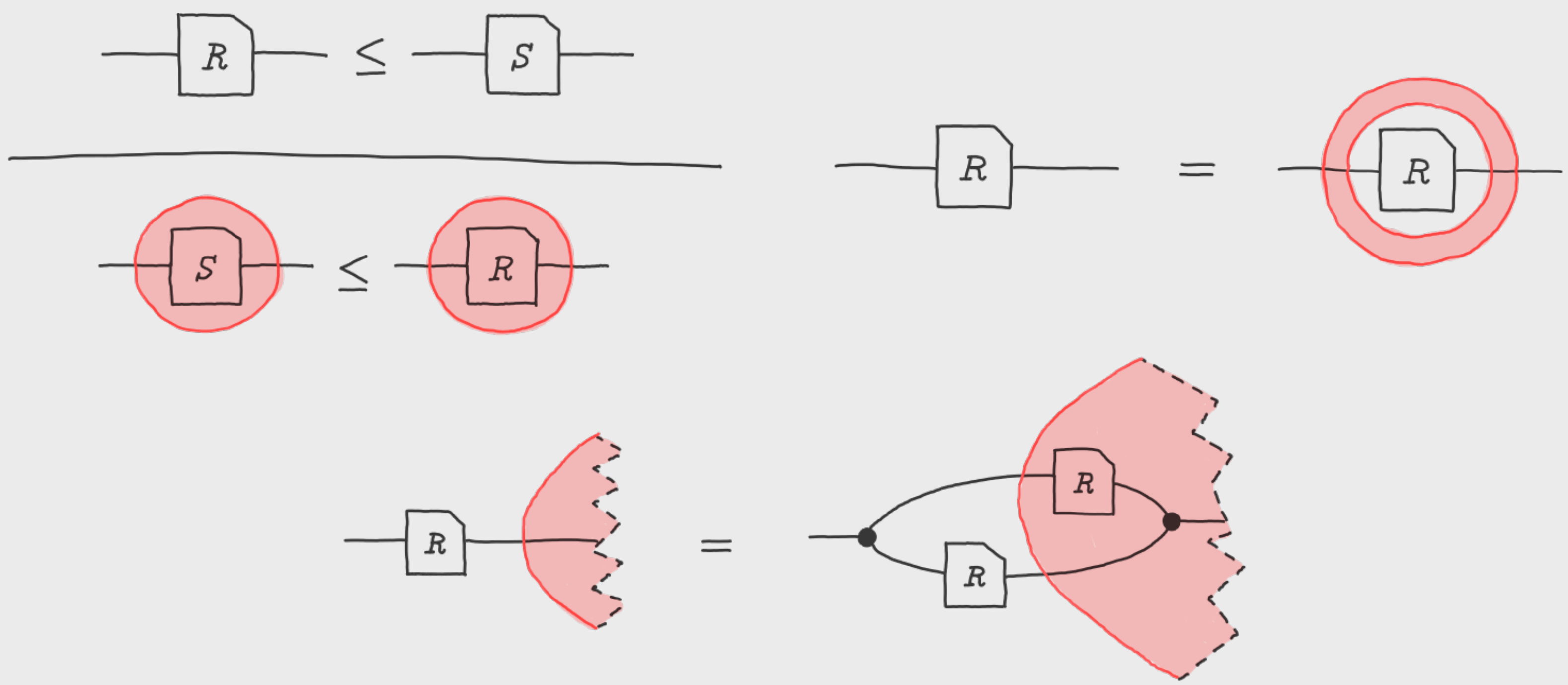

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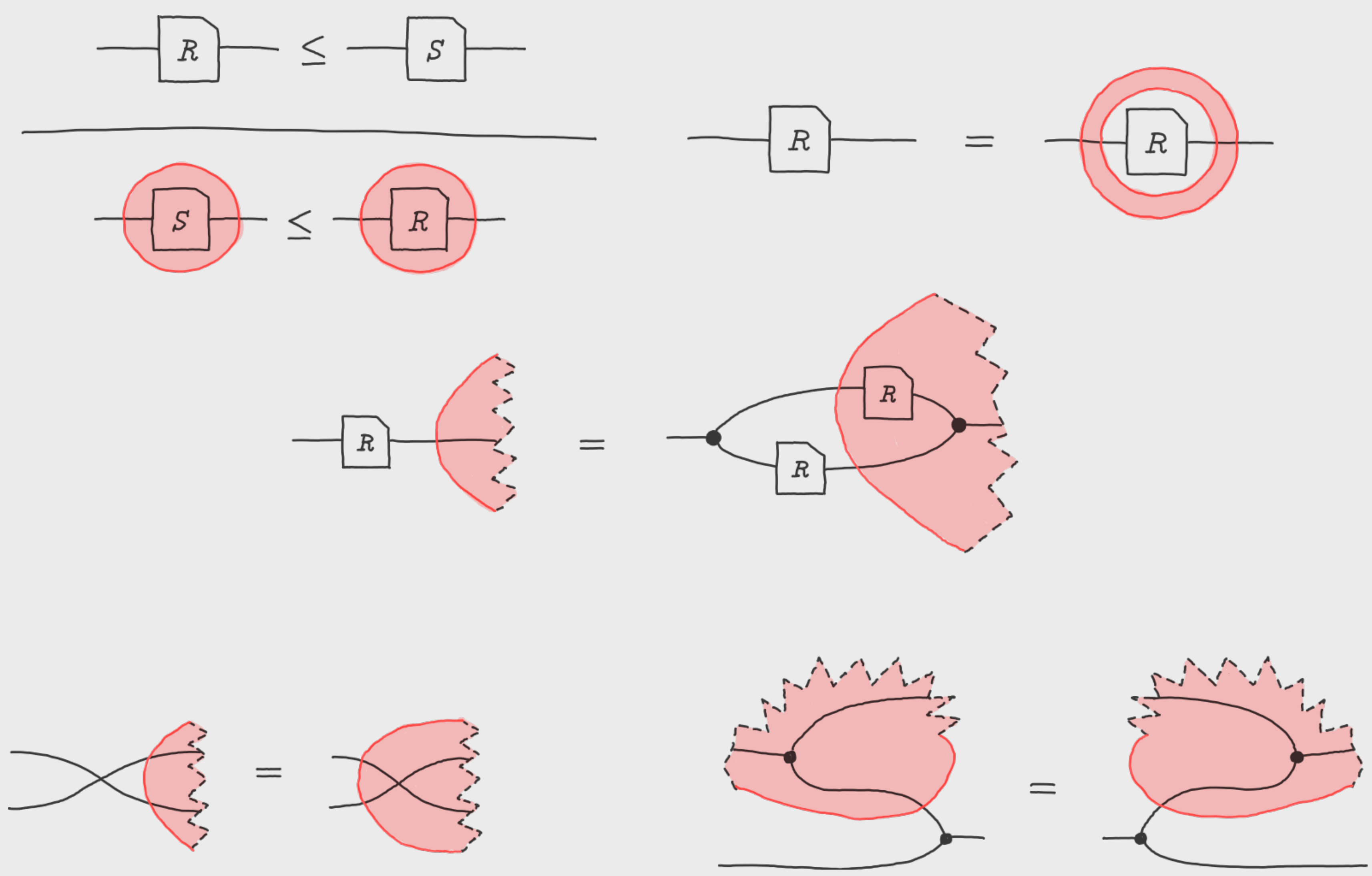

The standard account of Peirce's Existential Graphs correspond to…

… the string diagrammatic presentation of cartesian bicategories of relations (CBR)…

… with negation.

[Modern EG_1]

Syntax	Conventions	Inference Rules
Asserted Relations (P , Q , R…)	‘sheet of assertion’	Erasure/Insertion
‘Cut’	Juxtaposition as ‘conjunction’	Iteration/Deiteration
‘Line of Identity'	‘Cut’ as negation	Add/Remove ‘Double-Cut’
	‘Line of Identity’ as something exists	(Principle of Contraposition)

Cartesian bicategories with Peirce’s ‘Cut’ (first-order logic with equality).

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Cartesian bicategories with Peirce’s ‘Cut’ (first-order logic with equality).

See Haydon and Sobocinski ‘Compositional Diagrammatic First-Order Logic’ (2020).

Outline

- ~~introduce Peirce’s Existential Graphs (à la regular logic and cartesian bicategories)~~
- move to the Neo-Peircean Calculus of Relations (à la residuation and cyclic bilinear logic)
- demonstrate topological advantages

Negation in the reconstruction above is taken as primitive. Can we motivate another account?…

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… perhaps…

… using the `scroll'?

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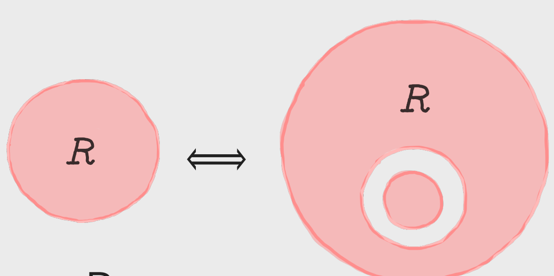
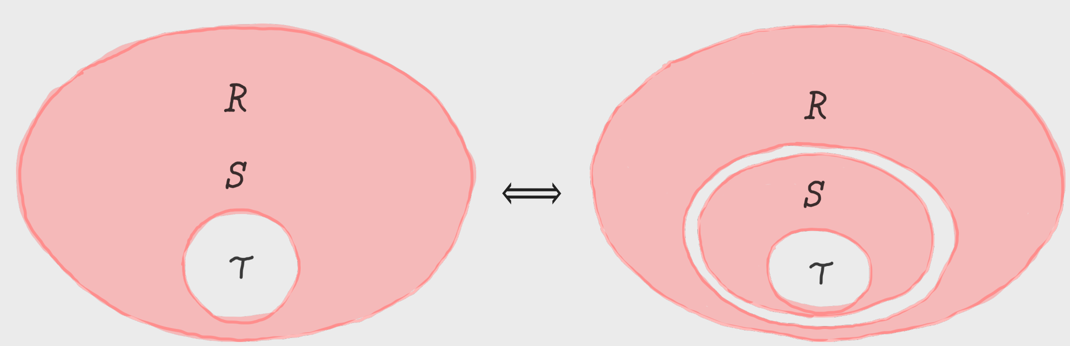


$\neg R$ $\quad$ $R \leq \bot$



$R \wedge S \leq T$ $\quad$ $R \leq (S \leq T)$

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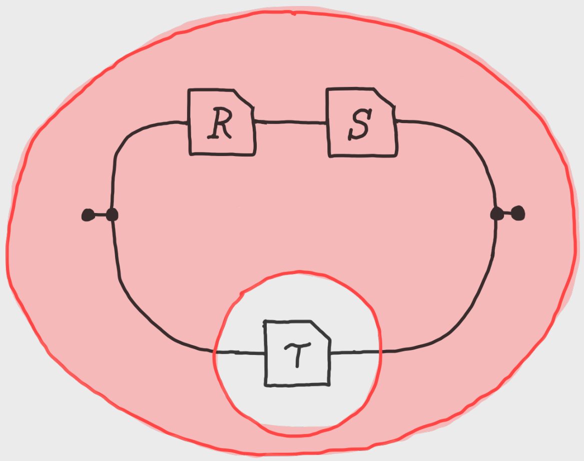

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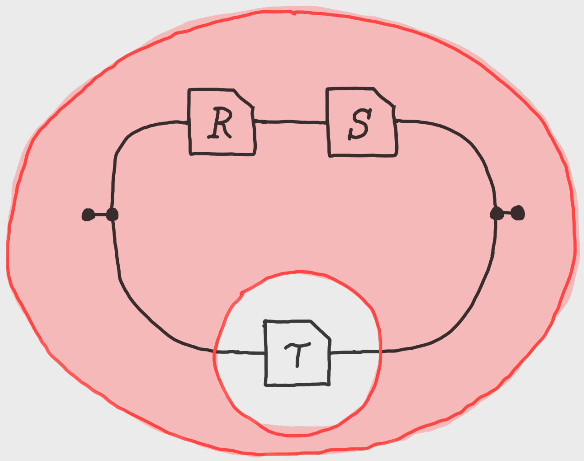

$R; S \leq T$

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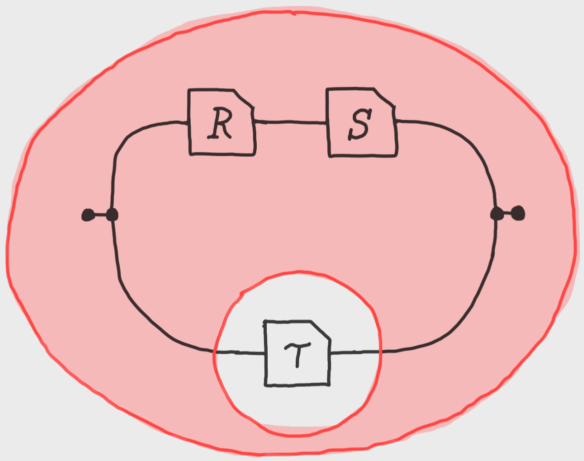

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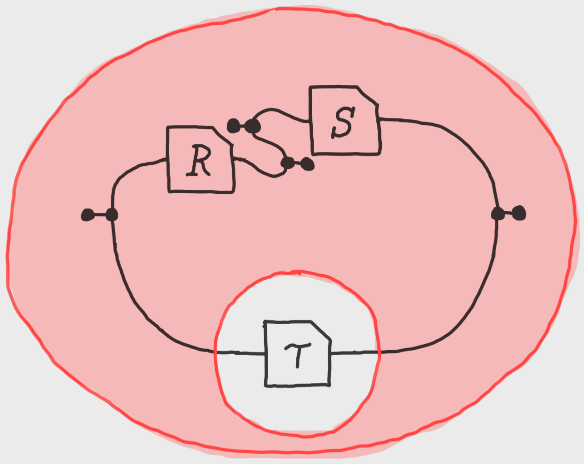

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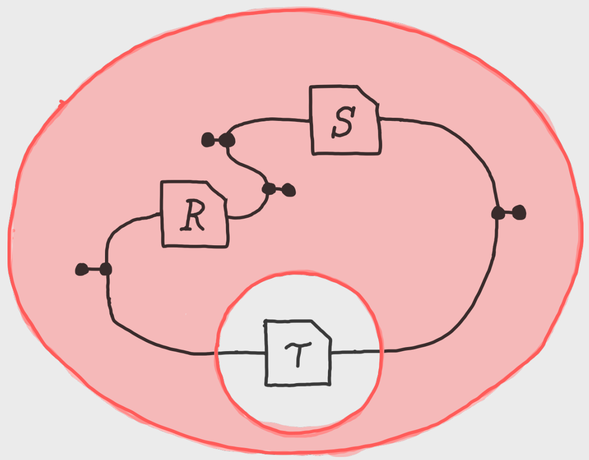

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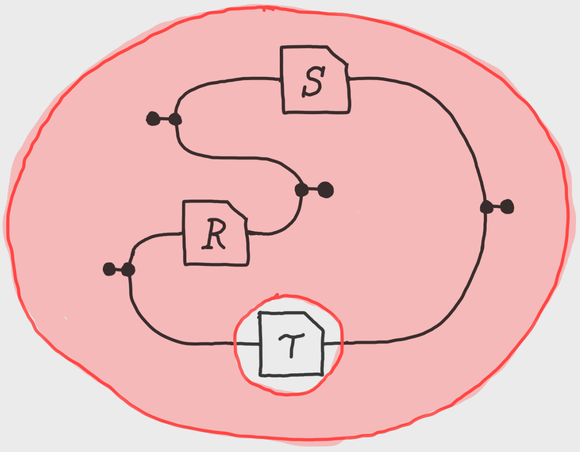

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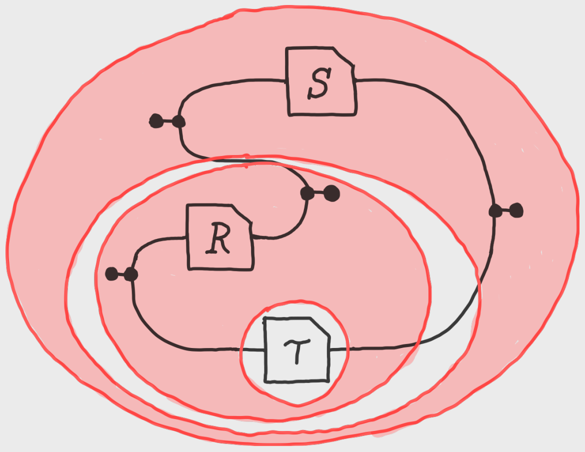

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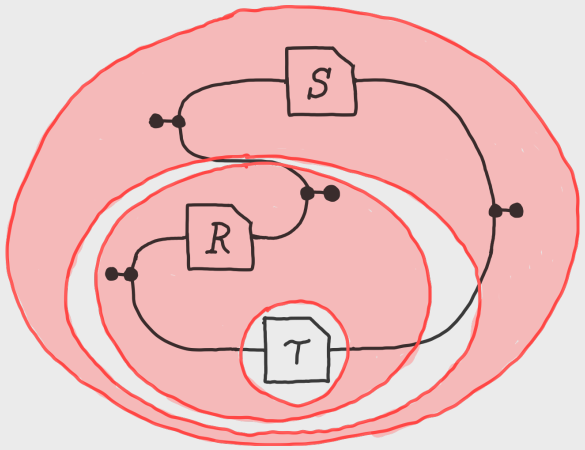

$$S \leq (\breve{R}; \bar{T})^{-}$$

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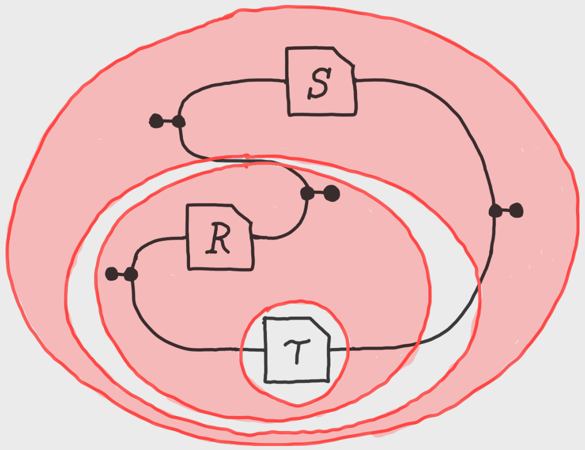

$$S \leq (\breve{R}; \bar{T})^{-}$$

$$\parallel$$

$$R \backslash T$$

Residuation Equivalences:

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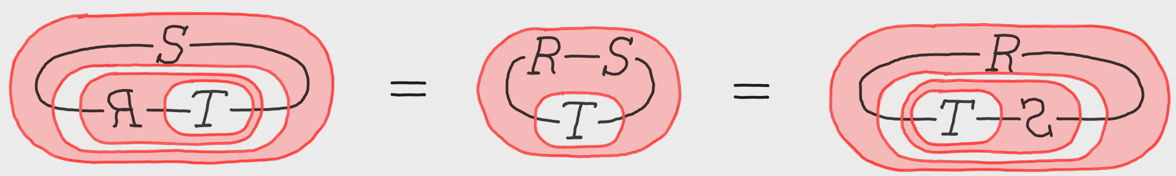
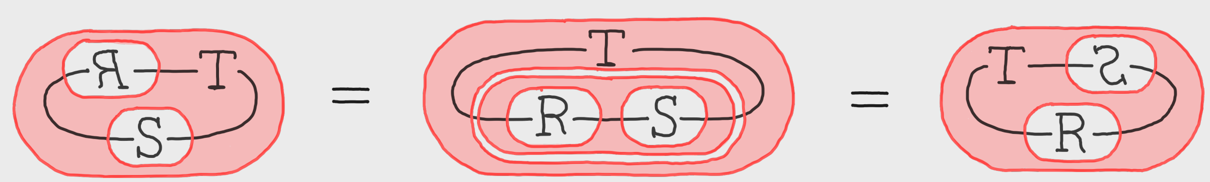


$[\ S \leq R \backslash T\ ]$ $\quad$ $[\ R; S \leq T\ ]$ $\quad$ $[\ R \leq S \backslash T\ ]$

Coresiduation Equivalences:



$[\ R^{\perp}; T \leq S\ ]$ $\quad$ $[\ T \leq R \dagger S\ ]$ $\quad$ $[\ T; S^{\perp} \leq R\ ]$

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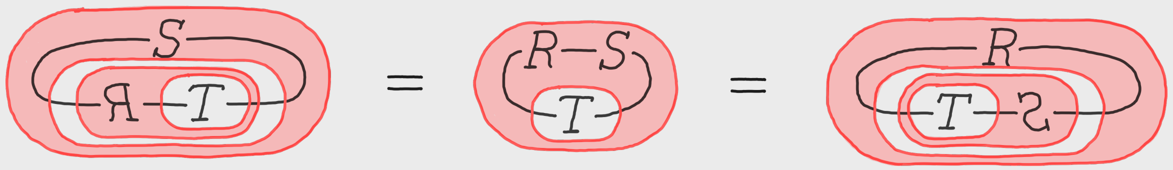

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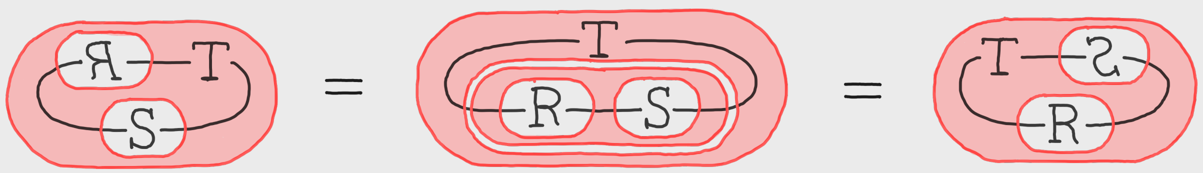

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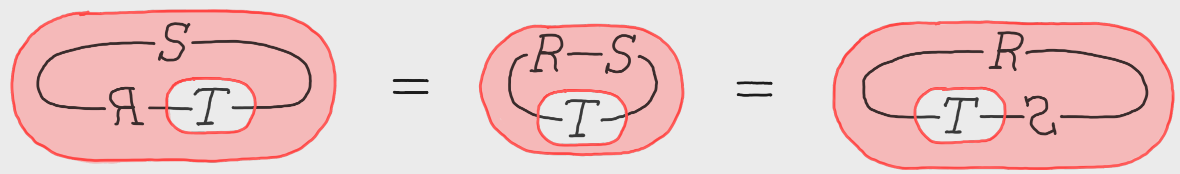

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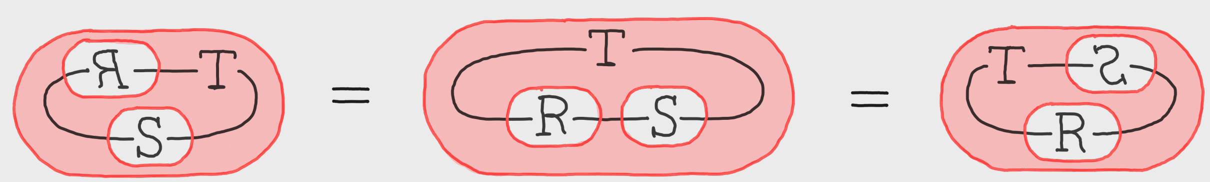

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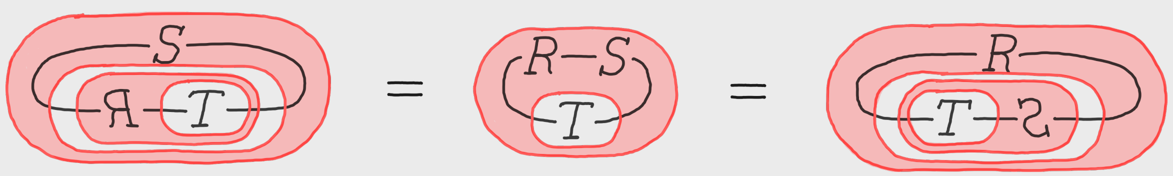

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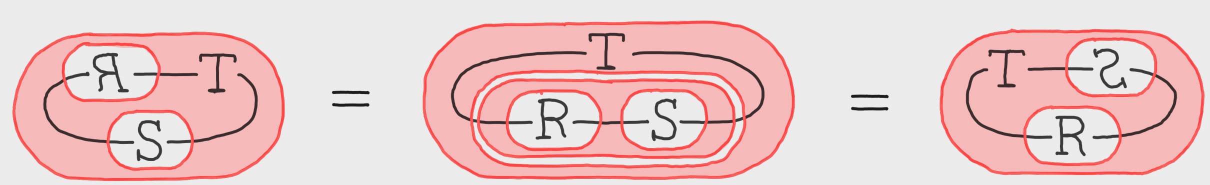

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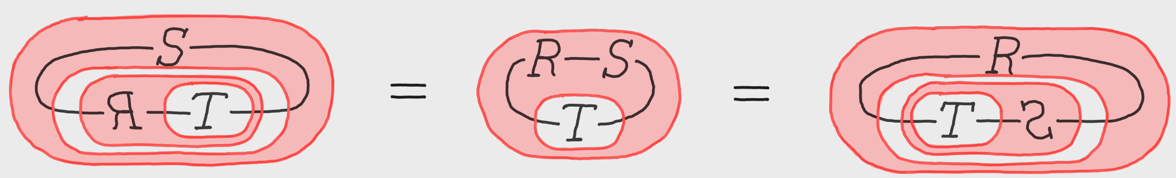

$[\ S \leq R \backslash T\ ]$ $[\ R ; S \leq T\ ]$ $[\ R \leq S \backslash T\ ]$

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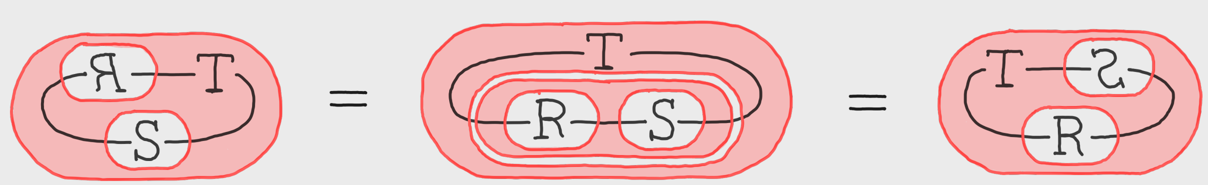

$[\ R^{\perp} ; T \leq S\ ]$ $[\ T \leq R \dagger S\ ]$ $[\ T ; S^{\perp} \leq R\ ]$

Residuation Equivalences:

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“Hence the rule is that having a formula of the form [$R ; S \leq T \dagger U$], the letters may be cyclically advanced one place in the order of writing, those which are carried from one side of the copula to the other being both negatived and converted.”

([On the Logic of Relatives], 1882)

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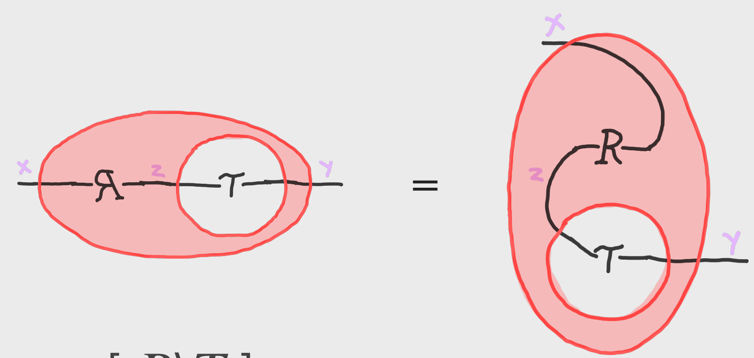

[$R \backslash T$]

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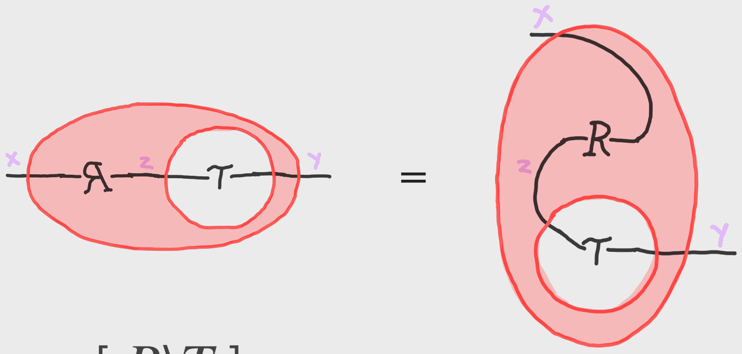

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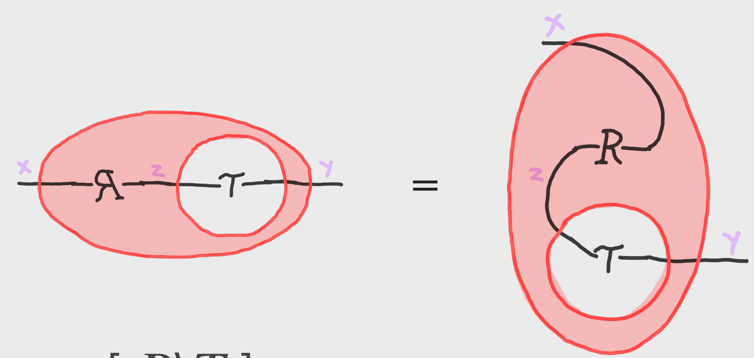

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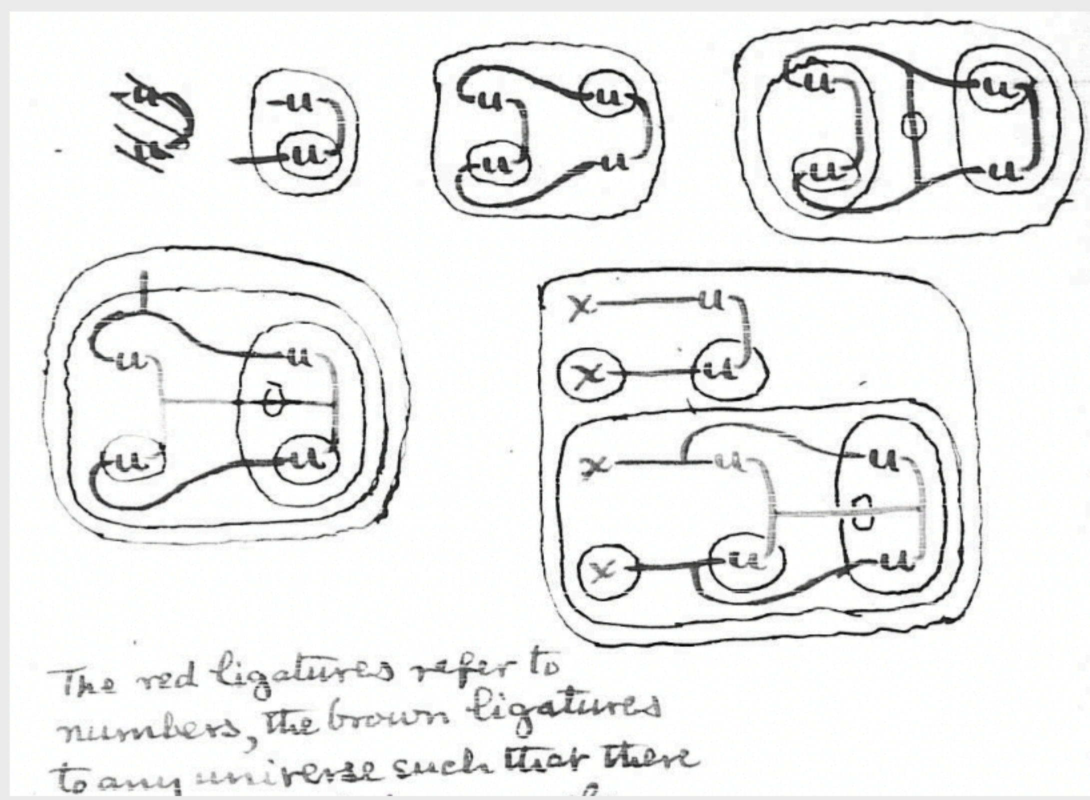

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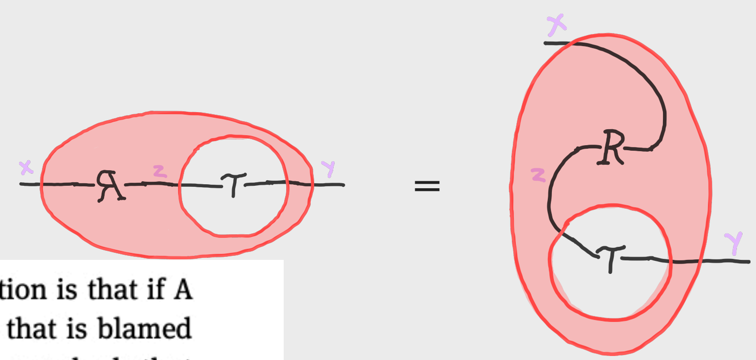
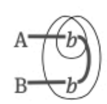
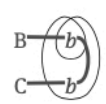


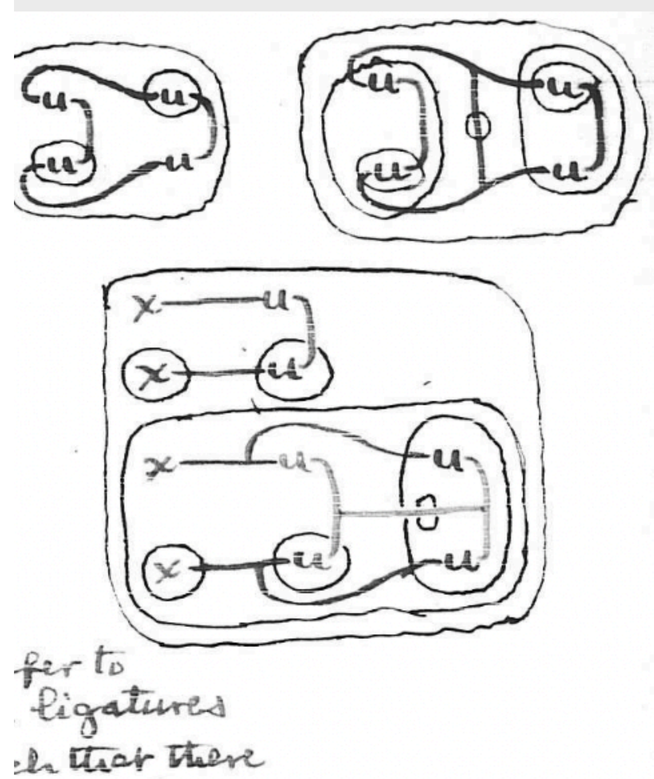
Taking any of these, say the monosyllabic "blames" my proposition is that if A blames everybody that is blamed by B and B blames everybody that is blamed by C, then A blames everybody that is blamed by C, then A blames everybody that is blamed by C. It is obviously so. But I will prove it by existential graphs. In order to express that A blames everybody blamed by B, we note this simply denies that there is anybody blamed by B and not blamed by A. Therefore we ~~write~~ scribe



So to say that B blames everybody that is blamed by C we scribe



Or dropping the capital letters



18 [The graphs in this fragment are inscribed in red (the lines and letters for spots), and in black or blue ink (the continuous and broken cuts and the scrolls).]

Residuation Equivalences:

Taking any of these, say the monosyllabic "b
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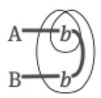

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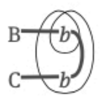

Or dropping the capital letters

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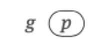

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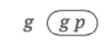
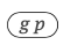
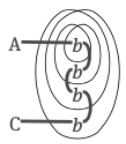


and may then erase the outside replica by the rule of ~~omission~~ erasure and insertion

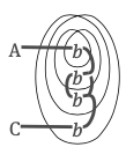


So that ~~we have the rule~~ it comes to this that any graph not in an odd number of cuts can be shoved into any additional cuts that may be *already made*.

Acting on this rule, let us shove the whole of the upper graph into the inner cut of the lower one; thus:



Now whatever rule holds good for every graph is true of every line of identity, since a line of identity is a graph. We therefore have a right to iterate the lower right hand line in one more cut thus



Now by the rule of omission and insertion within three cuts, three being an odd number we can make any insertion we like. We can therefore join the two lines there thus

Residuation Equivalences:

Taking any of these, say the monosyllabic "b
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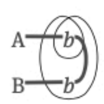

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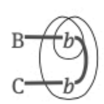

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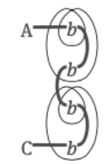

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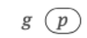

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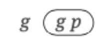

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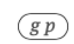

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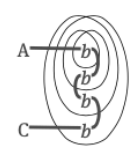

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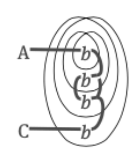

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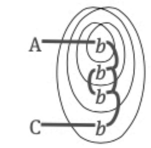
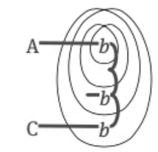
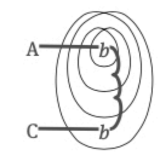


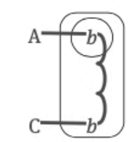
Now the two ~~inner~~ middle *b*'s having the same ~~connections~~ ligatures, we can ~~deiterate~~ erase the inner one by the rule of iteration and deiteration thus



And now, the middle *b* being in an even number of cuts can be erased, making

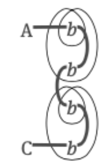


Finally, the two cuts with nothing between except traversing cuts destroy one another and we have



That is A blames everybody blamed by C which is what I undertook to prove.

Residuation Equivalences:



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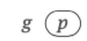

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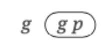

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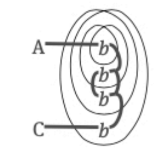

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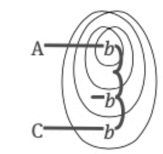

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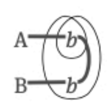

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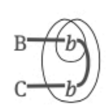

Or dropping the capital letters

18 [The graphs in this fragment are inscribed in red (th
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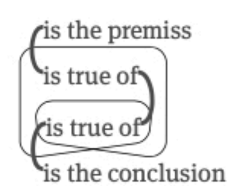
est importance for pure mathematics, since pure mathematics is precisely what results from abstracting from all the special meaning of necessary reasoning and considering only its forms. Of these four relations there is one which enters into the very ~~idea~~ definition of necessary reasoning. For necessary reasoning is that whose conclusion is true of whatever state of things there may be in which the premiss is true. Now this is expressed in a graph thus:



The pure mathematician substitutes for these logical terms defined symbols *x*, *y*, *z*, which are to mean whatever they may mean; and he thus gets this graph, which is precisely the graph of inclusion.

I cannot stop to consider the other three relations but must hurry back to the subject of numbers. The doctrine of ordinal numbers, then, is a theory of pure mathematics and, as matters stand today, is the most fundamental of all branches of pure mathematics after the mathematics of the pair of values which existential graphs illustrate. The doctrine of multitude is not pure mathematics. Pure math-

Peirce presented some of the earliest instances of string diagrams as we might recognize them today…

… and stated some essential features and laws.

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- Peirce studied residuation (around 1883) and did so extensively in the graphs
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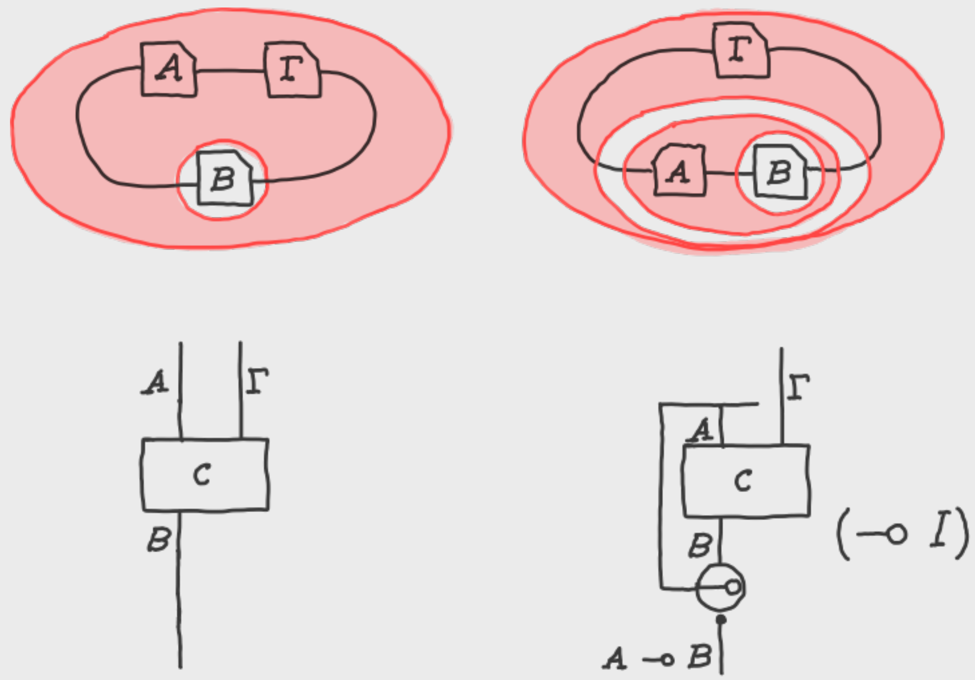
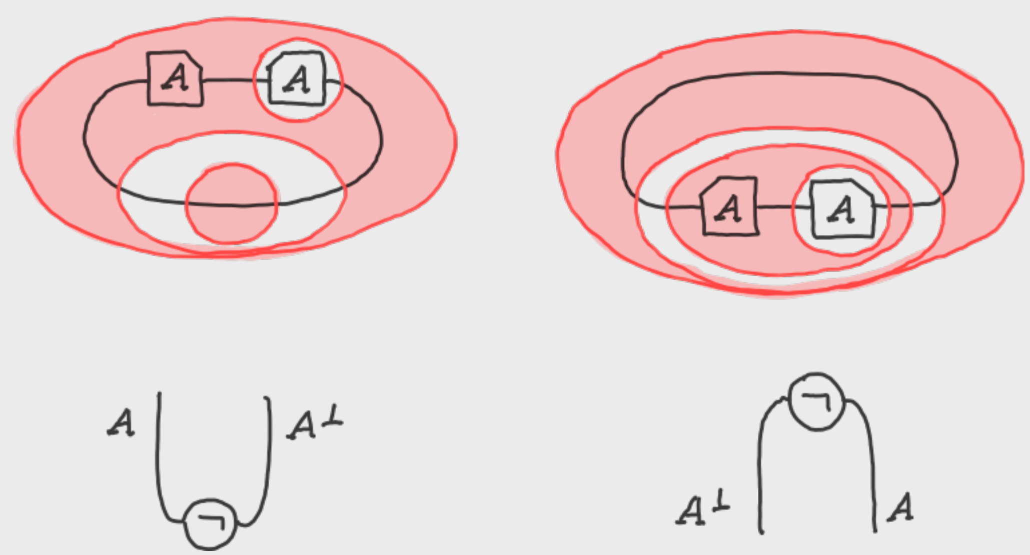
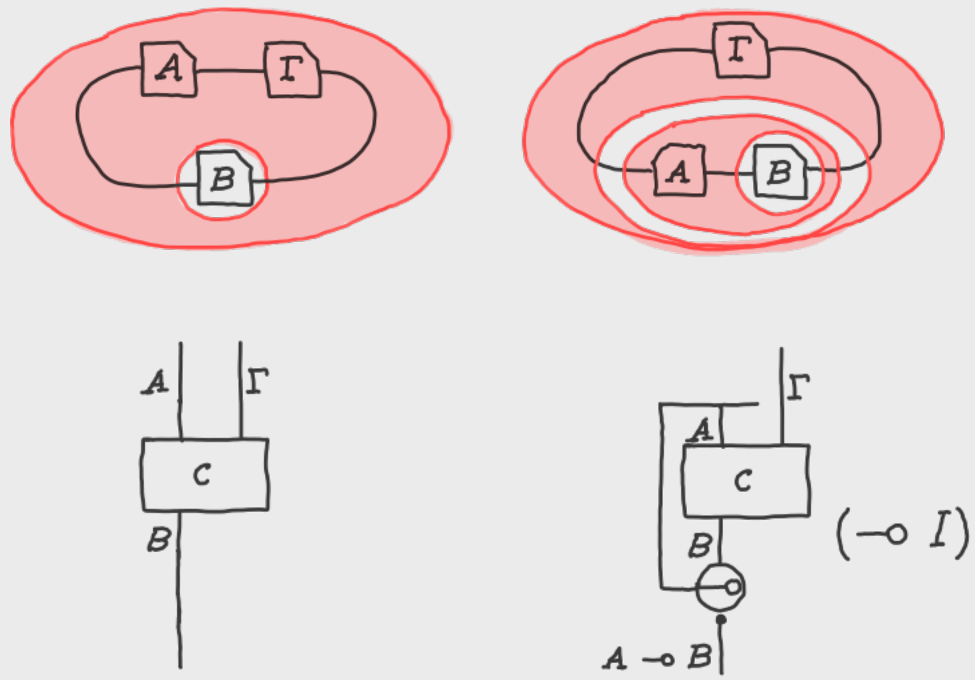
See Haydon and Pietarinen `Residuation in Peirce’s Existential Graphs’ (2021).

Residuation in the Existential Graphs coincides with other graphical approaches…



… such as Cockett and Seely's 'circuit diagrams' for bilinear logic…





… such as Cockett and Seely's 'circuit diagrams' for bilinear logic…

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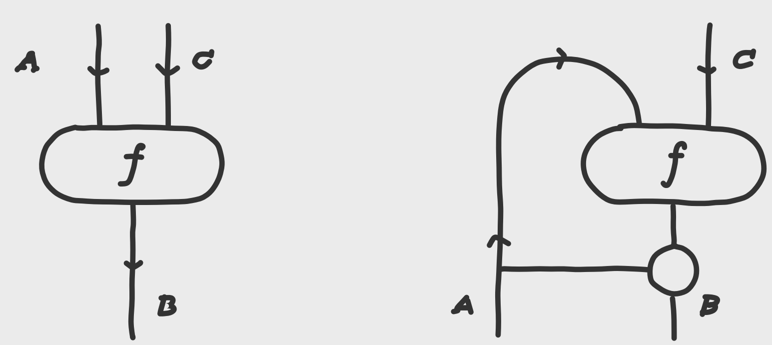

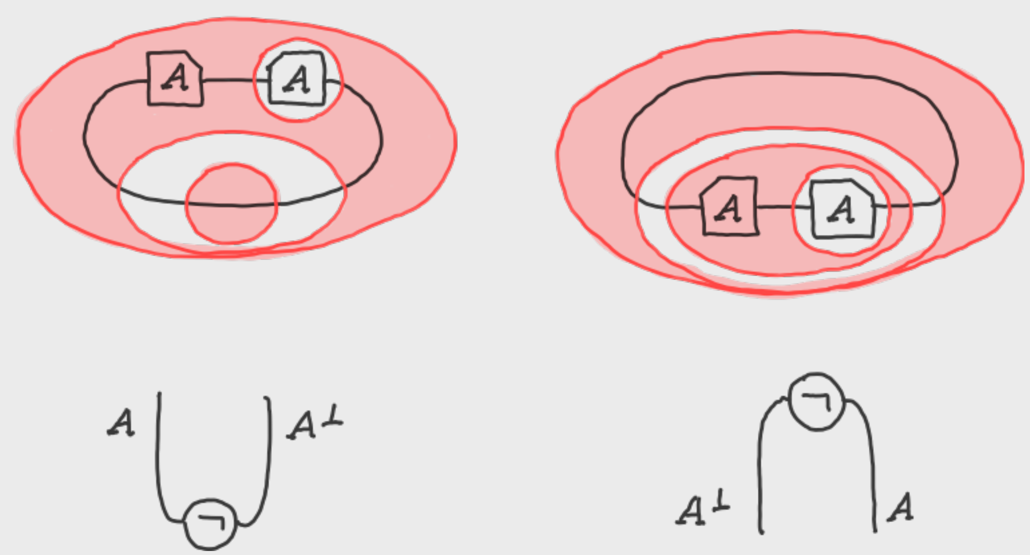

Negation in the reconstruction above is taken as primitive. Can we motivate another account?…

… perhaps…

… using the `scroll'?

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Residuation adds an important connective that gets us part of the way…

The Neo-Peircean Calculus of Relations…

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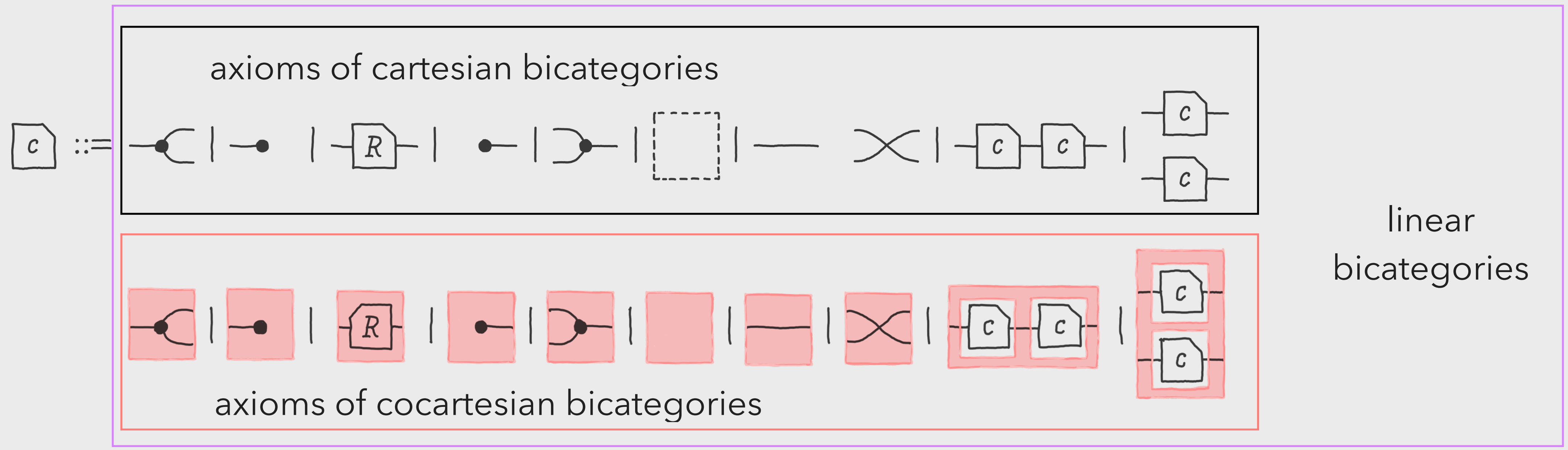
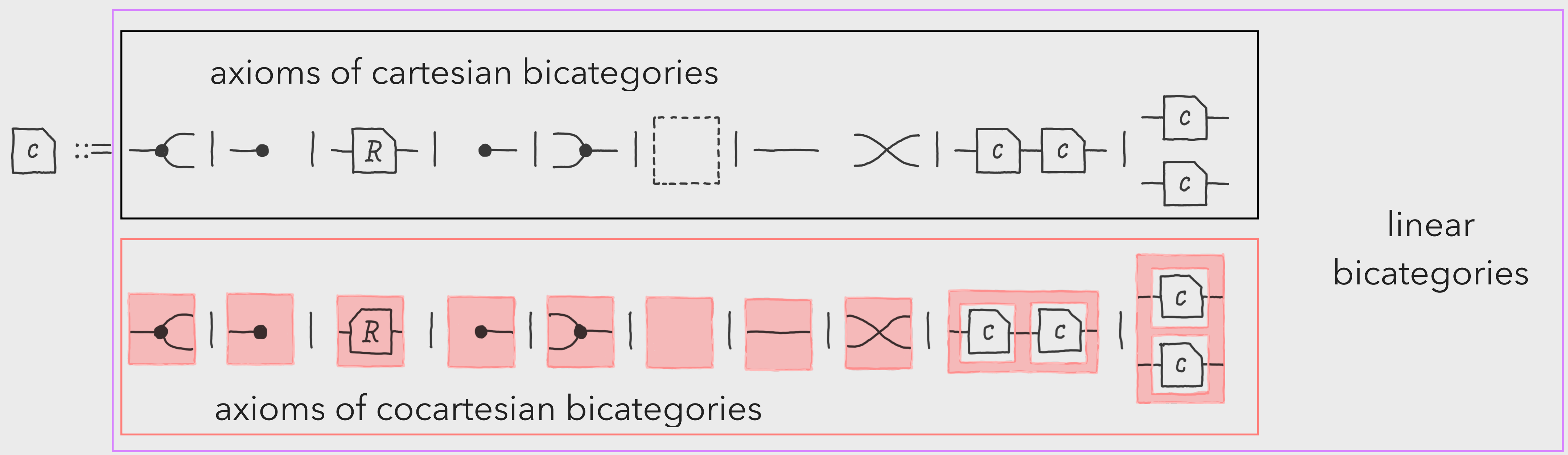
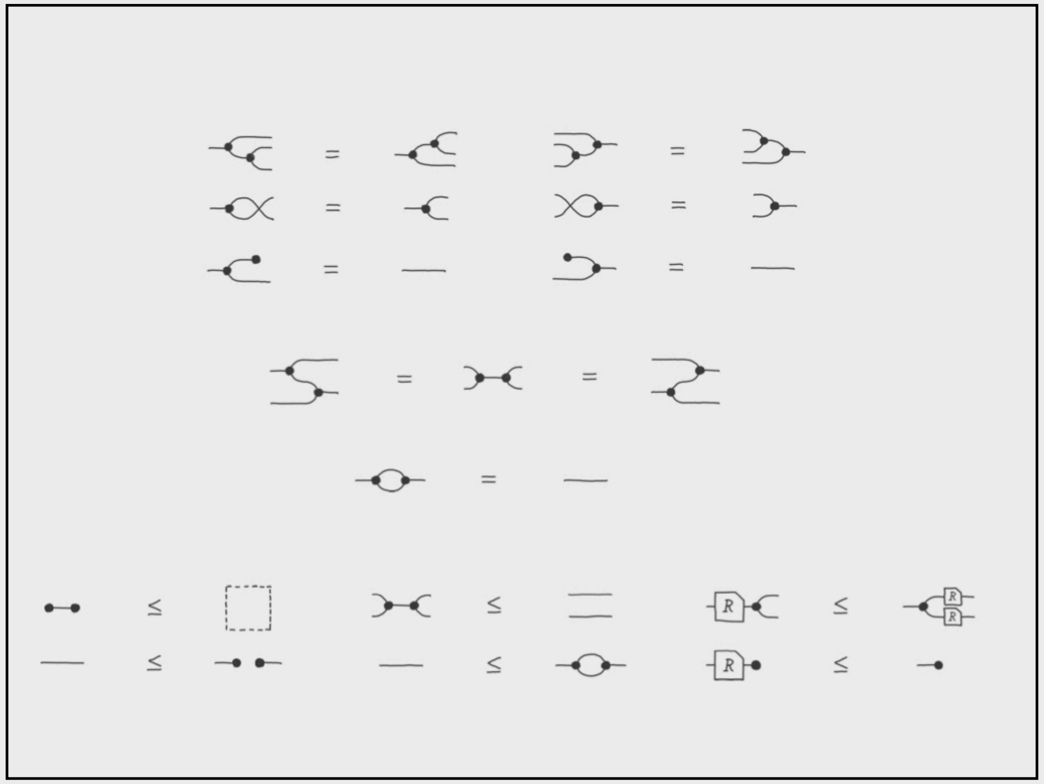


The Neo-Peircean Calculus of Relations

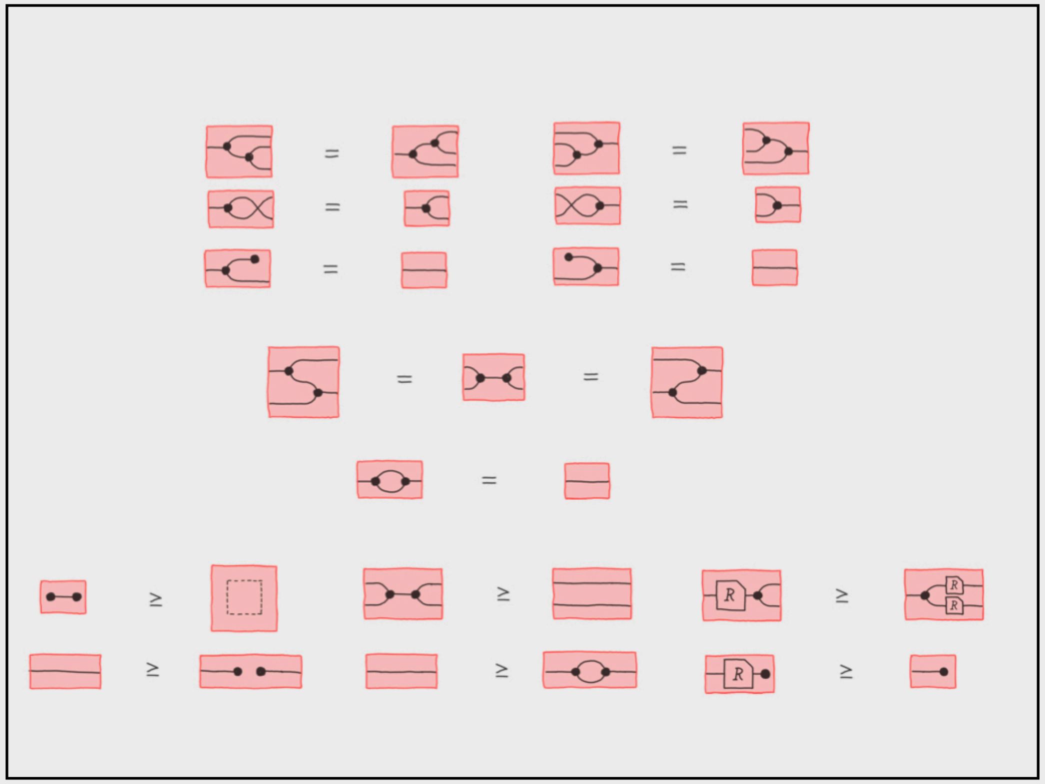


- regular and coregular fragment
- with the dual of relational composition
- these interact via linear negation and linear distributivity
- all inspired by Peirce's presentation of relations from 1883

What are the graphical rewrites, i.e. the inference rules?



Axioms of Cartesian bicategories



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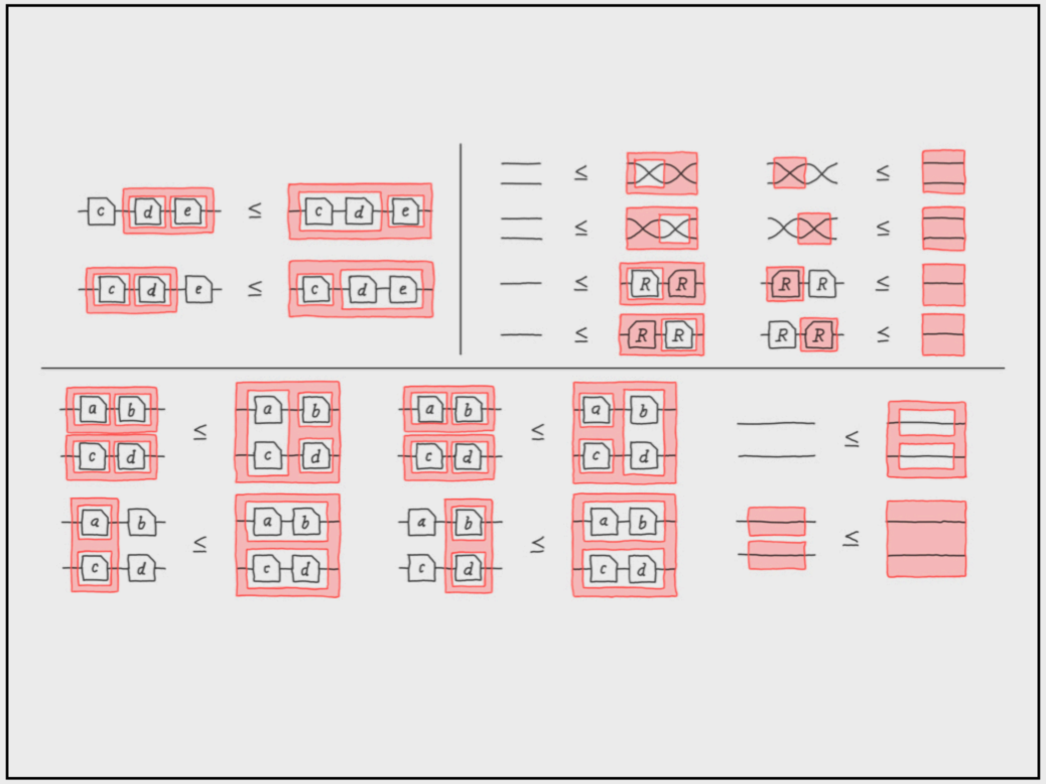
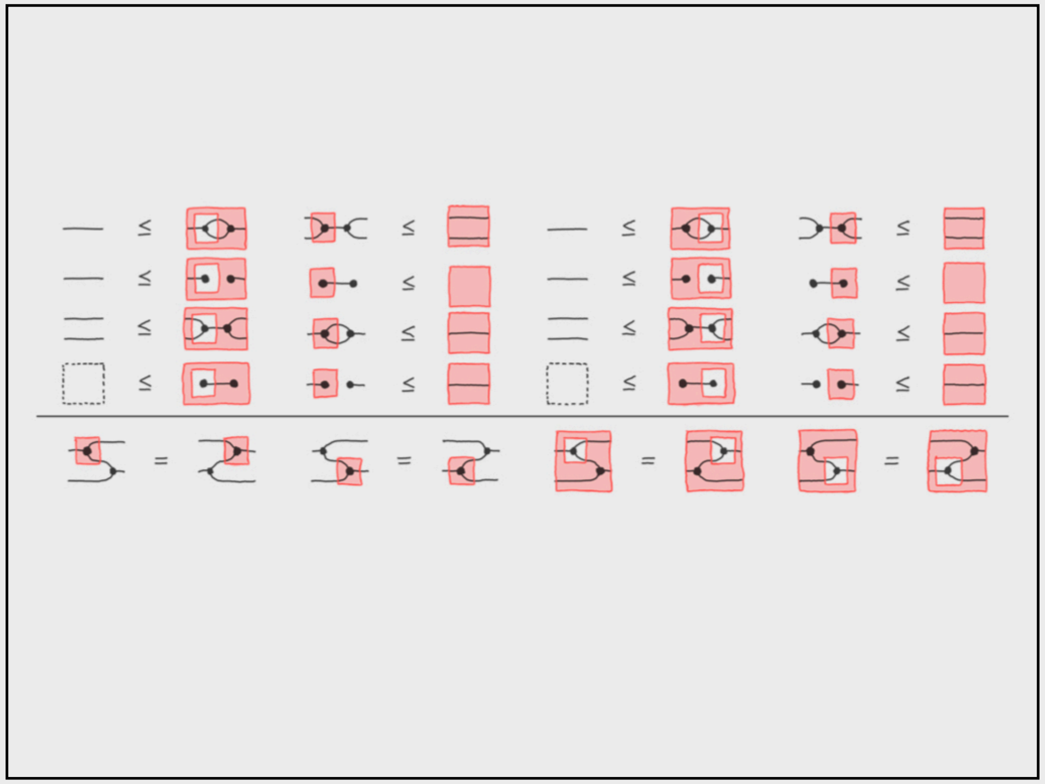
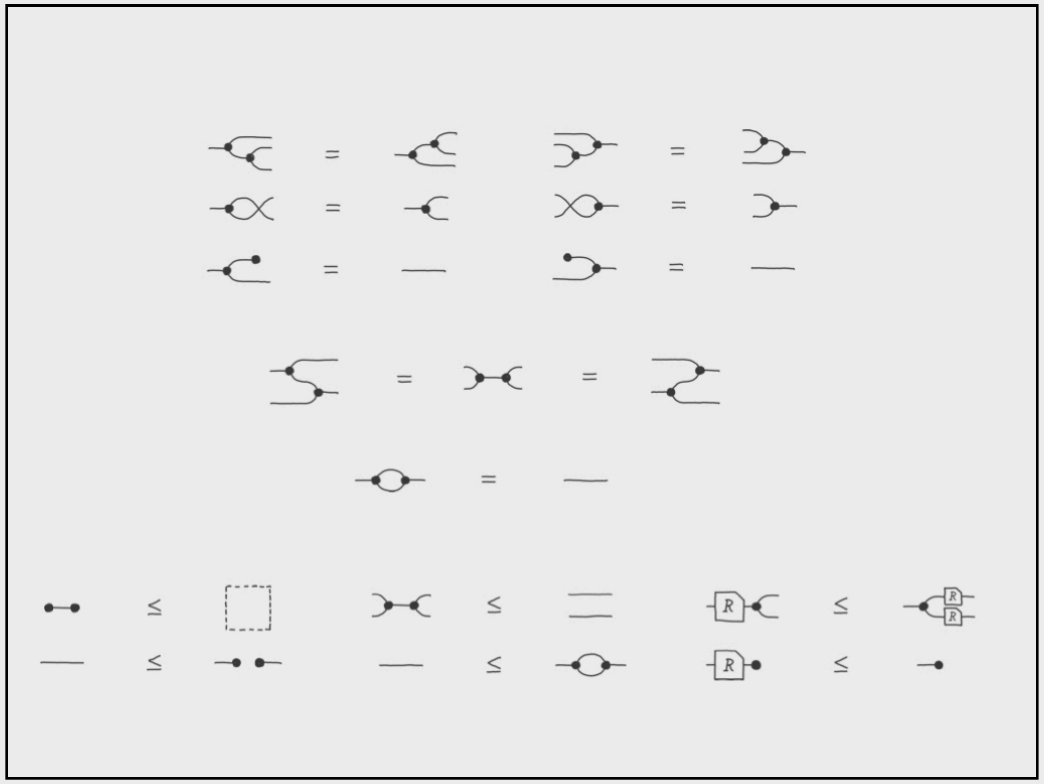


Axioms of closed symmetric monoidal linear bicategories



Additional axioms of NPR



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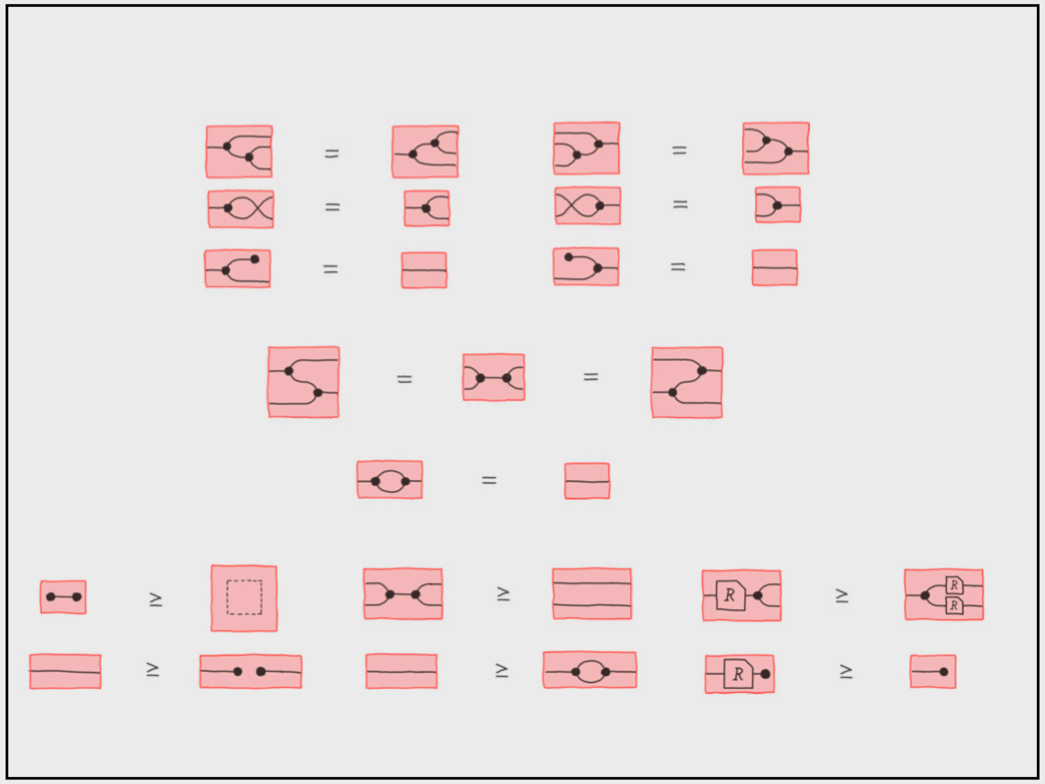

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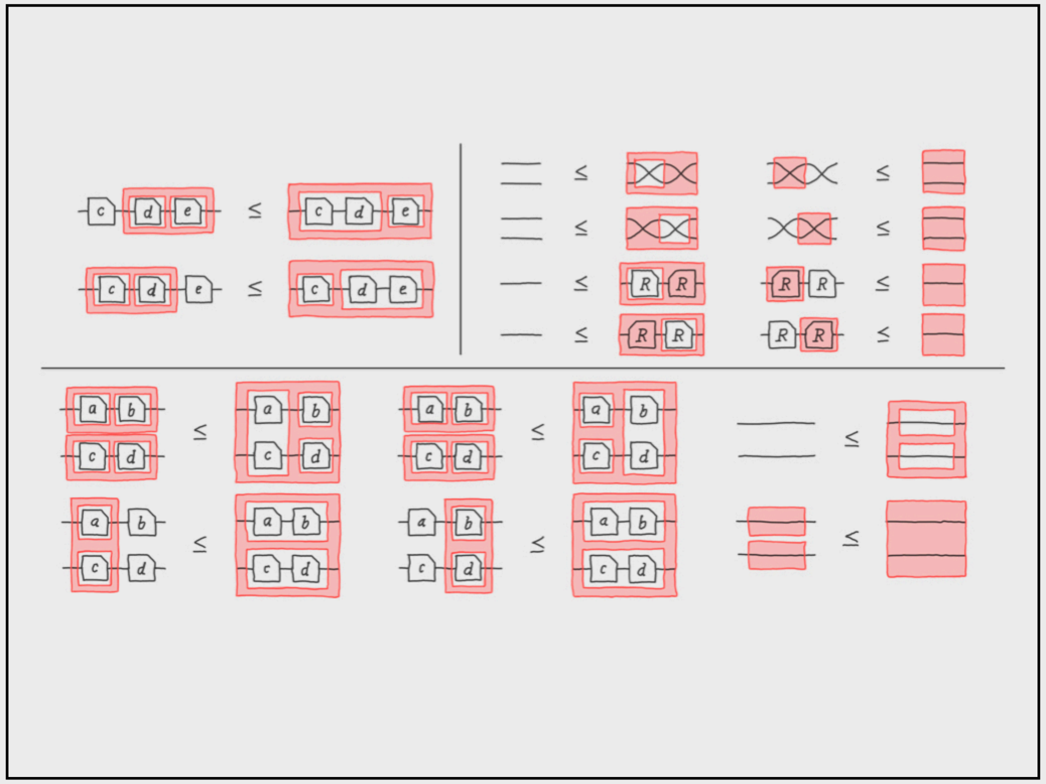

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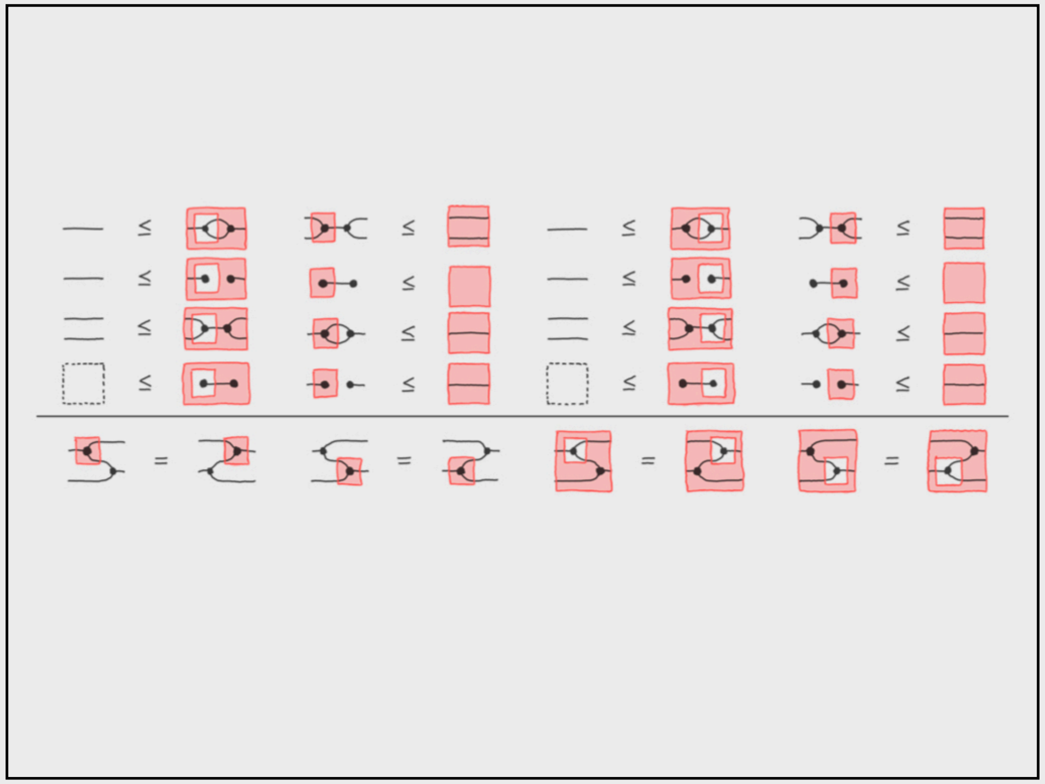

Additional axioms of NPR

[The closest extant theory is *classical (cyclic) bilinear logic*. See Lambek (1995).]

From Categorial Grammar to Bilinear Logic

Joachim Lambek

Department of Mathematics and Statistics, McGill University[1]

1 Introduction

The *syntactic calculus*, also known as 'bidirectional categorial grammar', is a kind of logic without any structural rules, other than the obligatory reflexive law and cut-rule. It had been inspired by multilinear algebra and non-commutative ring theory and was developed with applications to linguistics in mind. Here we shall confine attention to the associative version, although a non-associative version has also been studied [L 1961, Kandulski 1988, Došen 1988, 1989]. It differs from a very rudimentary form of Girard's linear logic [Girard 1987, 1989] by the absence of the interchange rule which licenses commutativity. Because of its roots in non-commutative algebra and syntax, all appearance of commutativity is forbidden. The no-

From Categorial Grammar to Bilinear Logic

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2 Recalling the Syntactic Calculus

The *syntactic calculus* deals with types, also called 'formulas', and arrows between them, which logicians think of as entailments or deductions and linguists as derivations. The types or formulas are constructed from basic ones by certain operations or connectives, of which we shall here consider three nullary and five binary ones, although not all of these operations occur in every exposition of the syntactic calculus. From a number of notational variants, we have here chosen the following:

$$\mathrm{I},\ \otimes,\ /,\ \backslash,\ \top,\ \wedge,\ \bot,\ \vee.$$

These operations are subject to the following axioms and rules of inference. (For purposes of this elementary exposition, we have not labeled the arrows, although a more advanced exposition would require this.)

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$$A \otimes \mathrm{I} \leftrightarrow A \leftrightarrow \mathrm{I} \otimes A, \quad (A \otimes B) \otimes C \leftrightarrow A \otimes (B \otimes C),$$

$$A \otimes B \to C \quad \text{iff} \quad A \to C/B \quad \text{iff} \quad B \to A\backslash C,$$

$$A \to \top, \quad A \wedge B \to A, \quad A \wedge B \to B, \quad \frac{C \to A \quad C \to B}{C \to A \wedge B},$$

$$\bot \to A, \quad A \to A \vee B, \quad B \to A \vee B, \quad \frac{A \to C \quad B \to C}{A \vee B \to C}.$$

Certain derived rules of inference are quite useful. For example, the rule

$$\frac{A \to B \quad C \to D}{A \otimes C \to B \otimes D}$$

expresses what categorists call the 'bifunctoriality' of the operation $\otimes$. In

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A weak form of bilinear logic, let us call it BL1, will be obtained from the syntactic calculus described earlier by adding the following new operations:

$$O, \ \oplus, \ \div, \ \div,$$

subject to the following new axioms and rules of inference:

$$(A \oplus B) \oplus C \leftrightarrow A \oplus (B \oplus C), \ A \oplus O \leftrightarrow A \leftrightarrow O \oplus A,$$

$$C \to A \oplus B \ \text{ iff } \ C \div B \to A \ \text{ iff } \ A \div C \to B.$$

The following derived rules are easily proved or just inferred from symmetry:

$$\frac{A \to B \quad C \to D}{A \oplus C \to B \oplus D}, \quad \frac{A \to B \quad C \to D}{A \div D \to B \div C}, \quad \frac{A \to B \quad C \to D}{B \div C \to A \div D}.$$

Models of BL1 might be called 'bi-residuated monoids' or 'bi-residuated lattices', the latter if the lattice operations are present. If, moreover, the

$$\bot \to A, \quad A \to A \vee B, \quad B \to A \vee B, \quad \frac{A \to C \quad B \to C}{A \vee B \to C}.$$

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$$\mathrm{O},\ \oplus,\ \div,\ \div,$$

In BL1 we have four negations

$$^{\perp}C = \mathrm{O}/C,\ \ C^{\perp} = C\backslash \mathrm{O}, {}^{\top}C = \mathrm{I} \div C,\ \ C^{\top} = C \div \mathrm{I},$$

which may be called *left annihilator, right annihilator, left creator* and *right creator* respectively, as they have the following properties:

$$(\#) \qquad {}^{\perp}C \otimes C \to \mathrm{O}, C \otimes C^{\perp} \to \mathrm{O}, \mathrm{I} \to {}^{\top}C \otimes C, \mathrm{I} \to C \otimes C^{\top}.$$

However, in BL1(a), ${}^{\top}C \to C^{\perp}$ and, in BL1(b), $C^{\perp} \to {}^{\top}C$. Thus, in BL2, $C^{\perp} \leftrightarrow {}^{\top}C$ and similarly $C^{\top} \leftrightarrow {}^{\perp}C$. Therefore, we may identify ${}^{\perp}C$ with

lattices', the latter if the lattice operations are present. If, moreover, the

$$\perp \to A,\ \ A \to A \vee B,\ \ B \to A \vee B,\ \ \frac{A \to C \qquad B \to C}{A \vee B \to C}.$$

Certain derived rules of inference are quite useful. For example, the rule

$$\frac{A \to B \qquad C \to D}{A \otimes C \to B \otimes D}$$

expresses what categorists call the 'bifunctoriality' of the operation $\otimes$. In

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of $\otimes, \mathrm{I}, \oplus, \mathrm{O}$ together with $^{\perp}$ and $^{\top}$. In this section, we shall ignore the connectives $\wedge, \top, \vee$ and $\perp$.

Clearly, the following axioms are necessary, in addition to the bifunctoriality of $\otimes$ and $\oplus$:

$$(A \otimes B) \otimes C \leftrightarrow A \otimes (B \otimes C), \qquad (A \oplus B) \oplus C \leftrightarrow A \oplus (B \oplus C),$$
$$A \otimes \mathrm{I} \leftrightarrow A \leftrightarrow \mathrm{I} \otimes A, \qquad A \oplus \mathrm{O} \leftrightarrow A \leftrightarrow \mathrm{O} \oplus A,$$
$$A \otimes (B \oplus C) \rightarrow (A \otimes B) \oplus C, \qquad (A \oplus B) \otimes C \rightarrow A \oplus (B \otimes C),$$
$$C^{\top} \otimes C \rightarrow \mathrm{O}, \qquad \mathrm{I} \rightarrow C \oplus C^{\top},$$
$$C \otimes C^{\perp} \rightarrow \mathrm{O}, \qquad \mathrm{I} \rightarrow C^{\perp} \oplus C.$$

To see that these axioms are also sufficient we shall check, for example, the combination of (a′) and (b):

$$(A \oplus B)/C \leftrightarrow (A \oplus B) \oplus C^{\top} \leftrightarrow A \oplus (B \oplus C^{\top}) \leftrightarrow A \oplus (B/C),$$

$C^{\perp} \leftrightarrow {}^{\top}C$ and similarly $C^{\top} \leftrightarrow {}^{\perp}C$. Therefore, we may identify ${}^{\perp}C$ with

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A \otimes (B \oplus C) \to (A \otimes B) \oplus C, & (A \oplus B) \otimes C \to A \oplus (B \otimes C), \\
C^{\top} \otimes C \to \mathrm{O}, & \mathrm{I} \to C \oplus C^{\top}, \\
C \otimes C^{\perp} \to \mathrm{O}, & \mathrm{I} \to C^{\perp} \oplus C.
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$$\mathrm{I} \to C \oplus C^{\top},$$
$$\mathrm{I} \to C^{\perp} \oplus C.$$

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$C^{\top}) \leftrightarrow A \oplus (B/C),$

dentify $^{\perp}C$ with
f, moreover, the

13 Cyclic Bilinear Logic

By BL3 we shall understand BL2 together with the following rules: if $A \otimes B \to \mathrm{O}$ then $B \otimes A \to \mathrm{O}$, if $\mathrm{I} \to A \oplus B$ then $\mathrm{I} \to B \oplus A$, or, equivalently,

$$\mathrm{O}/A \leftrightarrow A\backslash\mathrm{O}, \qquad \mathrm{I} \div A \leftrightarrow A \div \mathrm{I}.$$

These rules are not independent; according to the second formulation of BL2 they amount to $A^{\perp} \leftrightarrow A^{\top}$, so that we can discard $^{\top}$ altogether. In a sequent calculus presentation, they should be expressed as additional structural rules:

$$\frac{\Gamma\Delta \to}{\Delta\Gamma \to}, \qquad \frac{\to \Phi\Psi}{\to \Psi\Phi}.$$

These rules characterize Yetter's 'cyclic (non-commutative) linear logic'.

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rule

$$\frac{A \to B \quad C \to D}{A \otimes C \to B \otimes D}$$

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Clearly, the following axioms are necessary, in addition to the bifunctoriality of $\otimes$ and $\oplus$:

12 The Algebra of Binary Relations on a Set

We study binary relations on a set X, assuming that the underlying logic is classical. If $R \subseteq X \times X$, we write xRy for $(x, y) \in R$. Aside from the obvious lattice operations, we define the bilinear operations as follows:

aIb	iff	$a = b$
$a(R \otimes S)b$	iff	$\exists_z(aRz \wedge zSb)$
$a(R/S)b$	iff	$\forall_z(bSz \Rightarrow aRz)$
$a(S\backslash R)b$	iff	$\forall_z(zSa \Rightarrow zRb)$
aOb	iff	$a \neq b$
$a(R \oplus S)b$	iff	$\forall_z(aRz \vee zSb)$
$a(R \div S)b$	iff	$\exists_z(aRz \sim bSz)$
$a(S \div R)b$	iff	$\exists_z(zRb \sim zSa)$

where $\sim$ means 'but not'.

We claim that all the rules of BL2 are satisfied, with $\rightarrow$ interpreted as inclusion, here written $\leq$, and give four sample proofs.

13 Cyclic Bilinear Logic

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Peirce’s `Note B’ as a much earlier relational presentation…

‘Note B’ (1883)

We now come to the combination of relatives. Of these, we denote two by special symbols; namely, we write

lb for lover of a benefactor,

and

$l \dagger b$ for lover of everything but benefactors.

The former is called a particular combination, because it implies the *existence* of something *loved by* its relate and a *benefactor of* its correlate. The second combination is said to be *universal*, because it implies the *non-existence* of anything except what is either loved by its relate or a benefactor of its correlate. The combination lb is called a relative product, $l \dagger b$ a relative sum. The

The dual of relational composition…

‘Note B’ (1883)

Relative addition and multiplication are subject to the associative law. That is,

$$l \dagger (b \dagger s) = (l \dagger b) \dagger s,$$

$$l\,(bs) = (lb)\,s.$$

Two formulæ so constantly used that hardly anything can be done without them are

$$l\,(b \dagger s) \prec lb \dagger s,$$

$$(l \dagger b)\,s \prec l \dagger bs.$$

The former asserts that whatever is lover of an object that is benefactor of everything but a servant, stands to everything but servants in the relation of lover of a benefactor. The latter asserts that whatever stands to

…with the linear distributive laws…

'Note B' (1883)

To these partially correspond the following pair of highly important formulæ :—

$$1 \prec l \dagger \check{\bar{l}} \qquad l\check{\bar{l}} \prec n.$$

The logic of relatives is highly multiform ; it is characterized by innumerable immediate inferences, and by various distinct conclusions from the same sets of premises. An example of the first character is afforded by Mr. Mitchell's *F* following from *F*. As an instance

…the linear negation laws…

'Note B' (1883)

from subject to predicate as to make the subject 1. Thus, if we have given $l \prec b$, we may relatively add $\bar{\breve{l}}$ to both sides; whereupon we have

$$1 \prec l \dagger \bar{\breve{l}} \prec b \dagger \bar{\breve{l}}.$$

Every proposition will then be in one of the forms

$$1 \prec b \dagger l \qquad 1 \prec b\,l.$$

With a proposition of the form $1 \prec b \dagger l$, we have the right (1) to transpose the terms, and (2) to convert the terms. Thus, the following are equivalent:—

$$1 \prec b \dagger l$$

$$1 \prec l \dagger b \qquad 1 \prec \breve{b} \dagger \breve{l}$$

$$1 \prec \breve{l} \dagger \breve{b}.$$

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'Note B' (1883)

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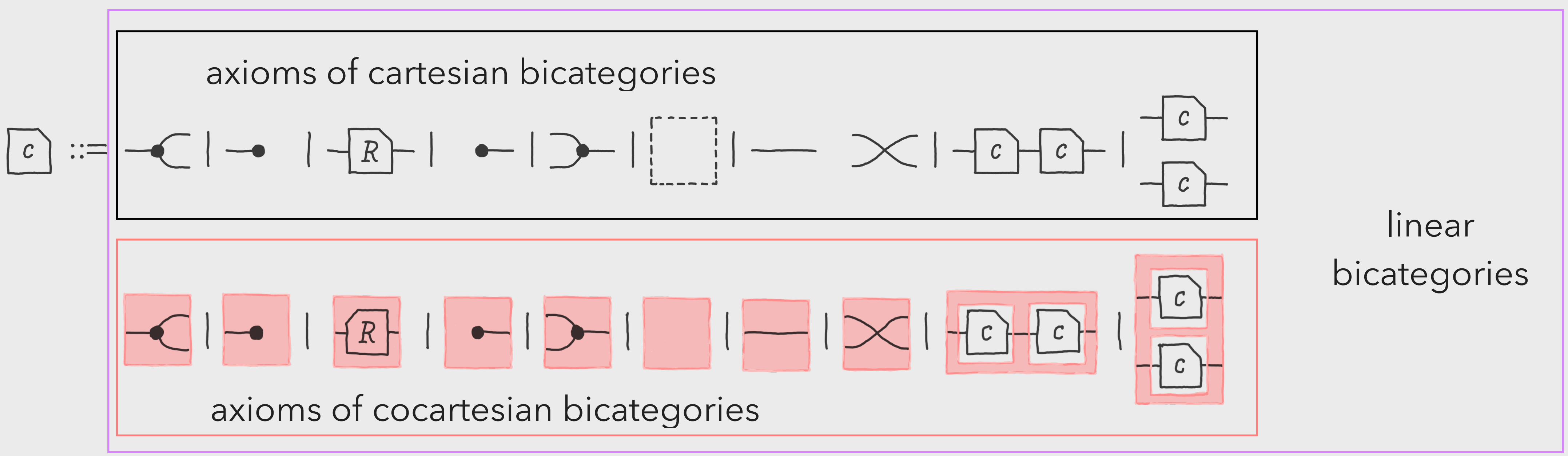
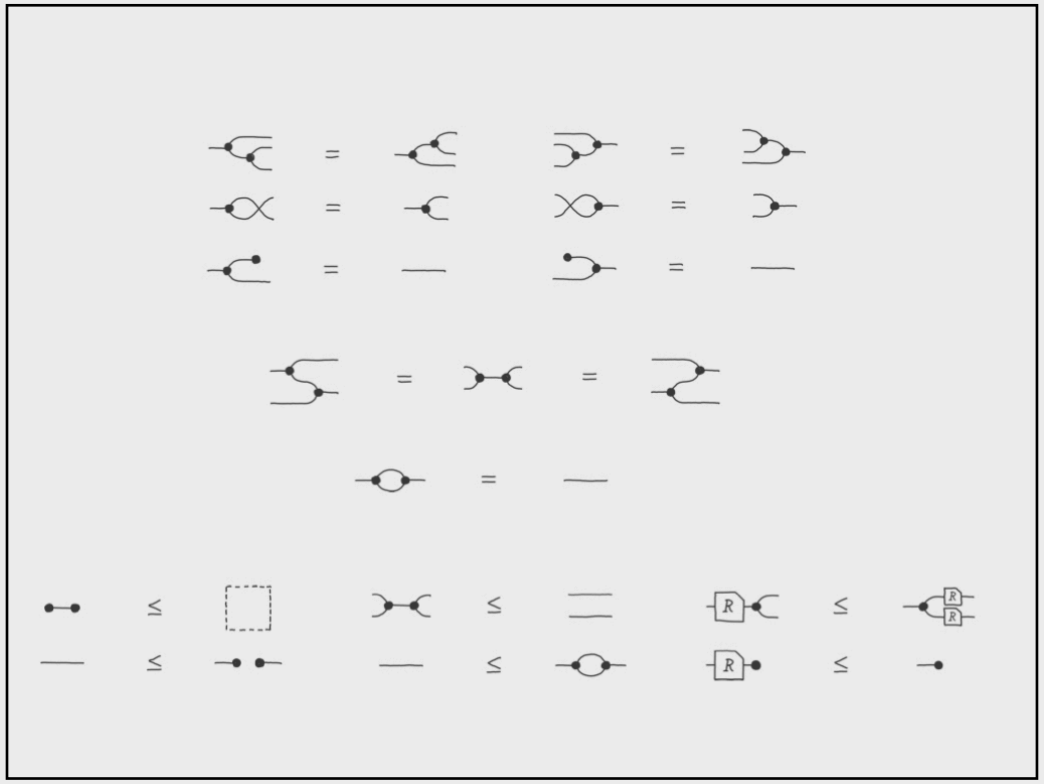
…and the cyclicity condition…

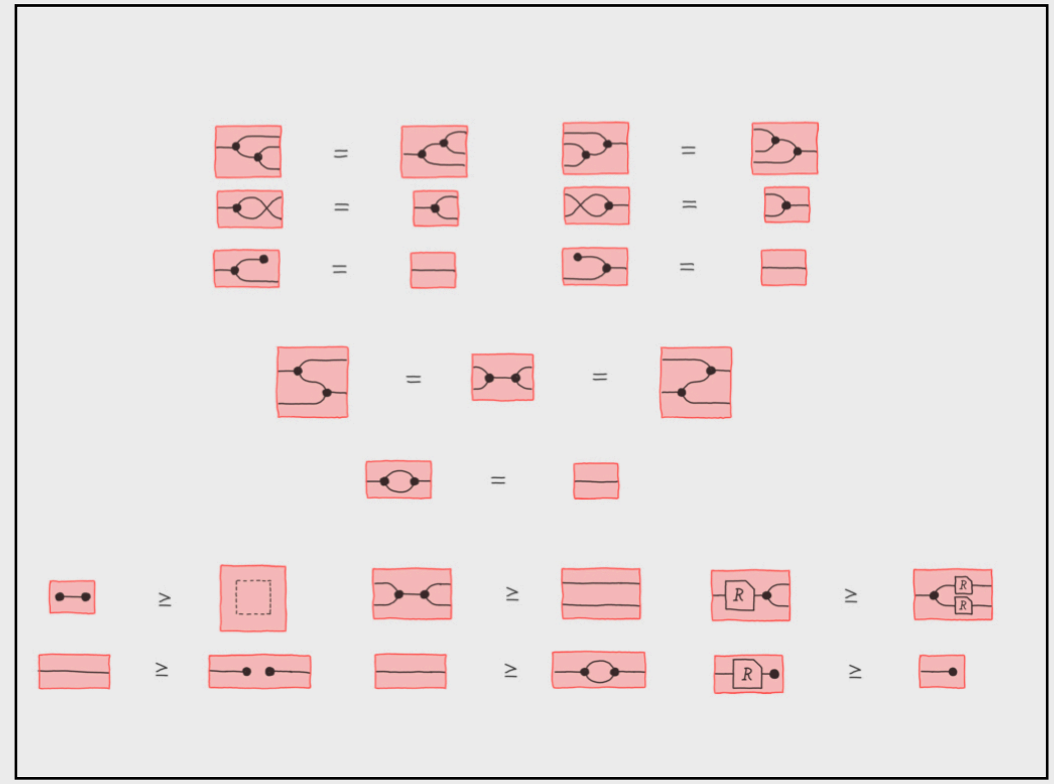
The Neo-Peircean Calculus of Relations



What are the graphical rewrites, i.e. the inference rules?



Axioms of Cartesian bicategories



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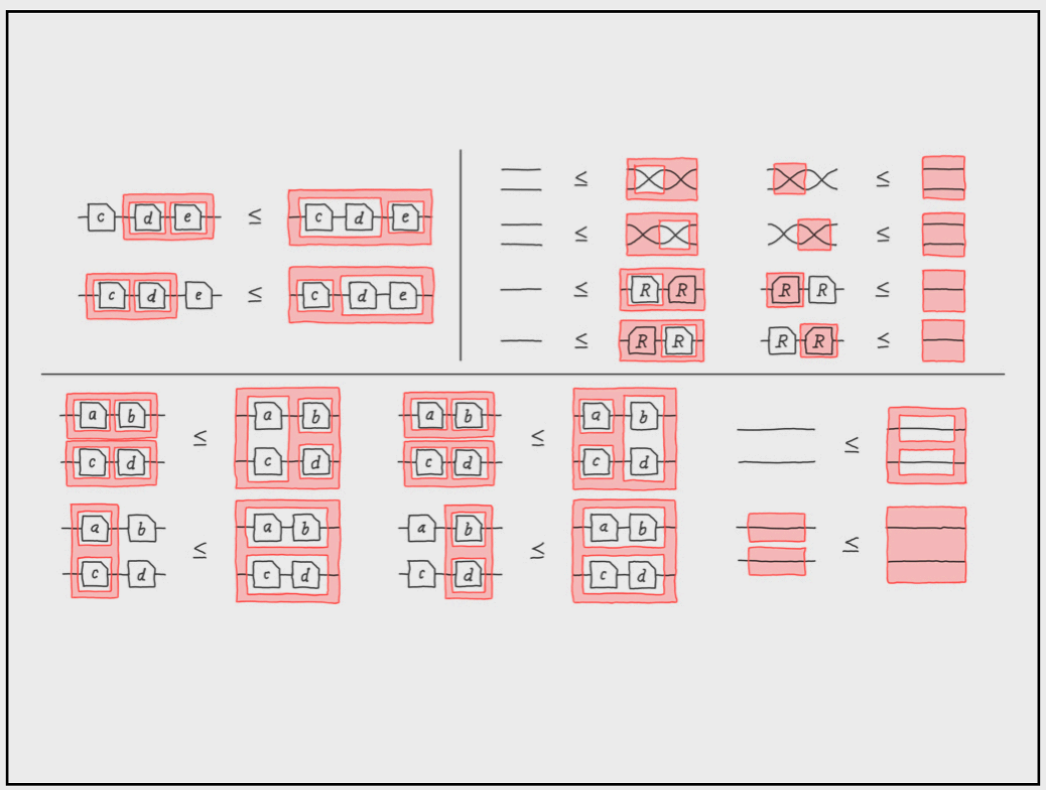
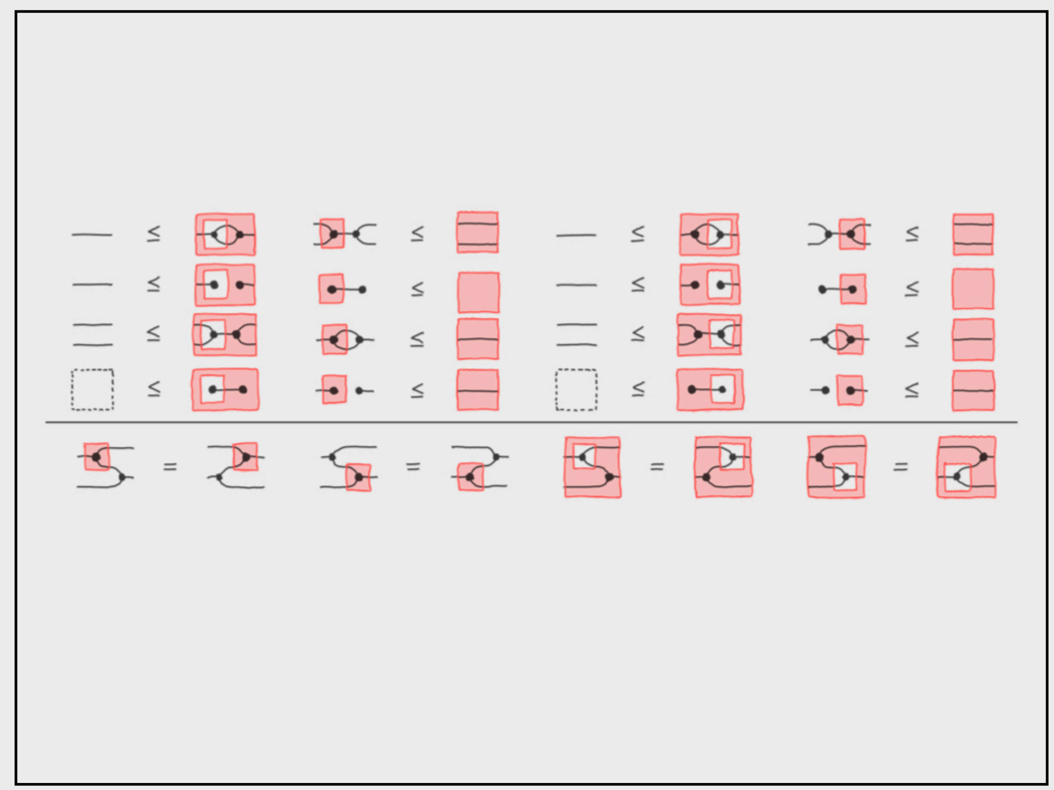
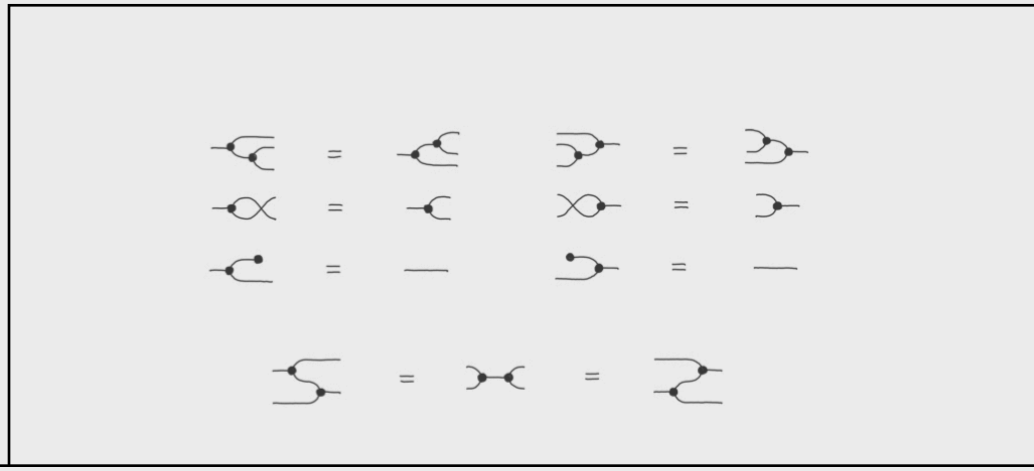
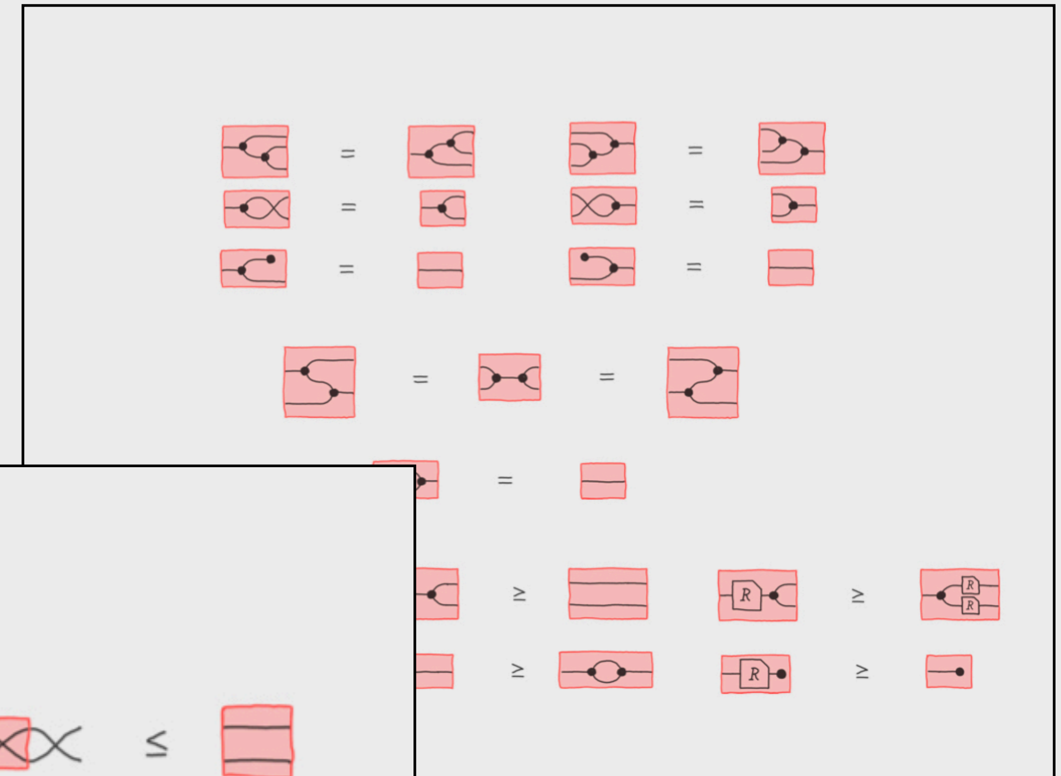


Axioms of closed symmetric monoidal linear bicategories

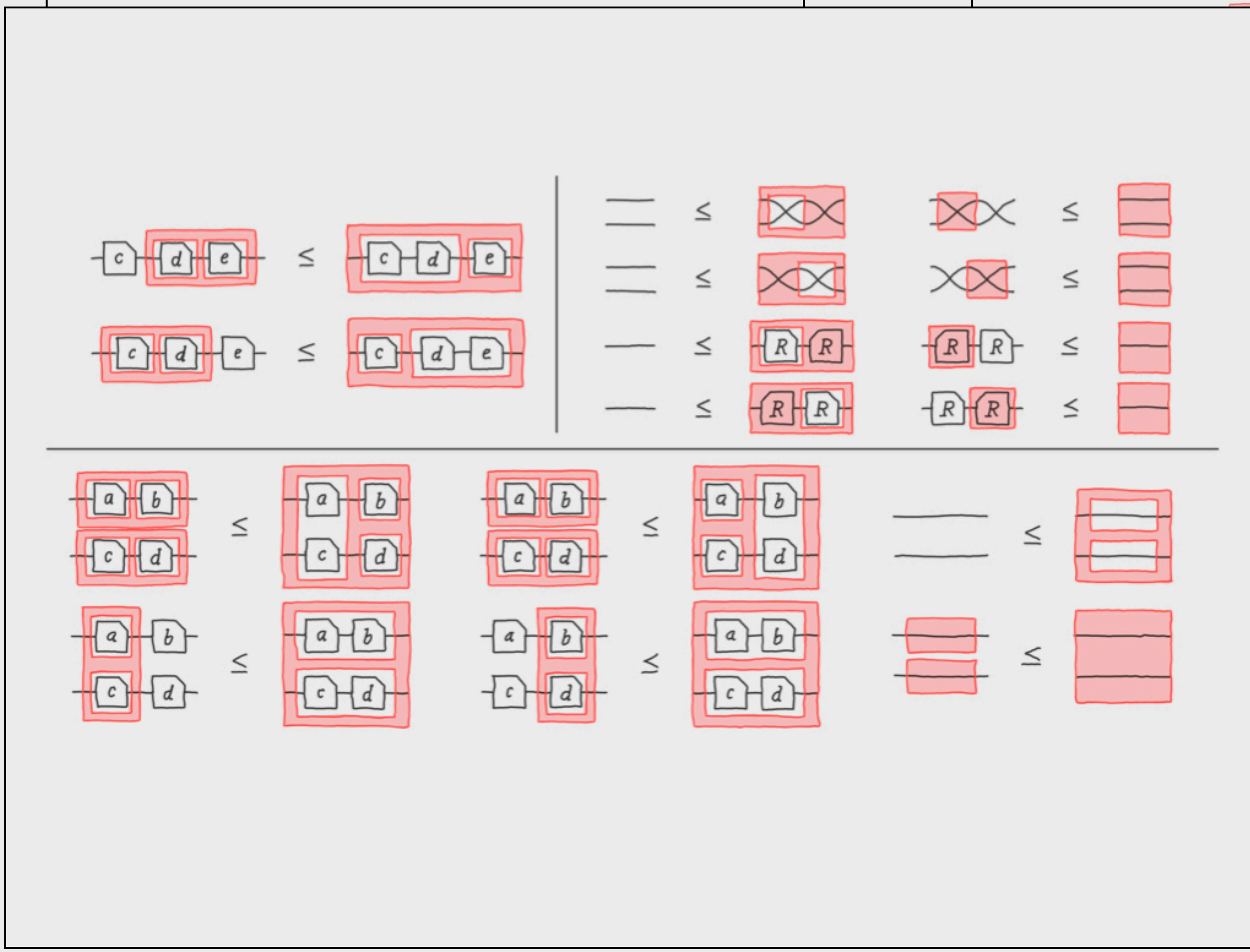


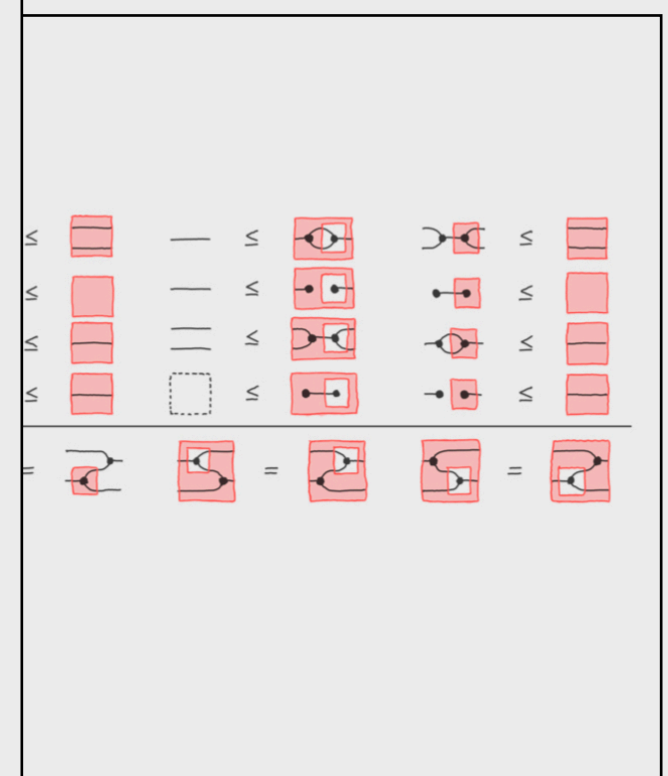
Additional axioms of NPR





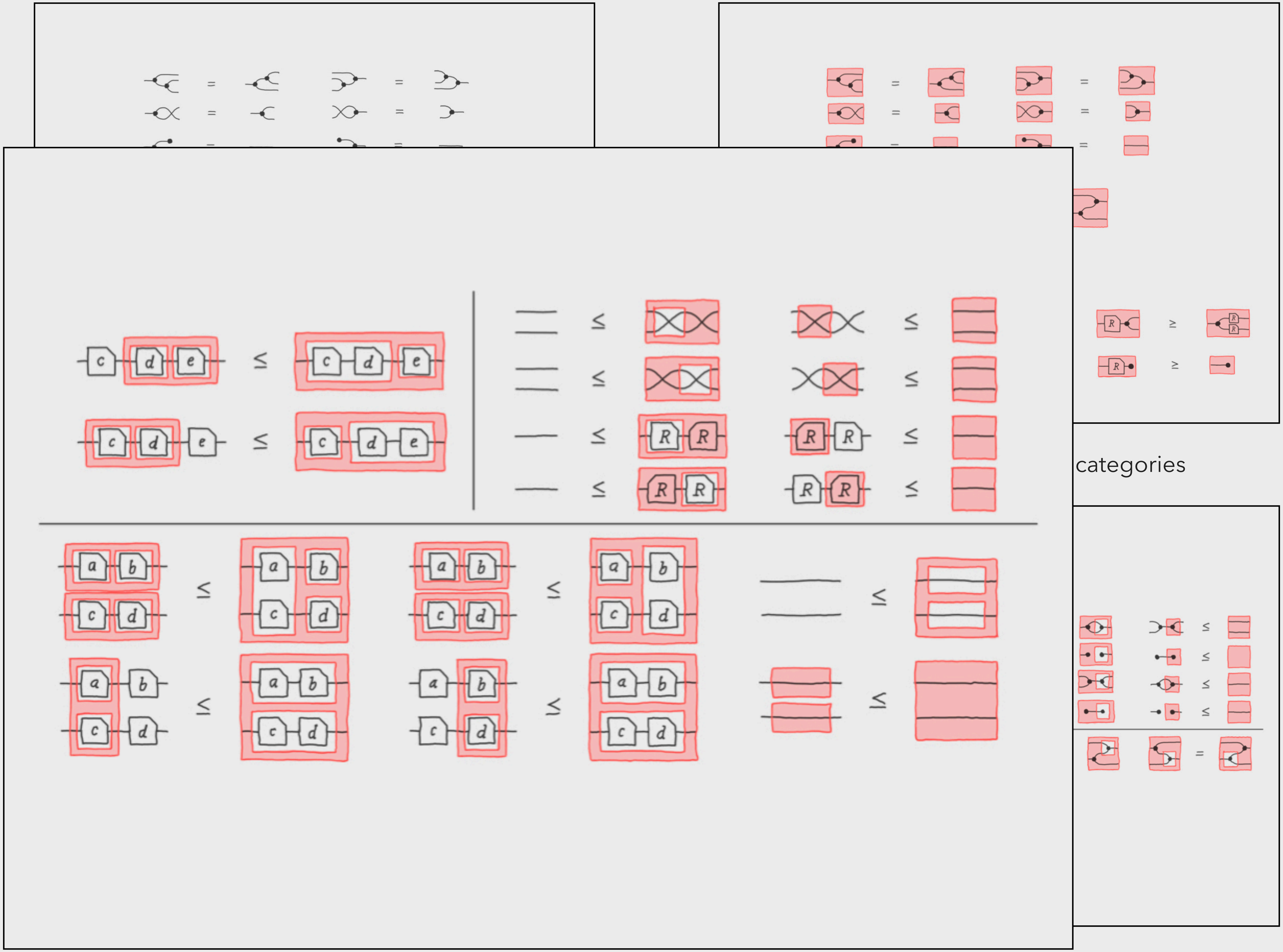
Cocartesian bicategories





Axioms of closed symmetric monoidal linear bicategories

Additional axioms of NPR



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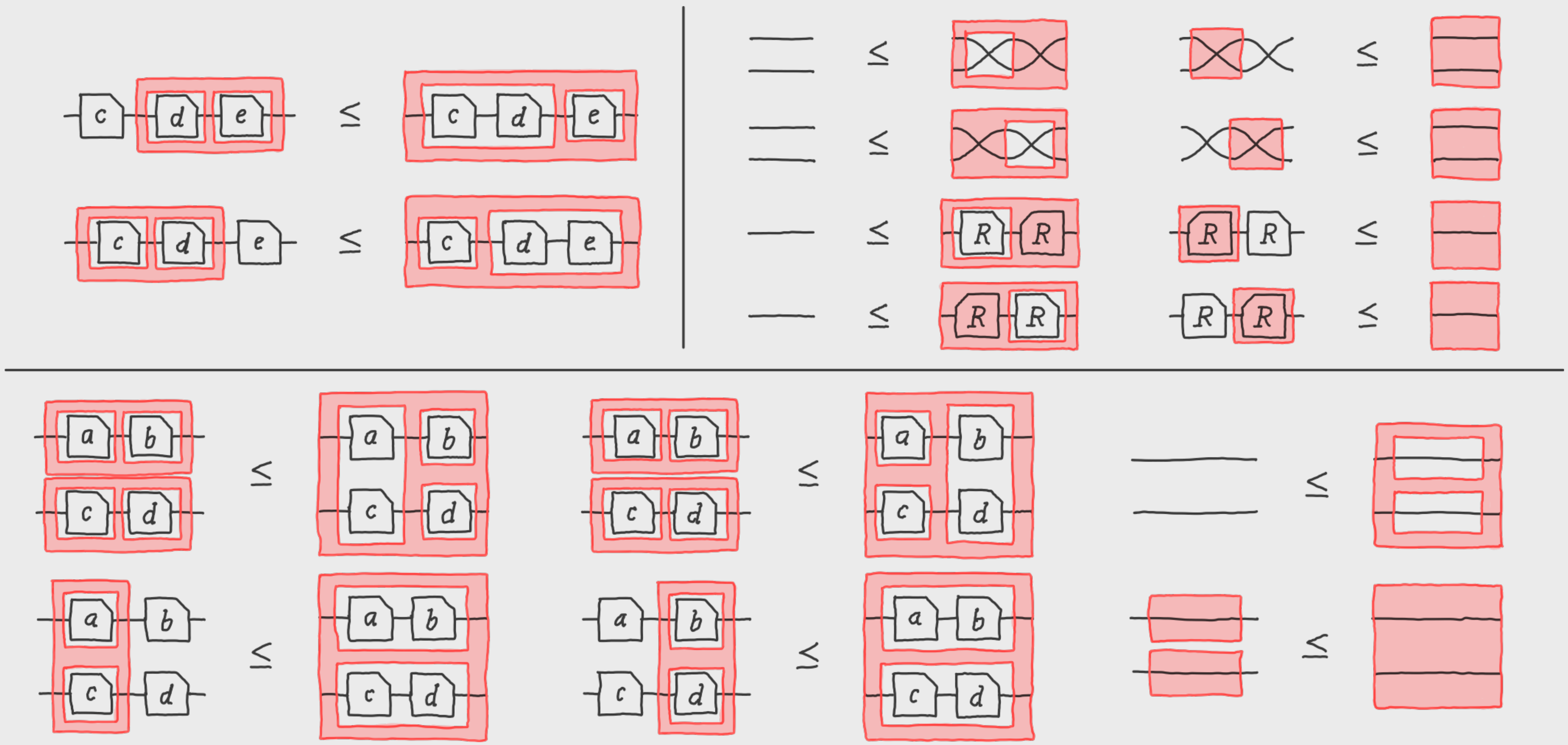

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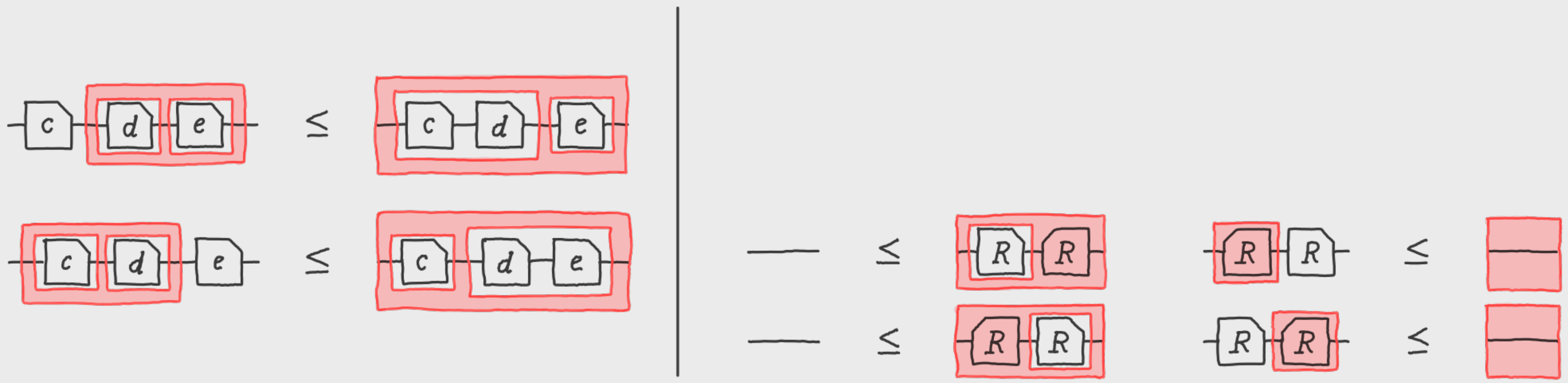

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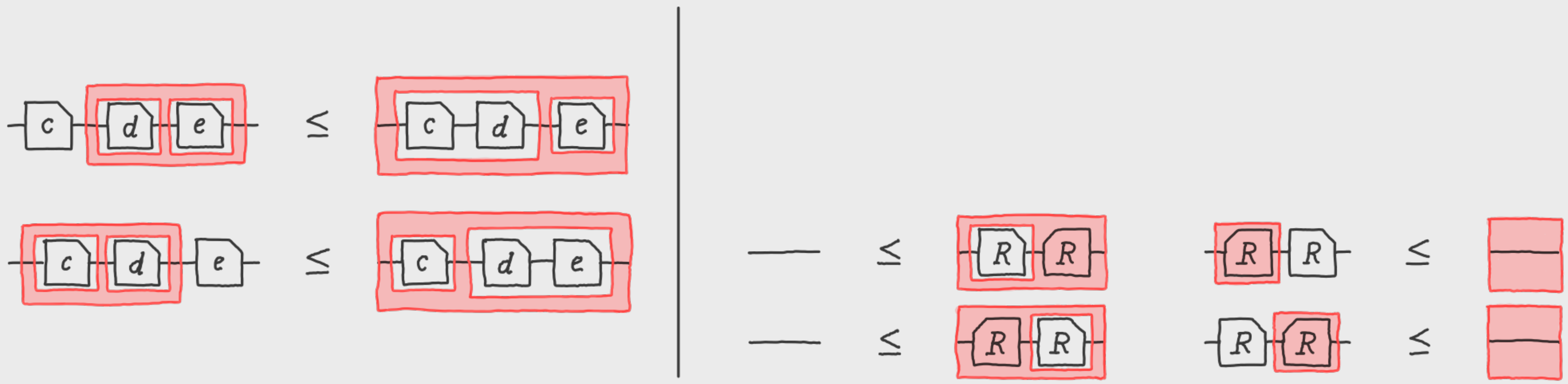
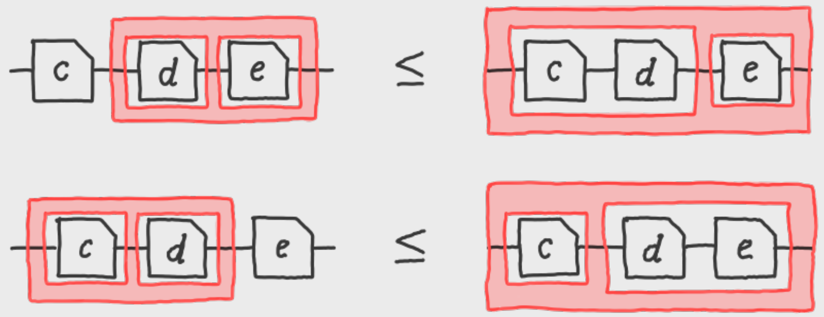


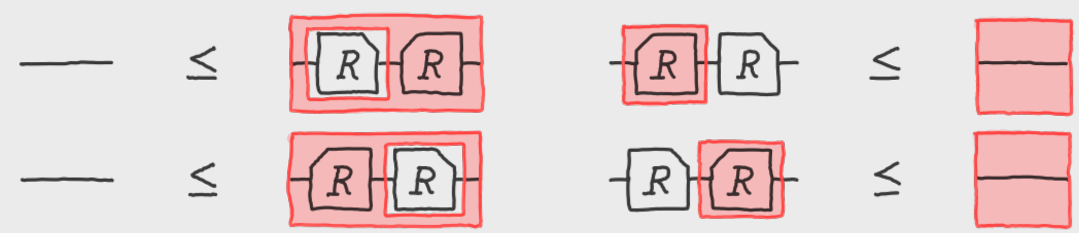
Left and right linear distributivity

Linear negation laws

Axioms of closed symmetric monoidal linear bicategories

$[\ c;(d \dagger e) \leq (c;d) \dagger e\ ]$



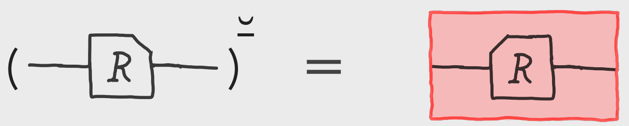


$[\ I \leq R \multimap R\ ]$ $[\ R;R^{\perp} \leq O\ ]$

Left and right linear distributivity

Linear negation laws

Axioms of closed symmetric monoidal linear bicategories



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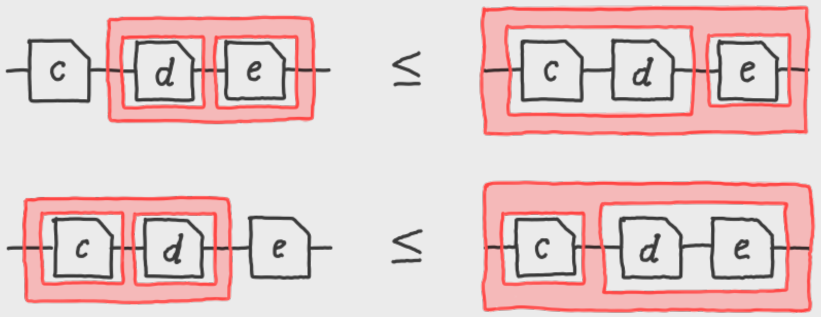

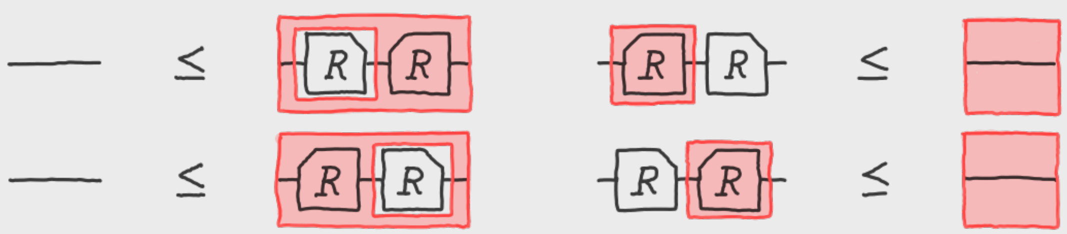

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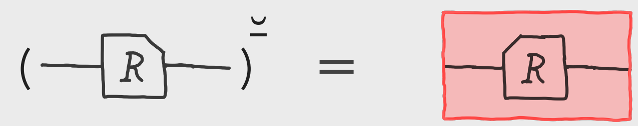

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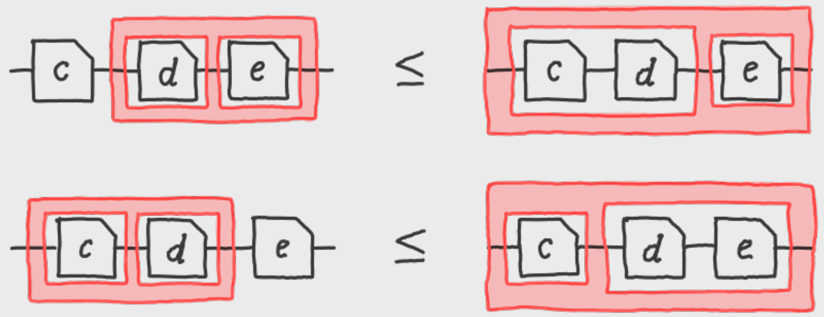

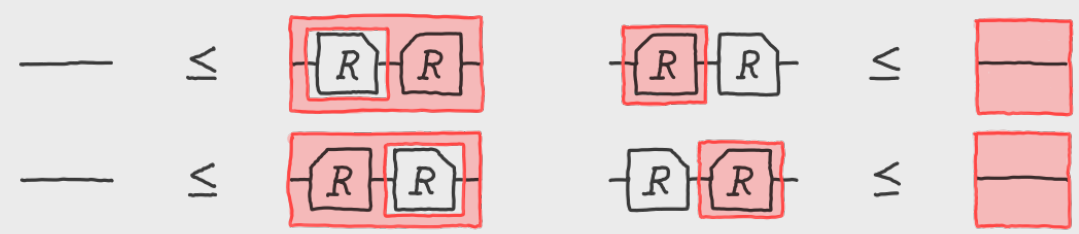

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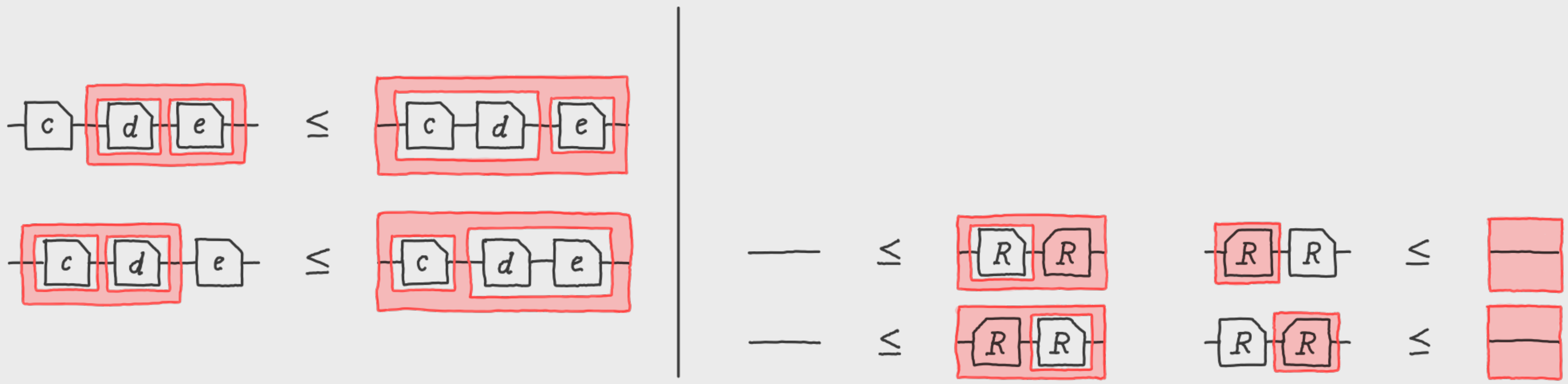
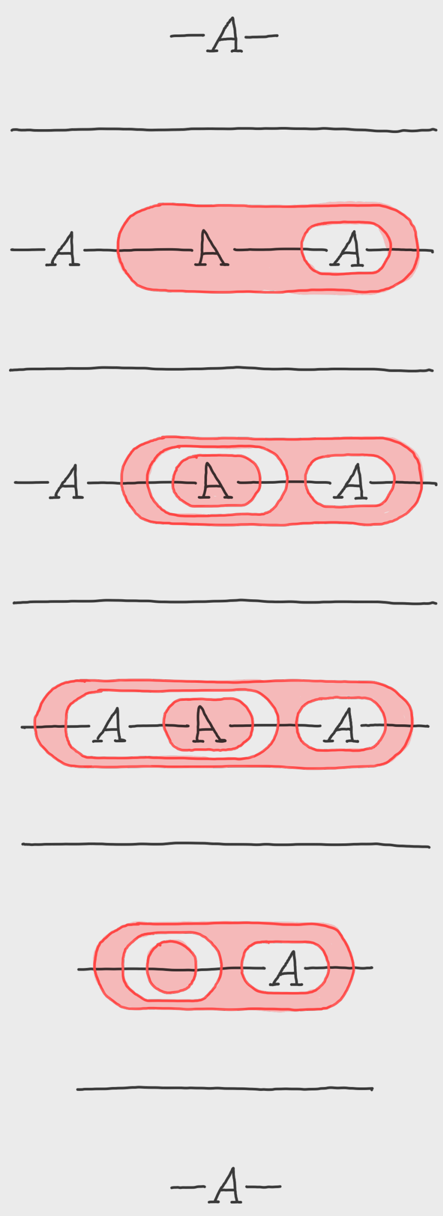
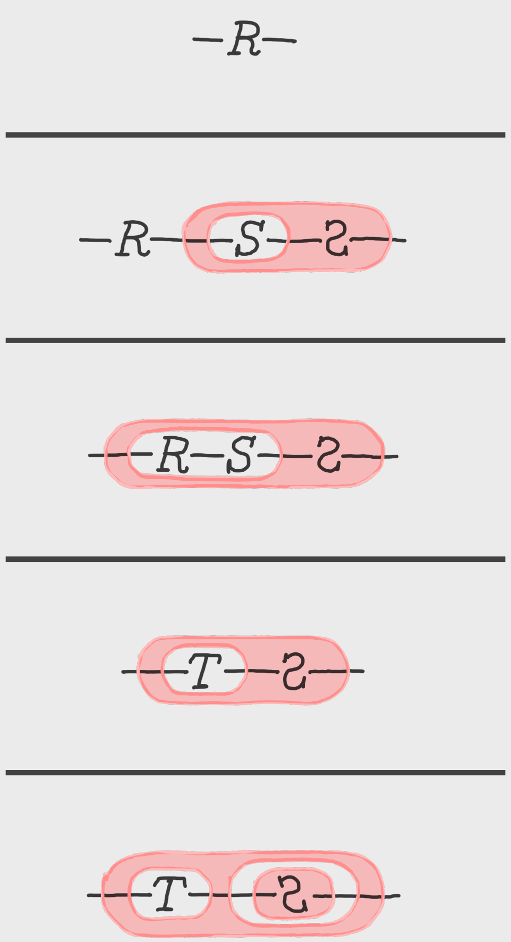


Axioms of closed symmetric monoidal linear bicategories

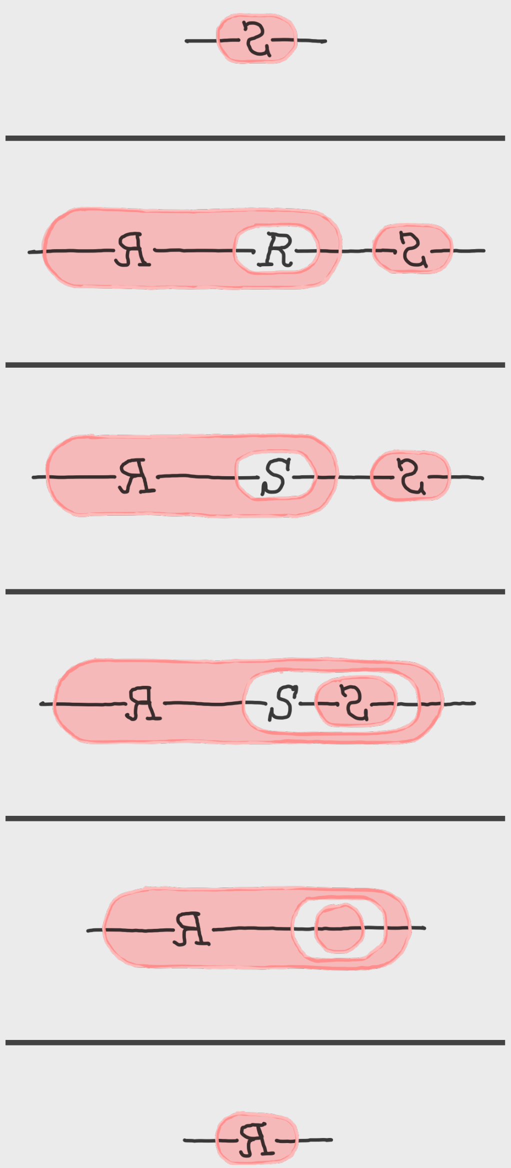
Example Derivation:



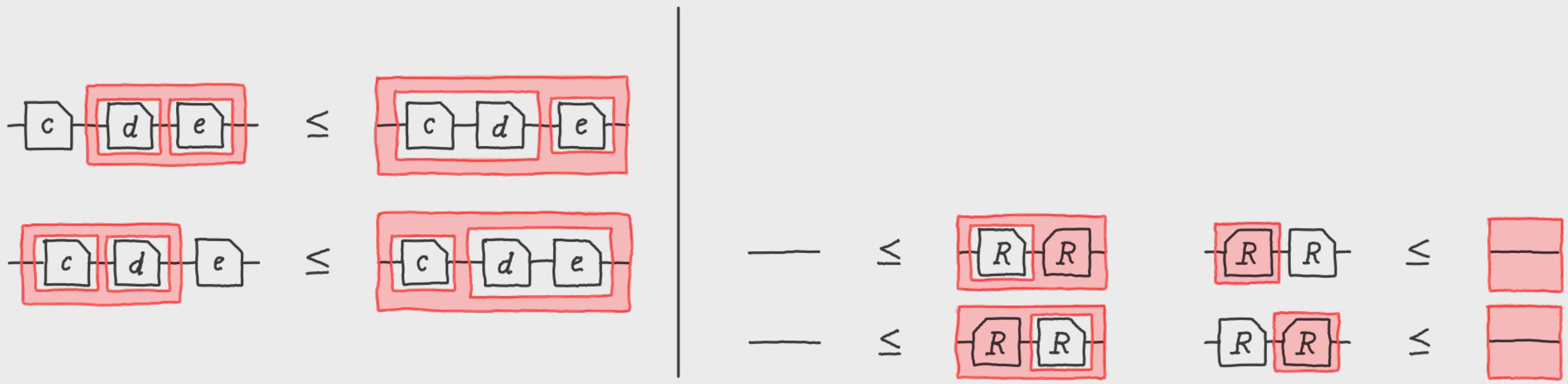
Example Derivation (more):



If $R;S \leq T$ then $R \leq T \dagger S^{\perp}$



If $R \leq S$ then $S^{\perp} \leq R^{\perp}$



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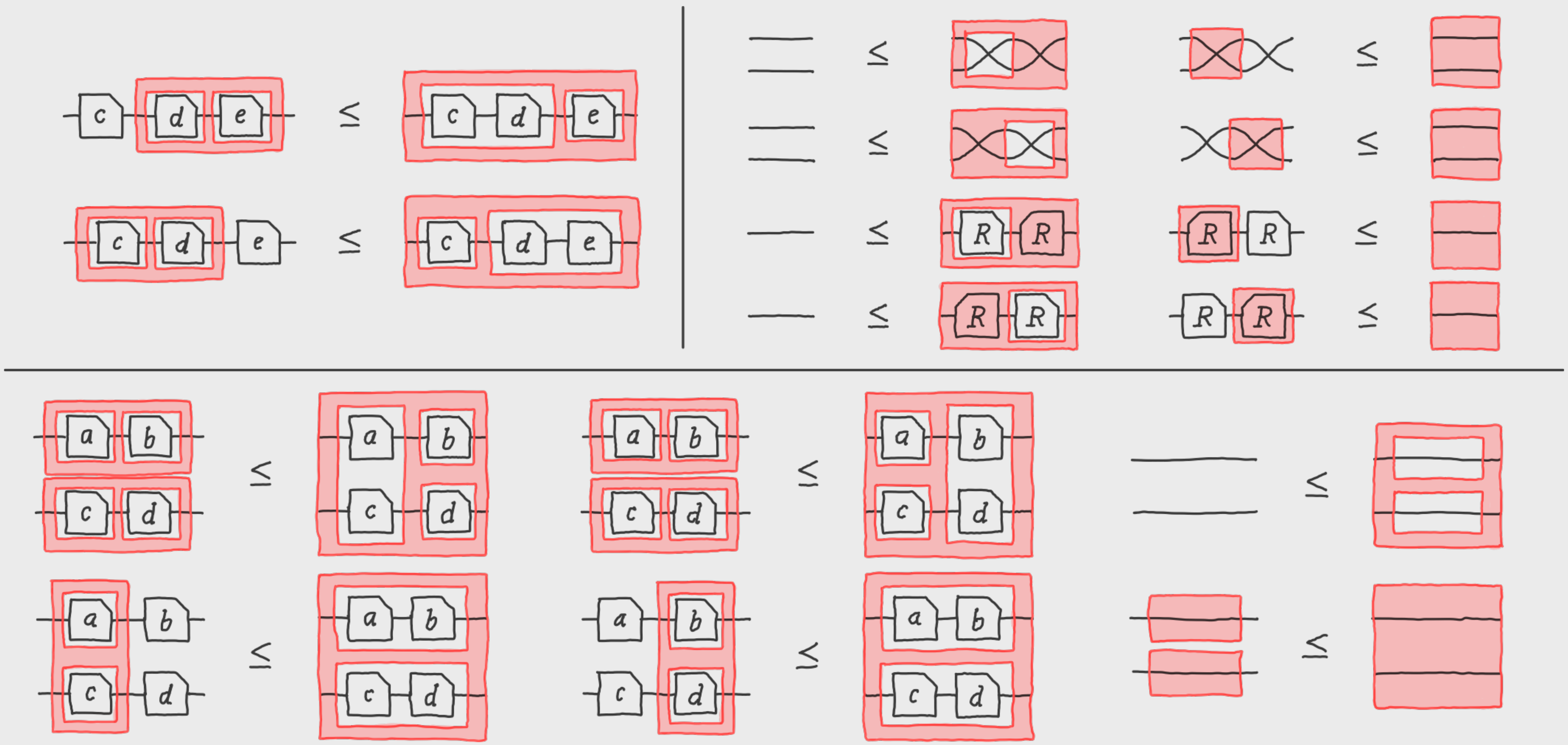

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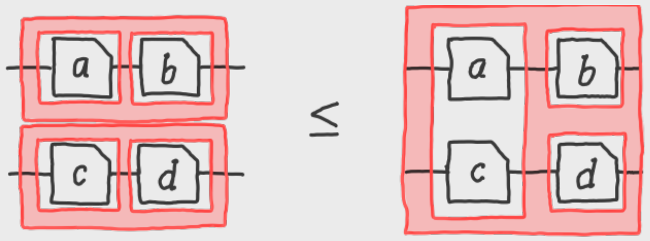

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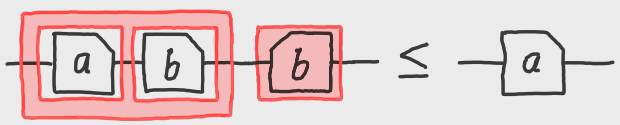
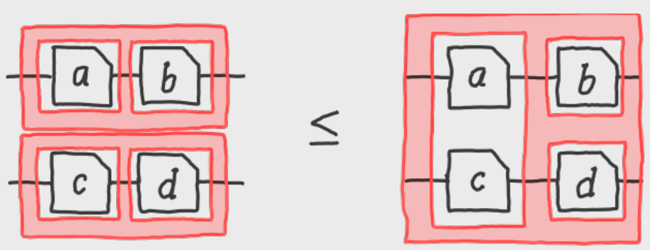
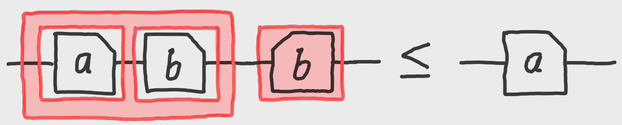
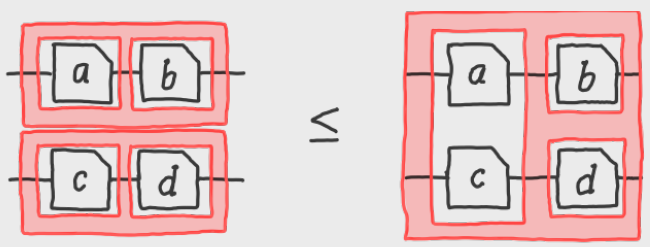




Axioms of closed symmetric monoidal linear bicategories

$[\ (a \dagger b); b^{\perp} \leq a]$





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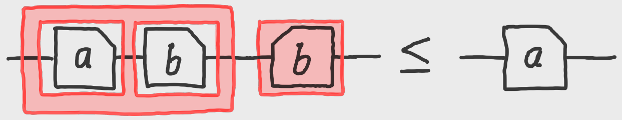

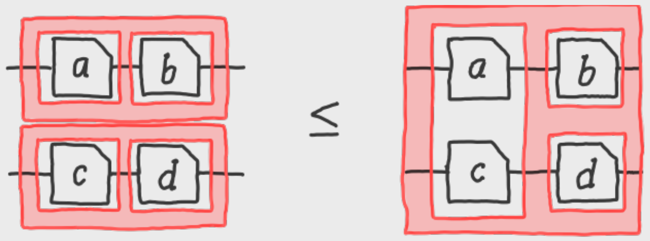

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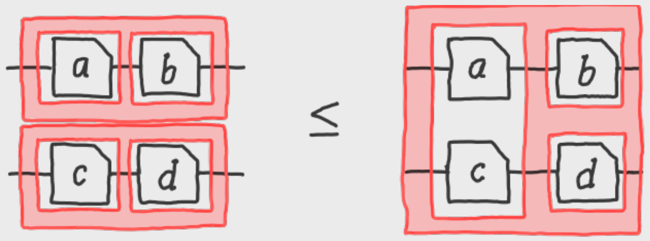

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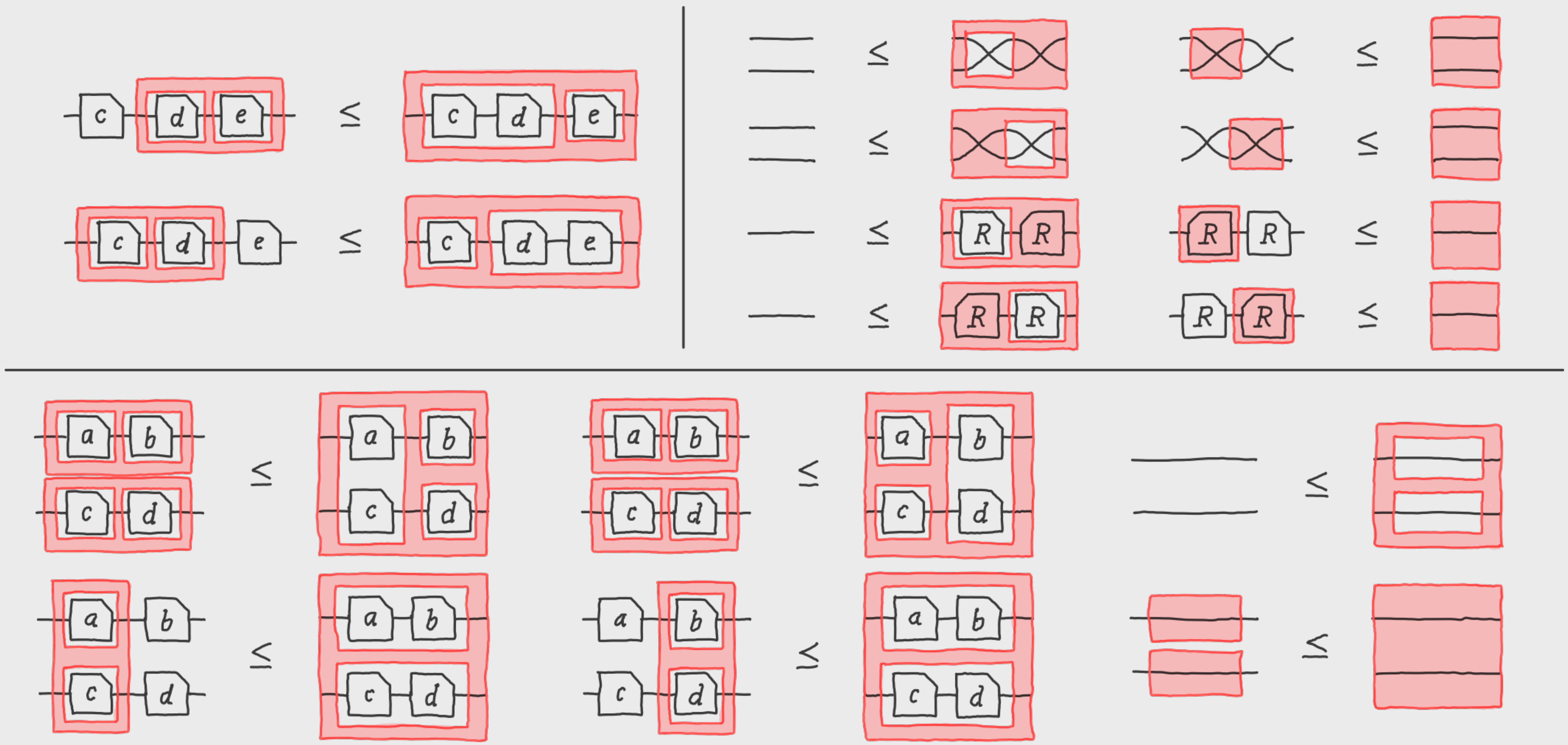
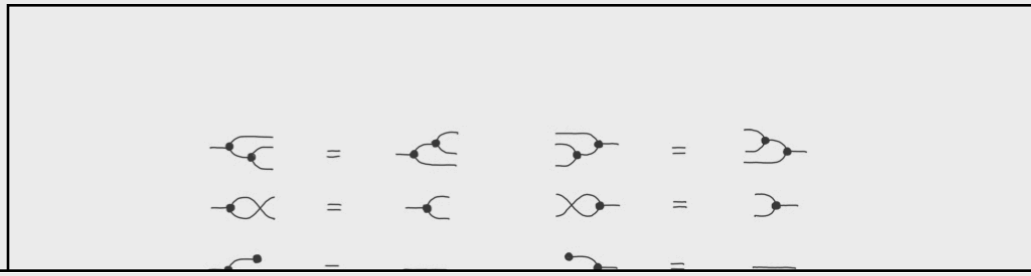
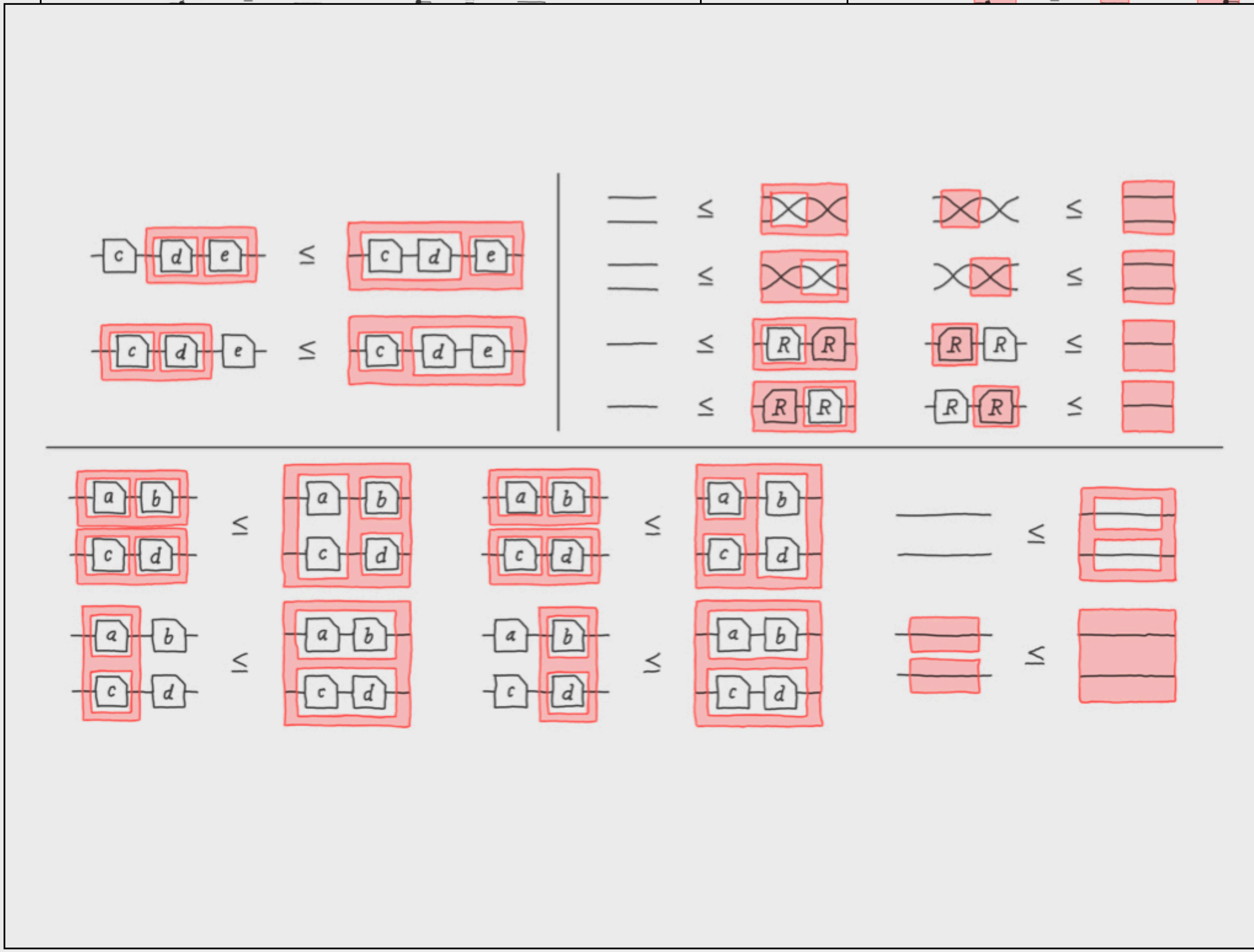
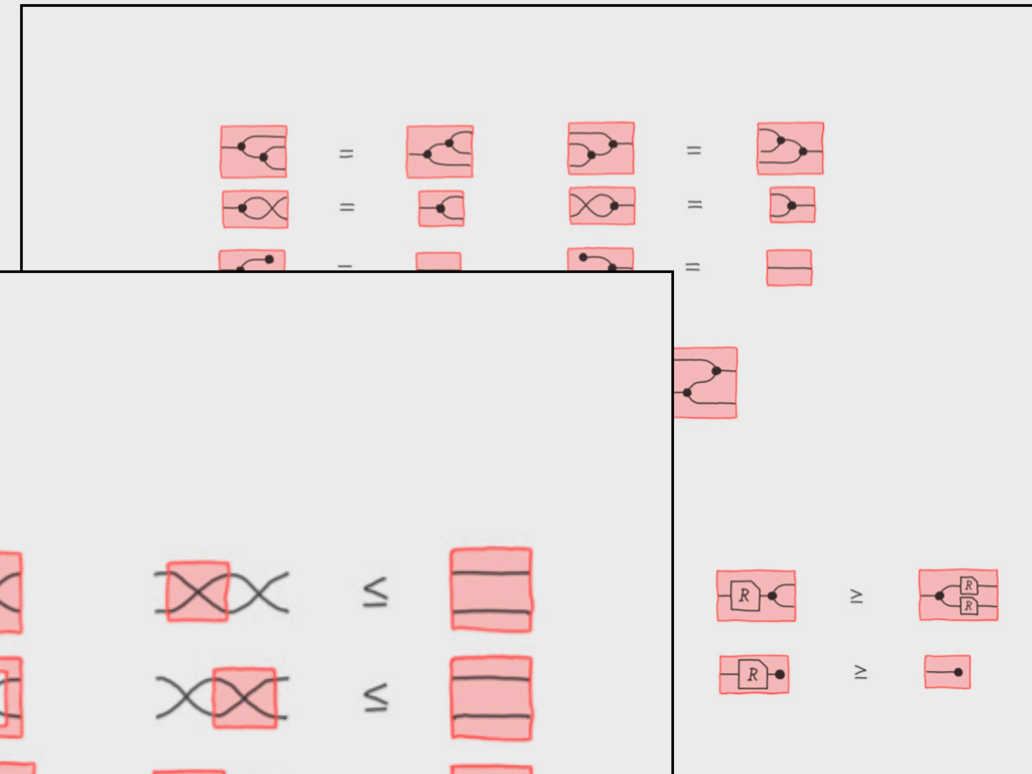


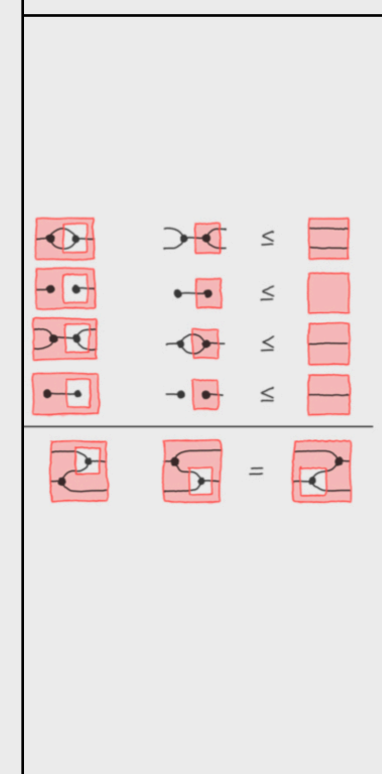
Axioms of closed symmetric monoidal linear bicategories





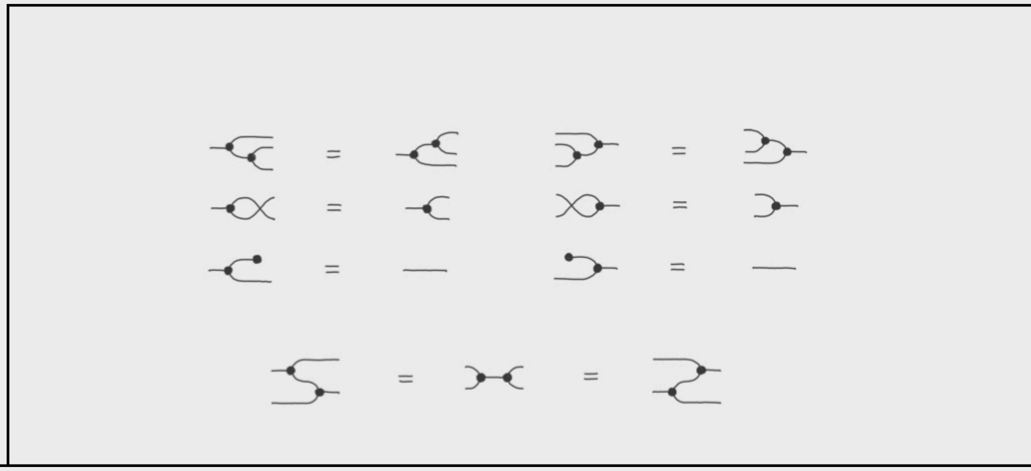


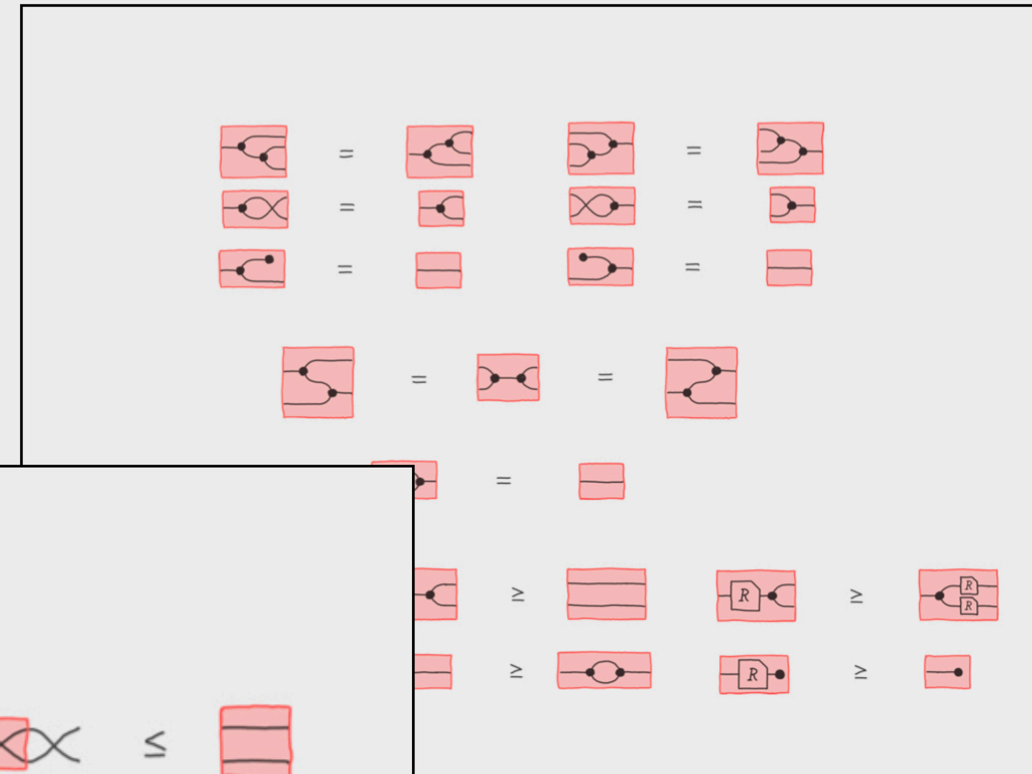
categories



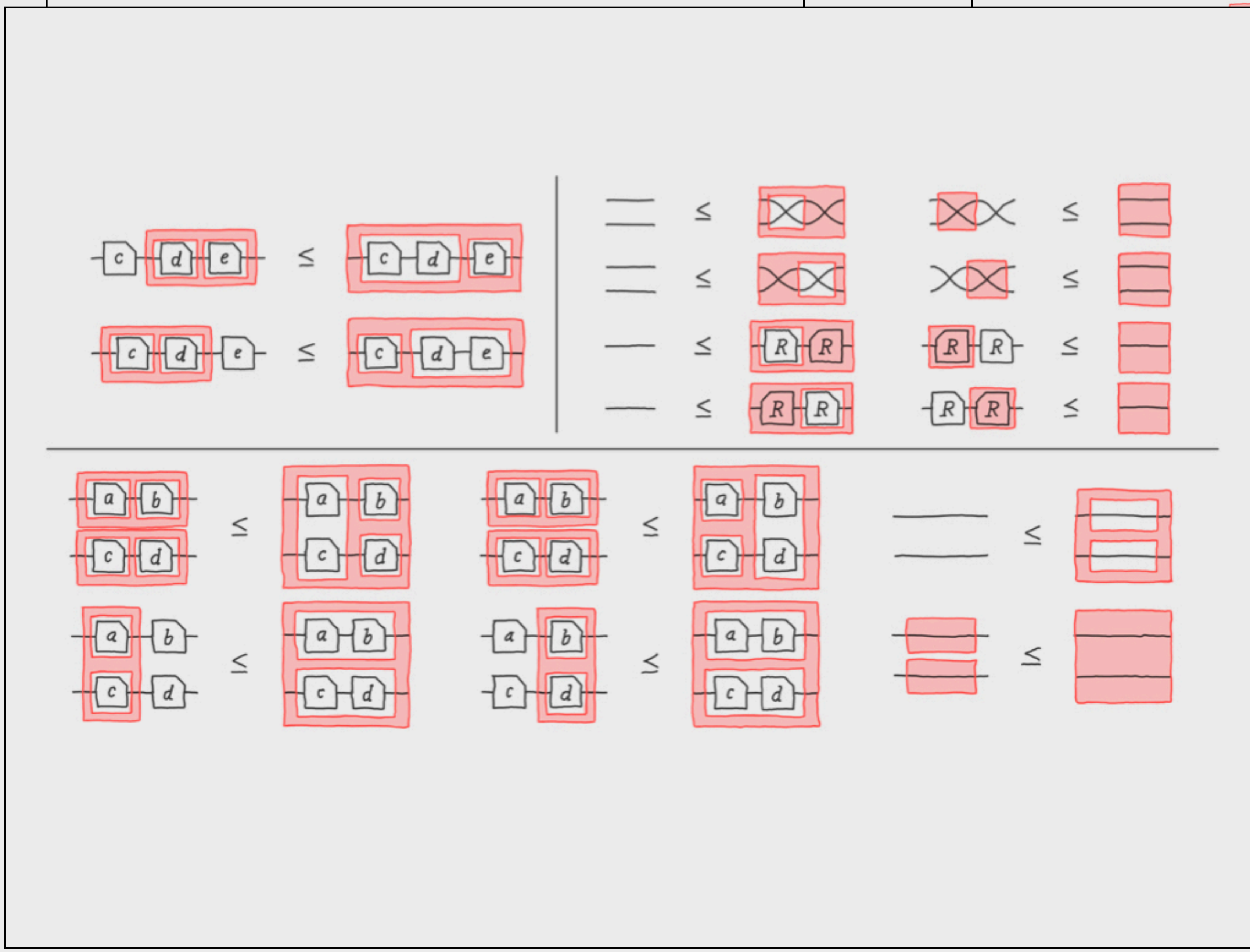
Axioms of closed symmetric monoidal linear bicategories

Additional axioms of NPR





Cocartesian bicategories



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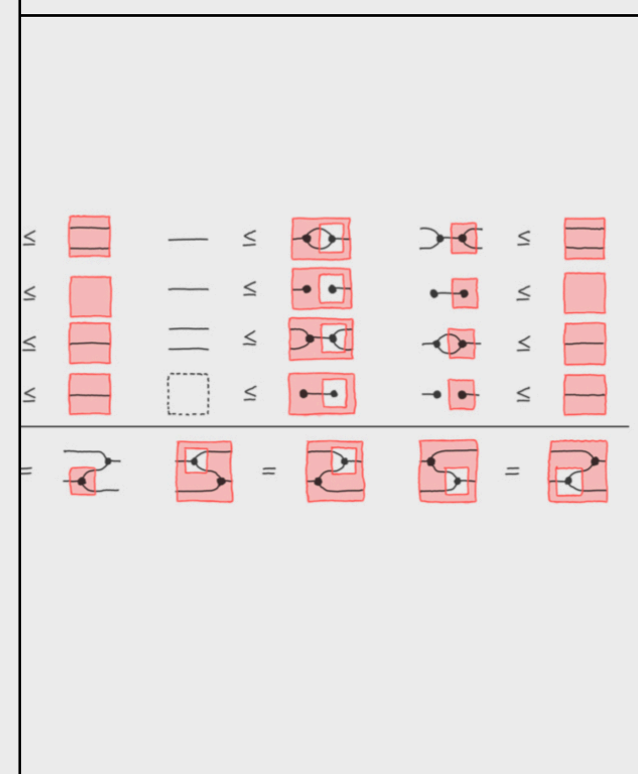

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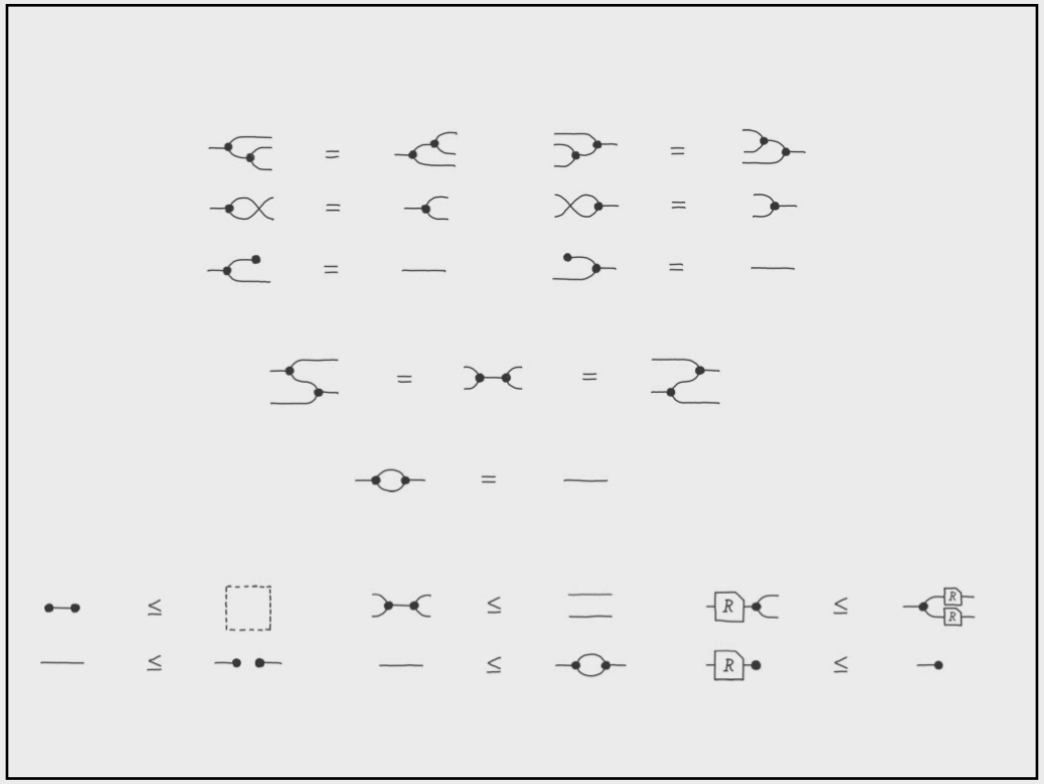
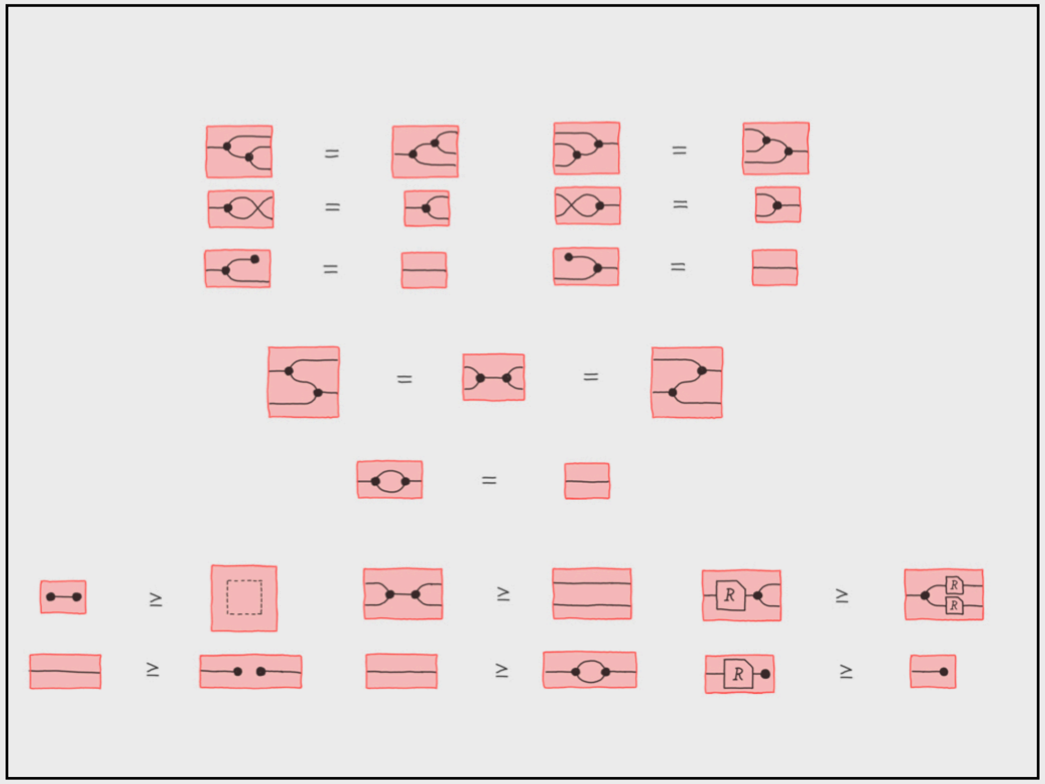


Axioms of Cartesian bicategories



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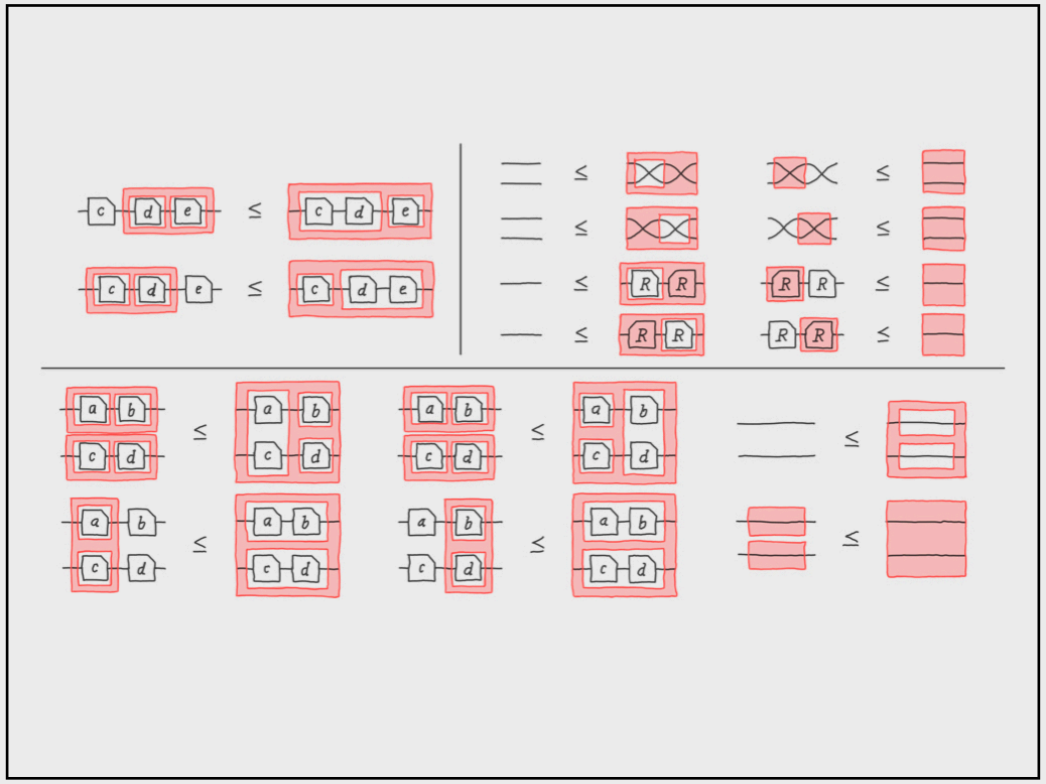

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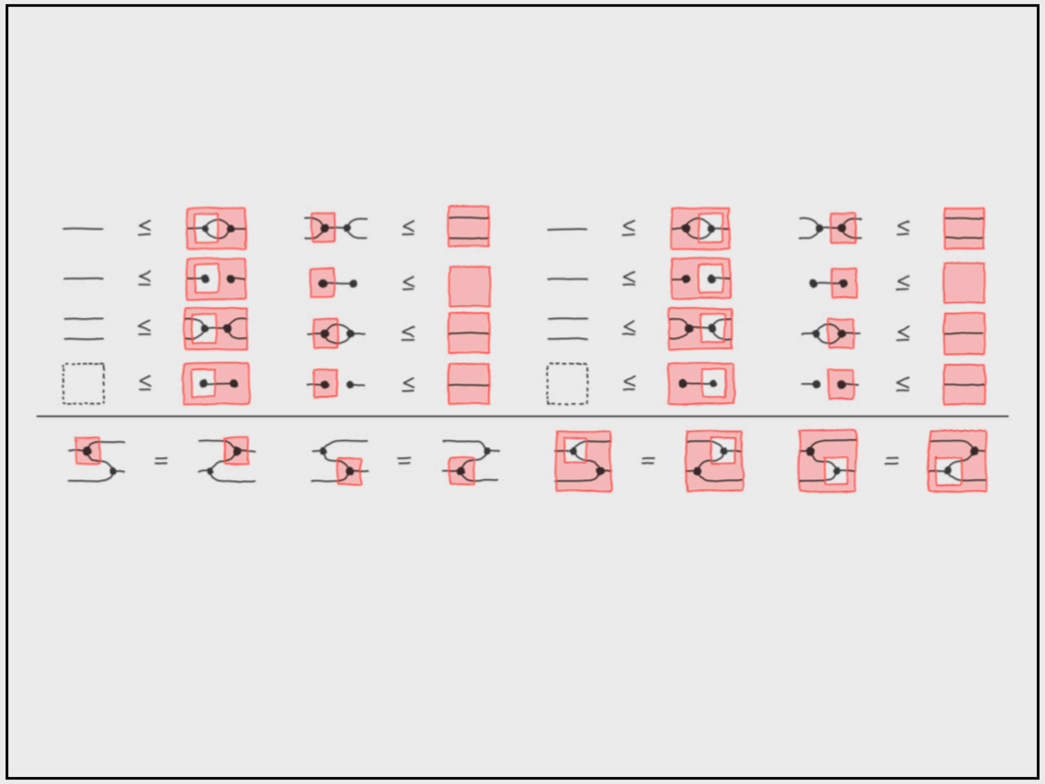

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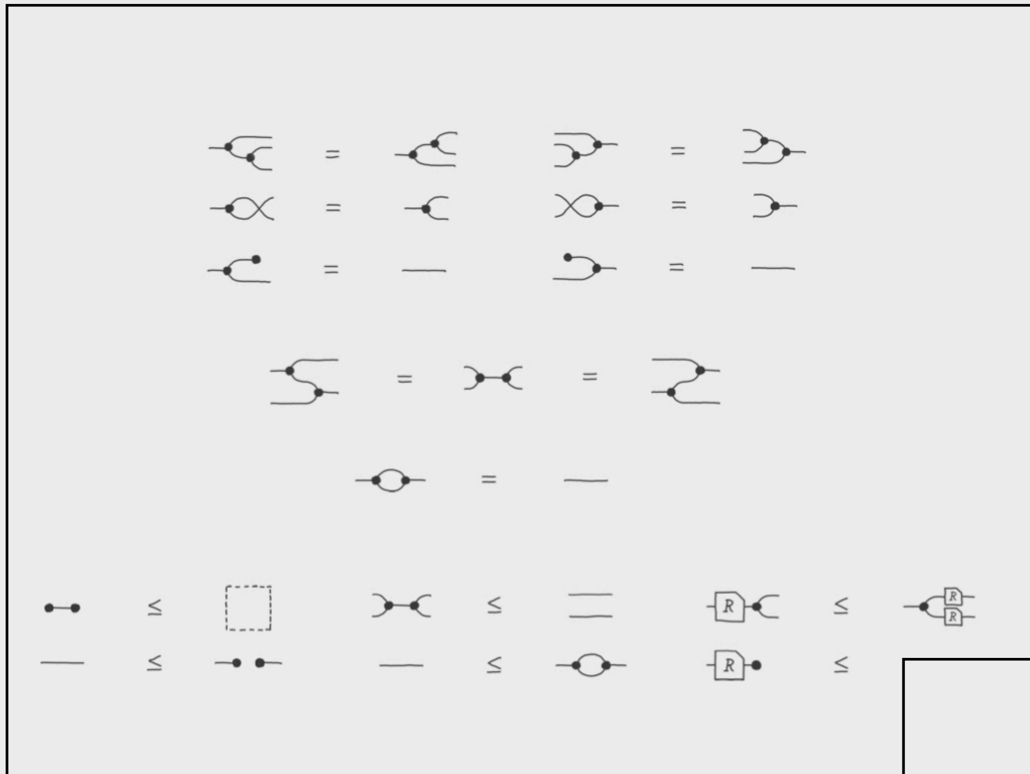

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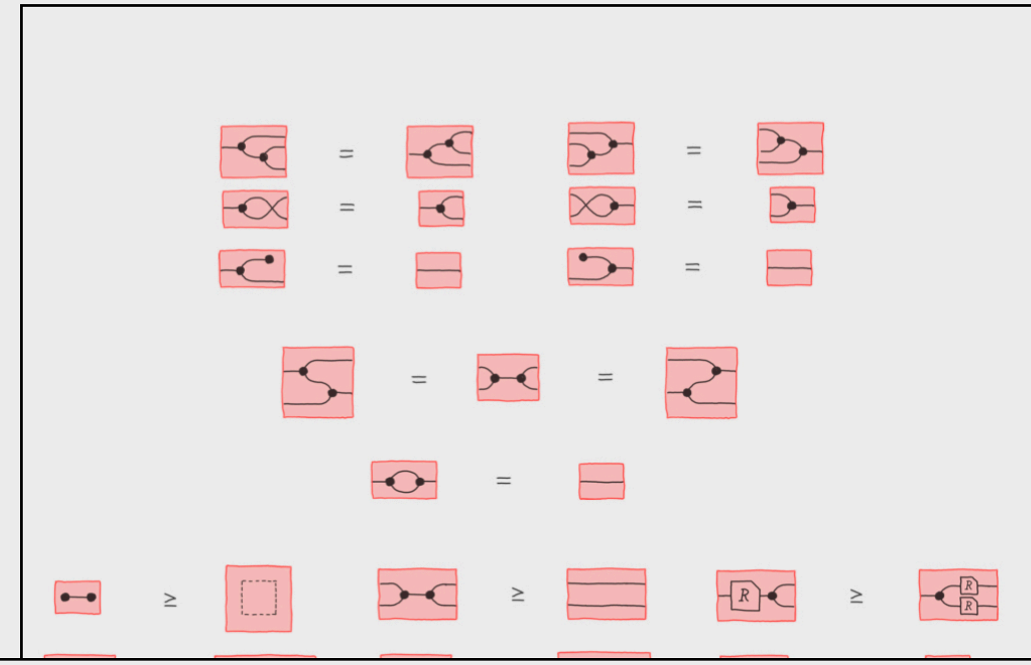

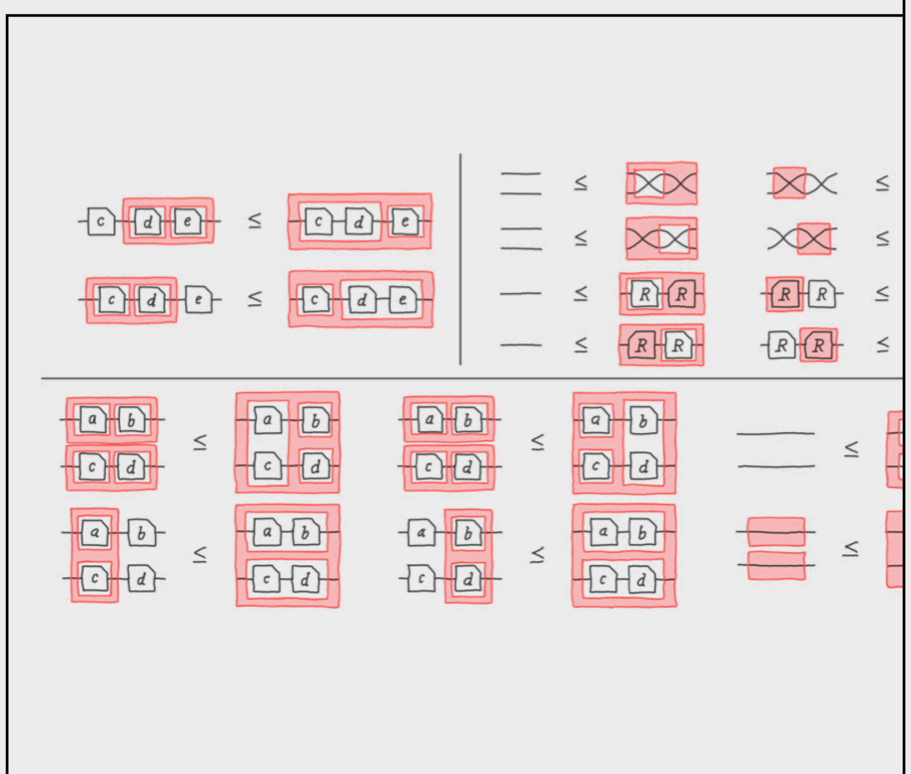

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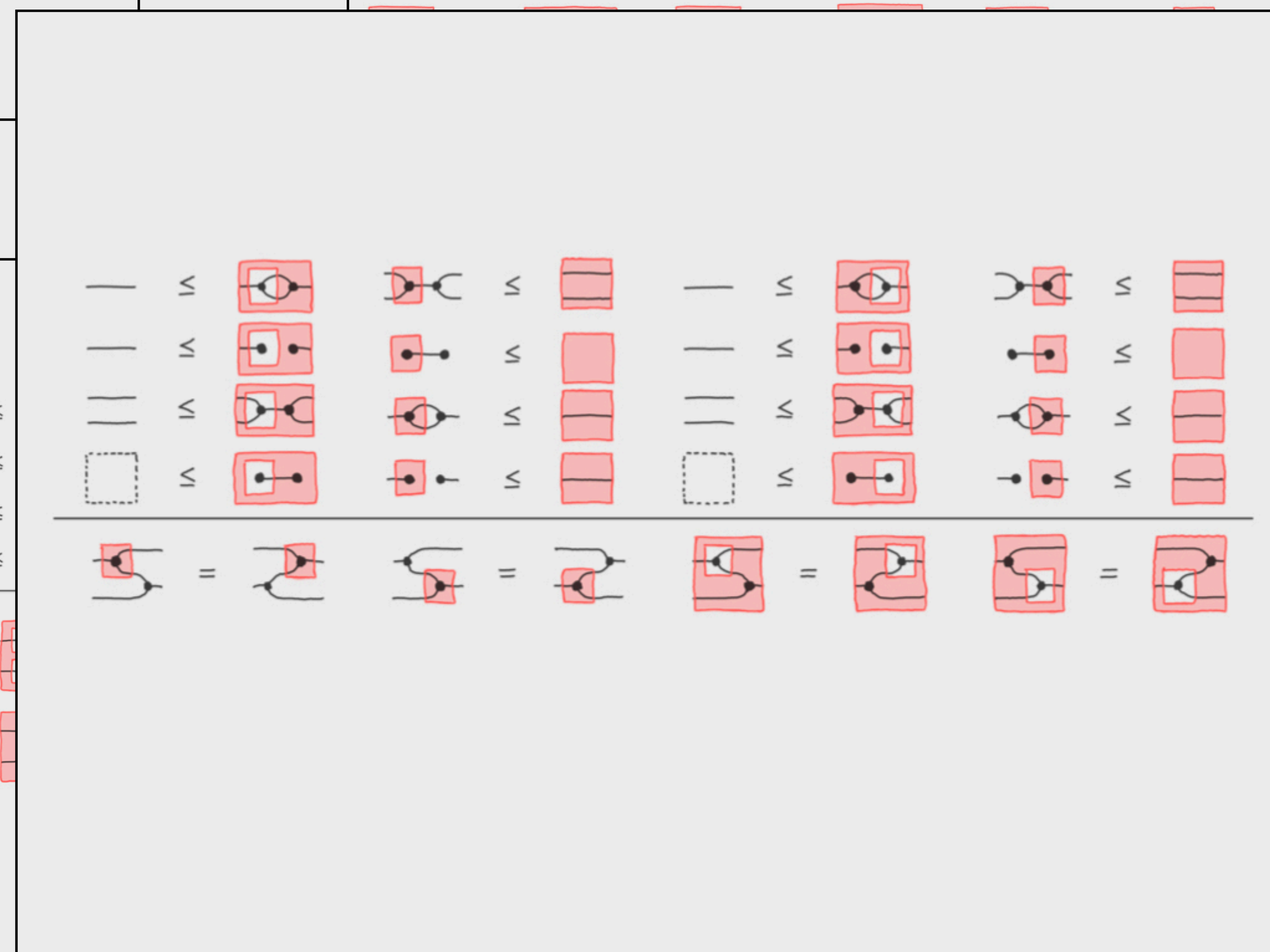

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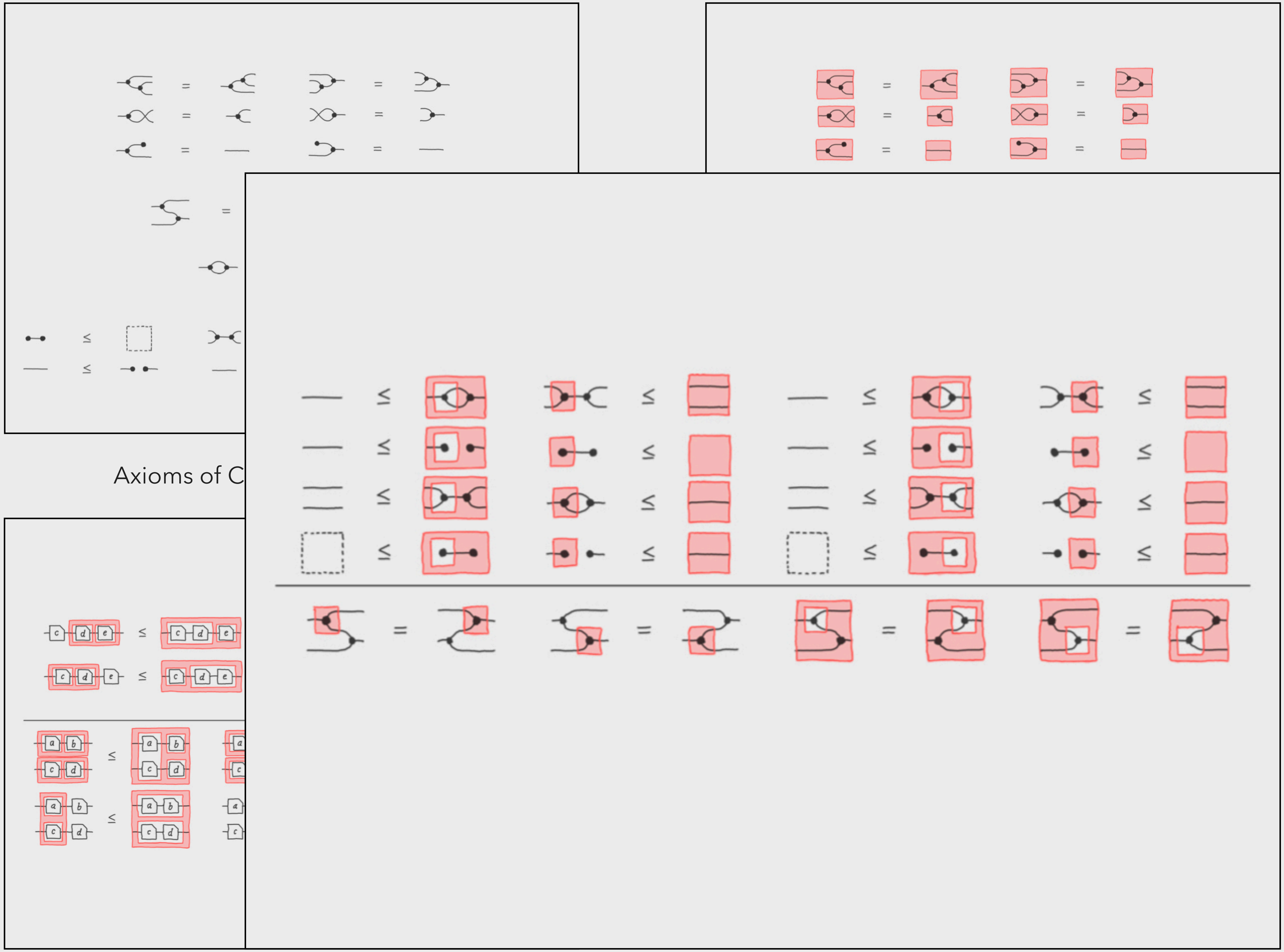

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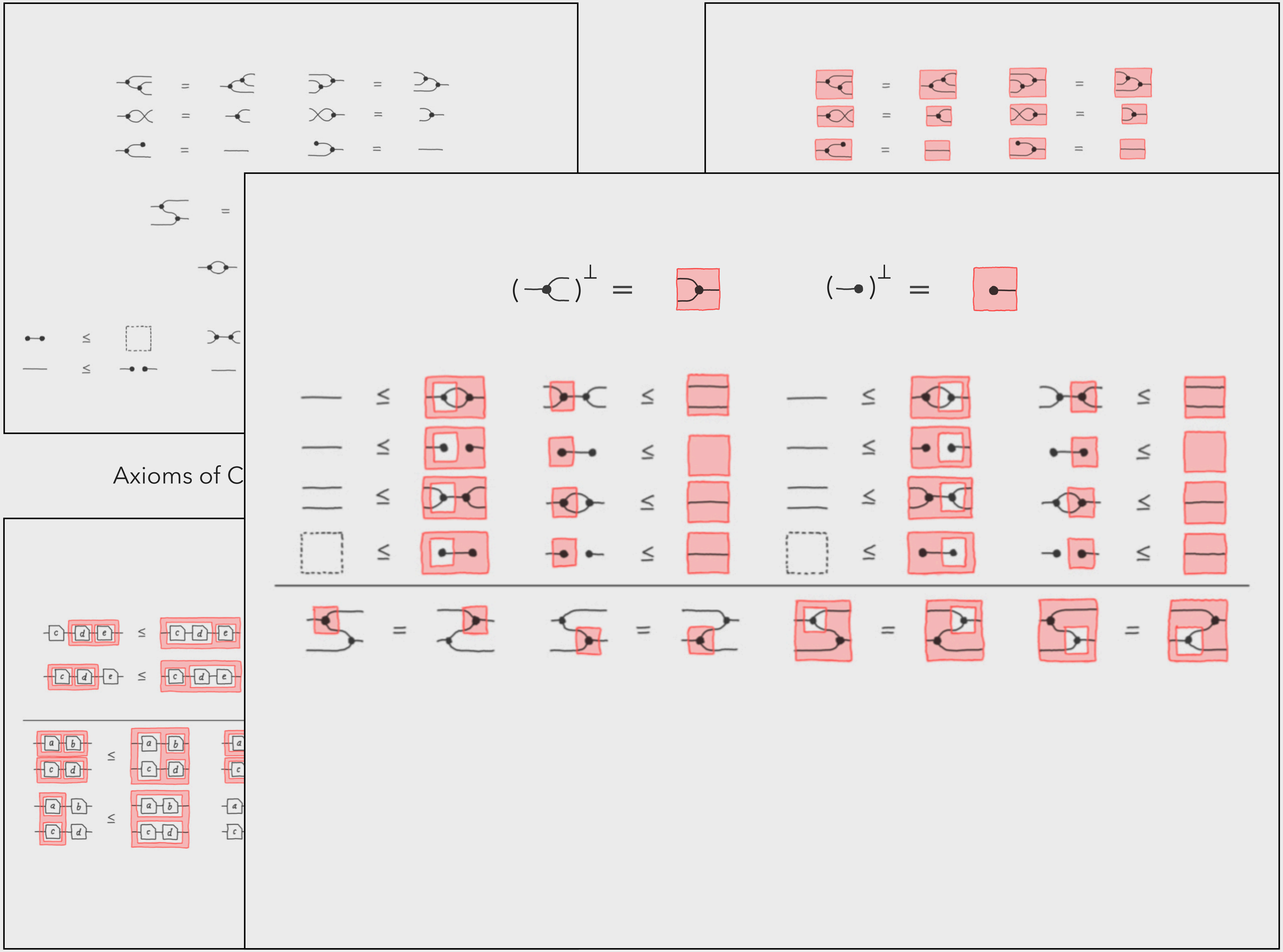

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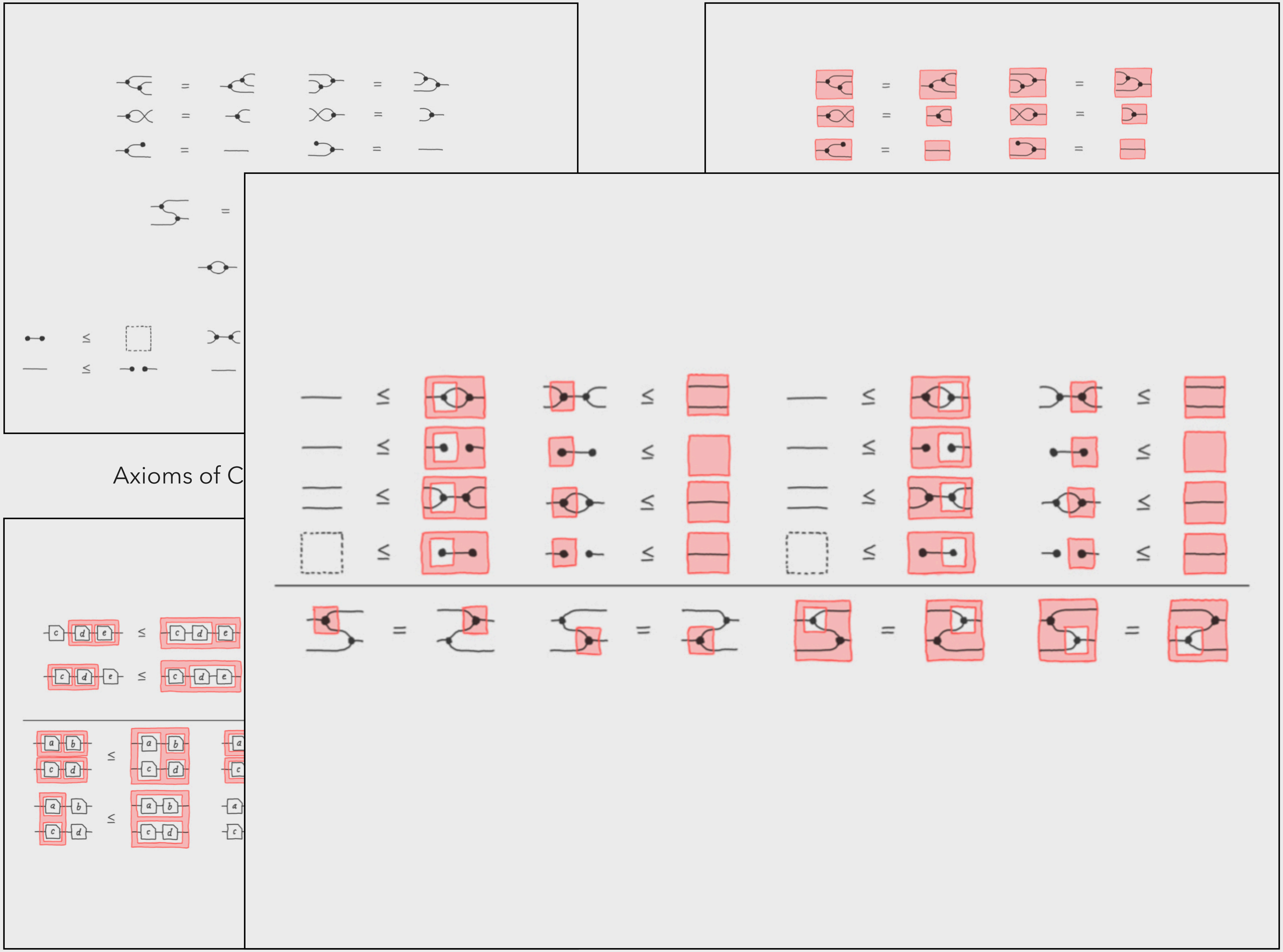

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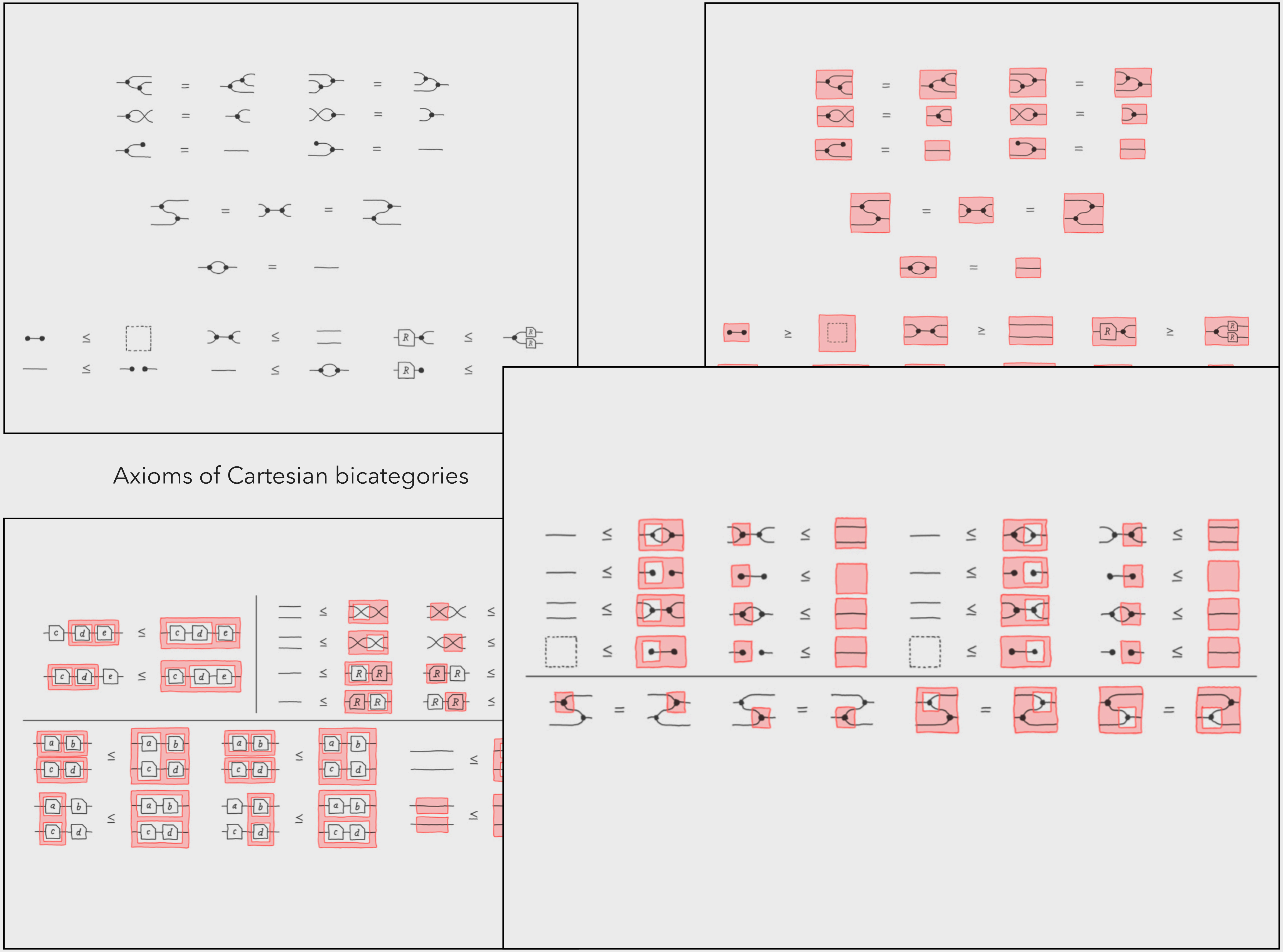

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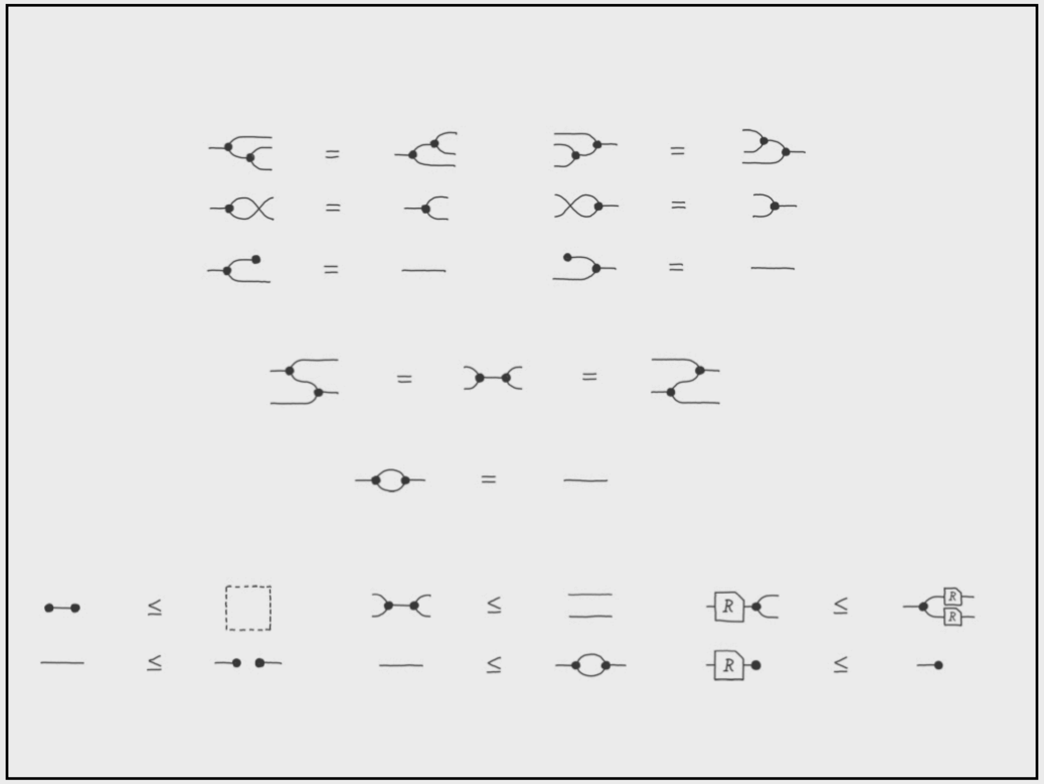

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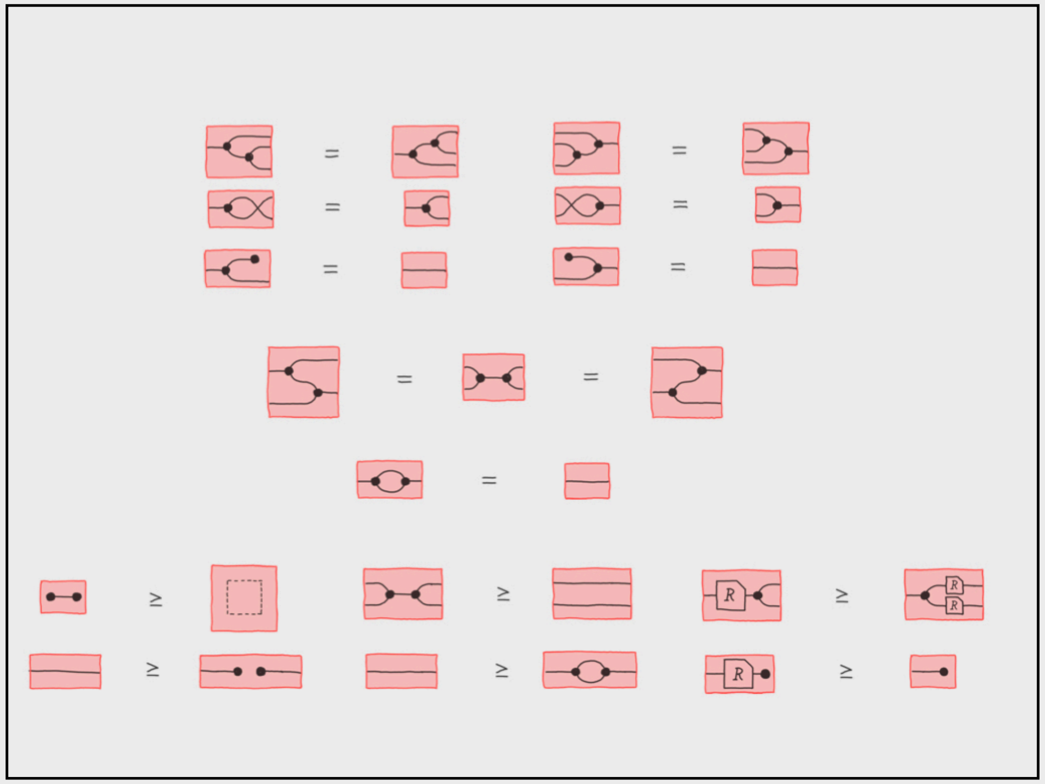

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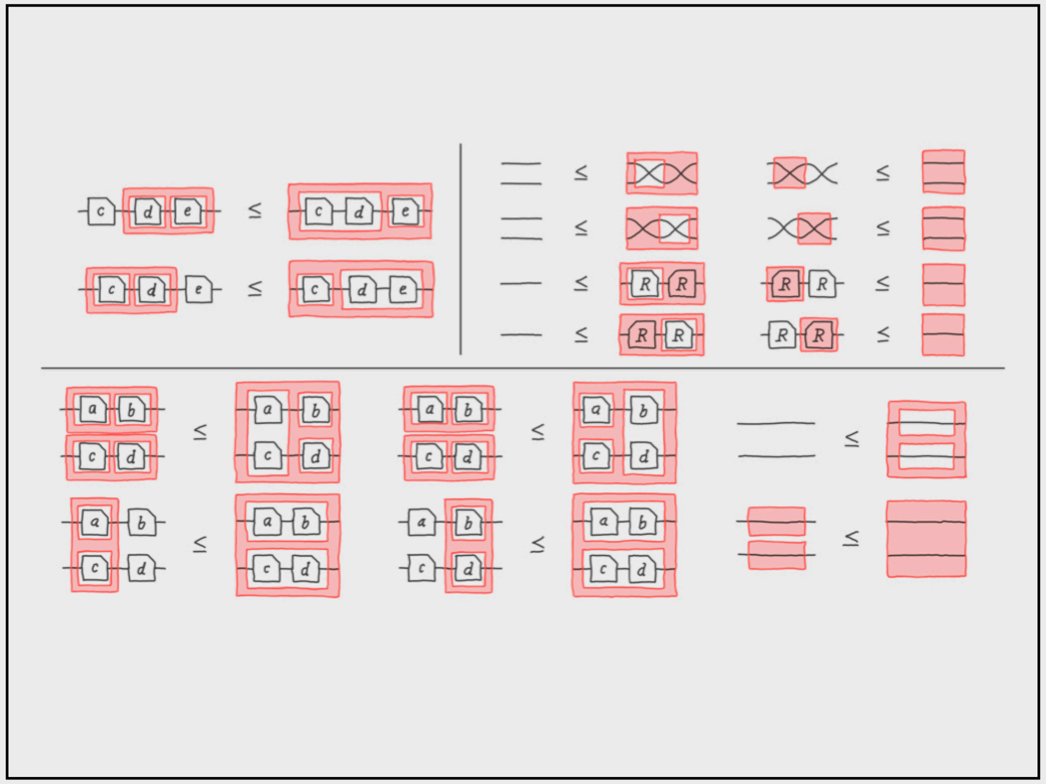

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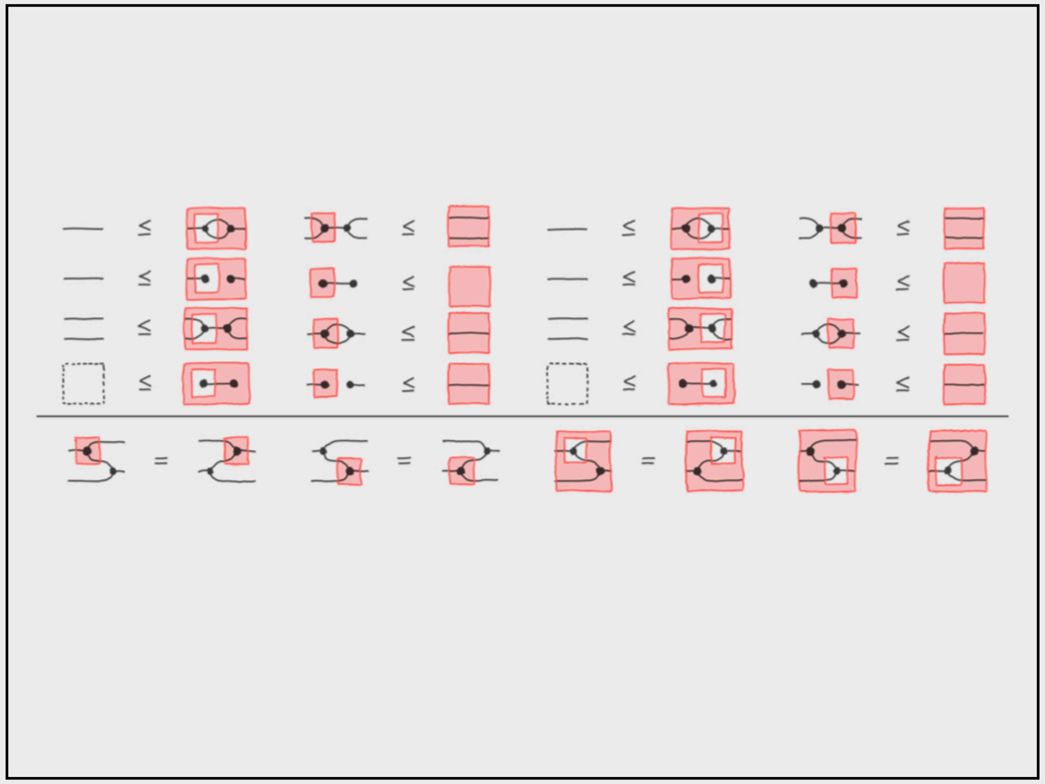

Additional axioms of NPR

[See Bonchi, Di Giorgio, Haydon, and Sobocinski 'Diagrammatic Algebra of First-Order Logic' (2024)]

Negation in the reconstruction above is taken as primitive. Can we motivate another account?…

… perhaps…

… using the `scroll′?

Residuation adds a contribution that gets us part of the way…

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Residuation adds a contribution that gets us part of the way…

… adding the dual to relational composition, linear distributivity, and the other linear negation laws from `Note B’ is sufficient!

Advantages:

- finite axiomization of full first-order logic,
- negation is a derived operation, with the linear negation, i.e. complement converse ($\bar{\breve{R}}$), now taken as primitive
- diagrammatic syntax: no complex rules for treating variables, simple inference rules, quantifier-free,
- comes with soundness and completeness results,
- comes with encodings into Tarski's relation algebra
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While the calculus is sufficient for classical FOL, its building blocks are not!

The closest extant theory is *classical (cyclic) bilinear logic*.

See Lambek (1995) and Cockett and Seely (1997).

Outline

- ~~introduce Peirce’s Existential Graphs (à la regular logic and cartesian bicategories)~~
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- demonstrate topological advantages

In the NPR calculus, the following are all equivalent:

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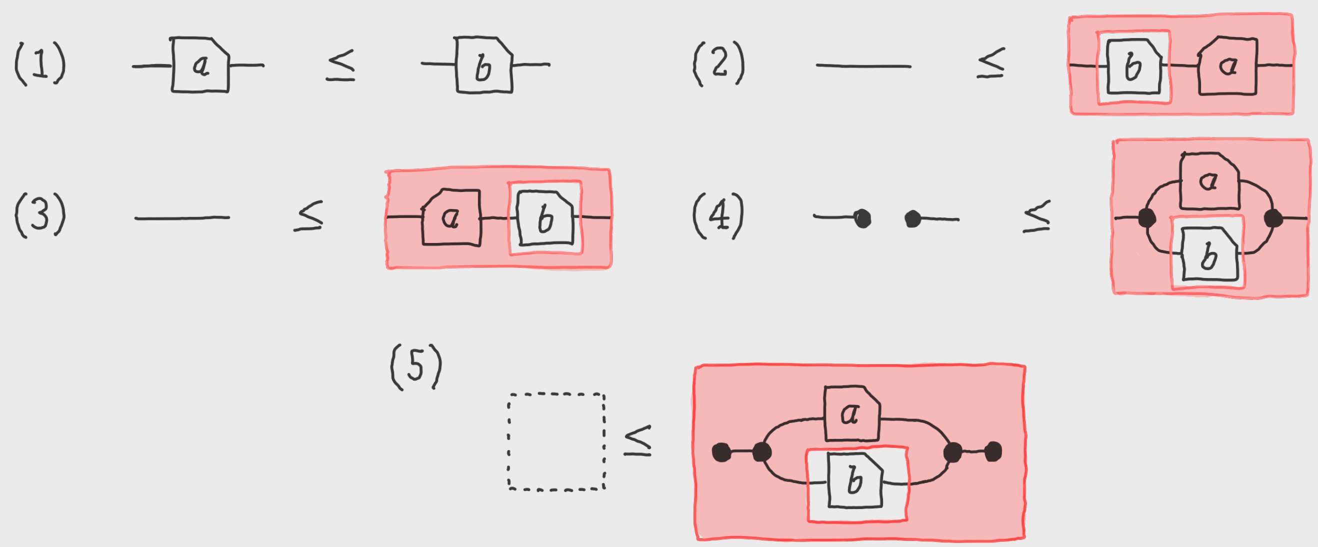

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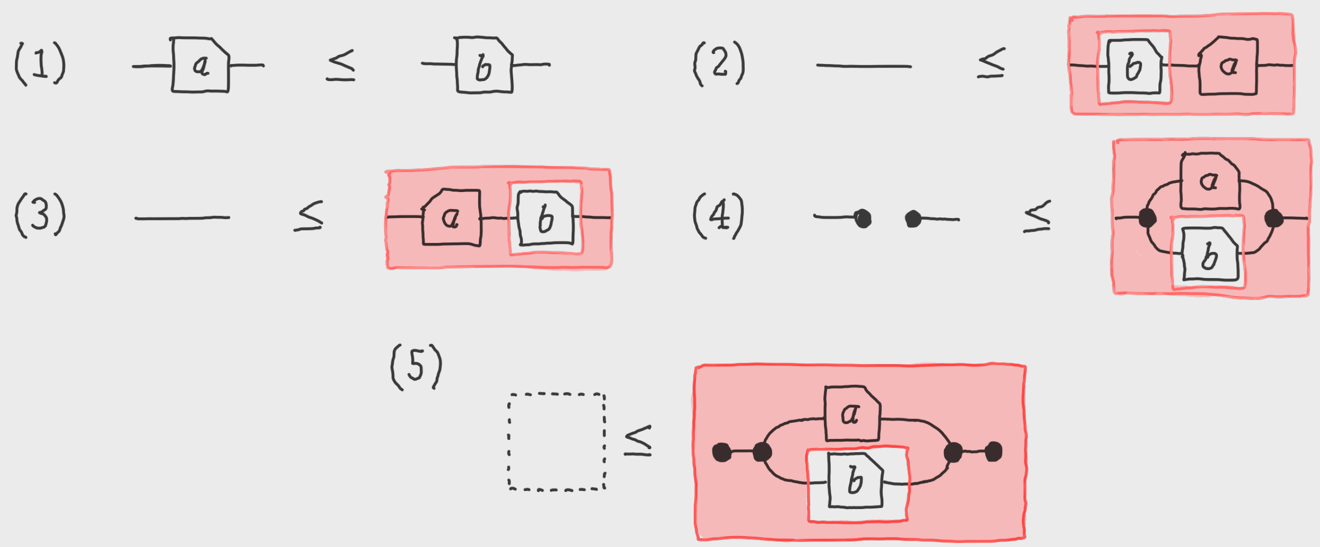

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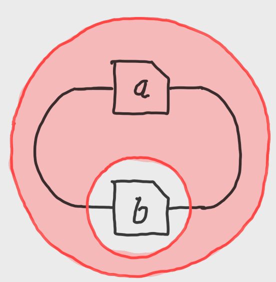

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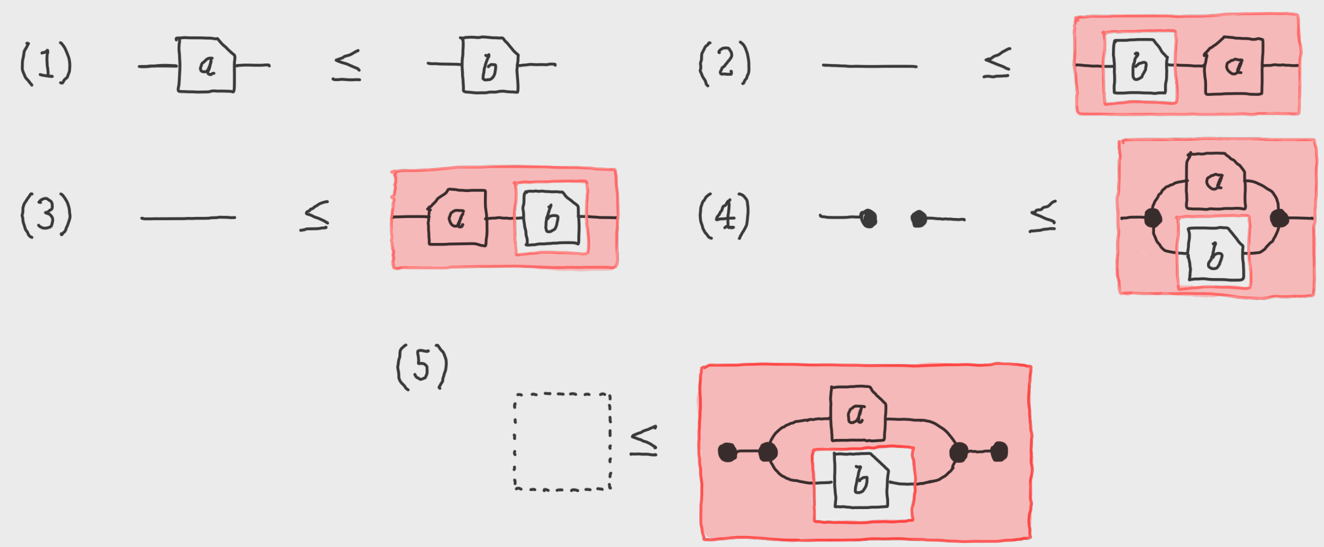

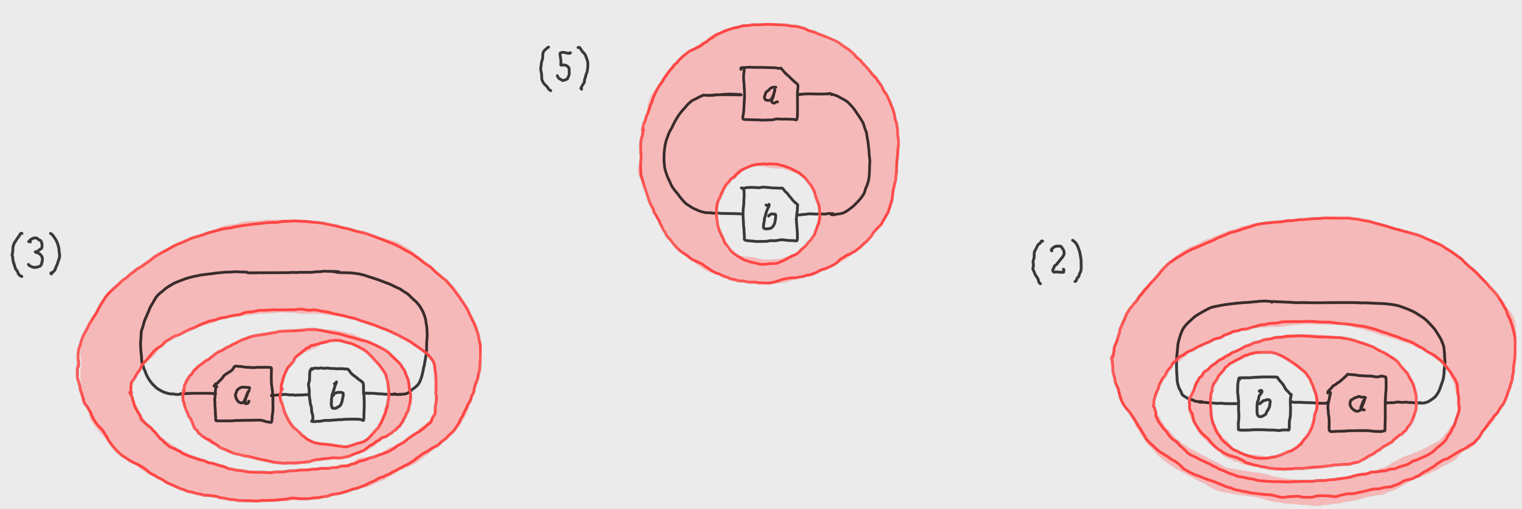

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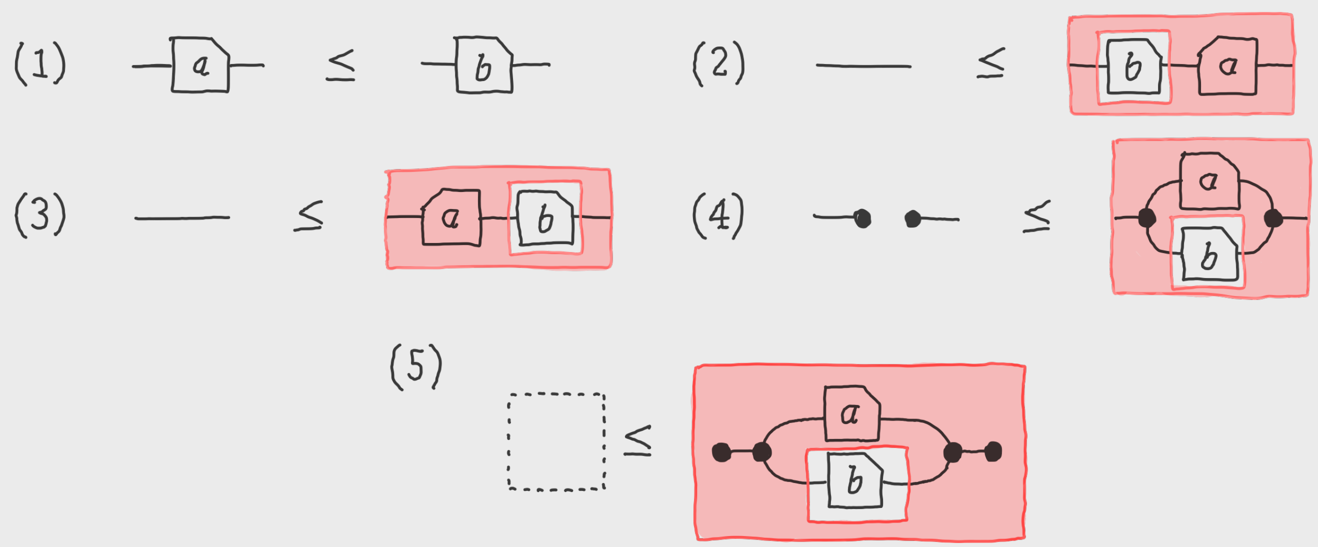

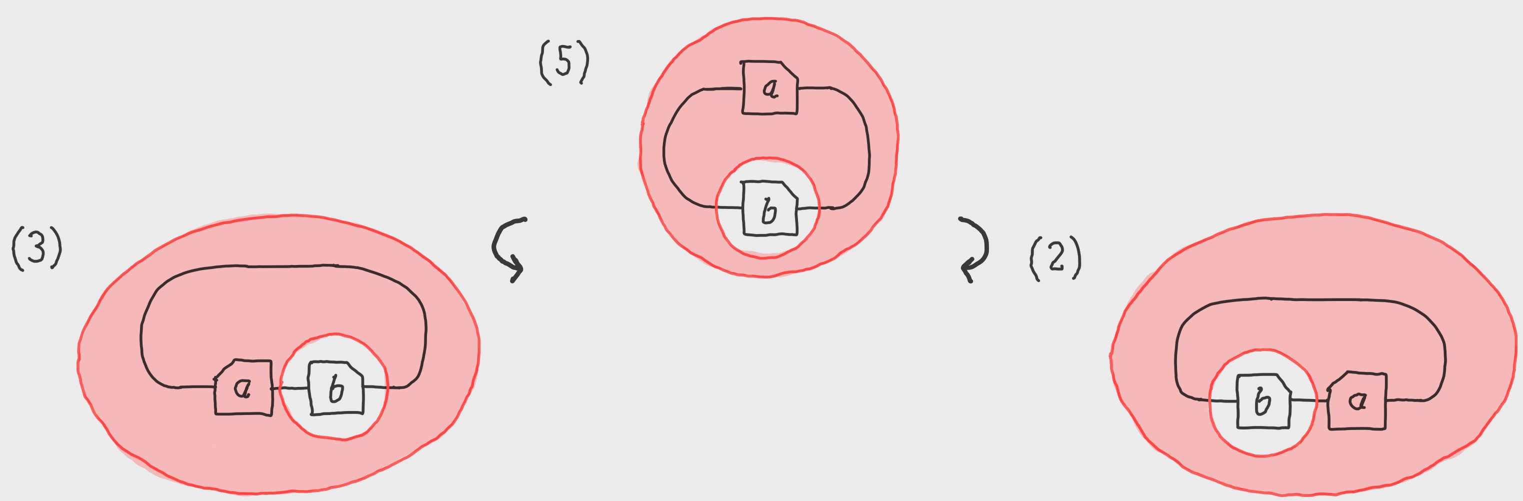

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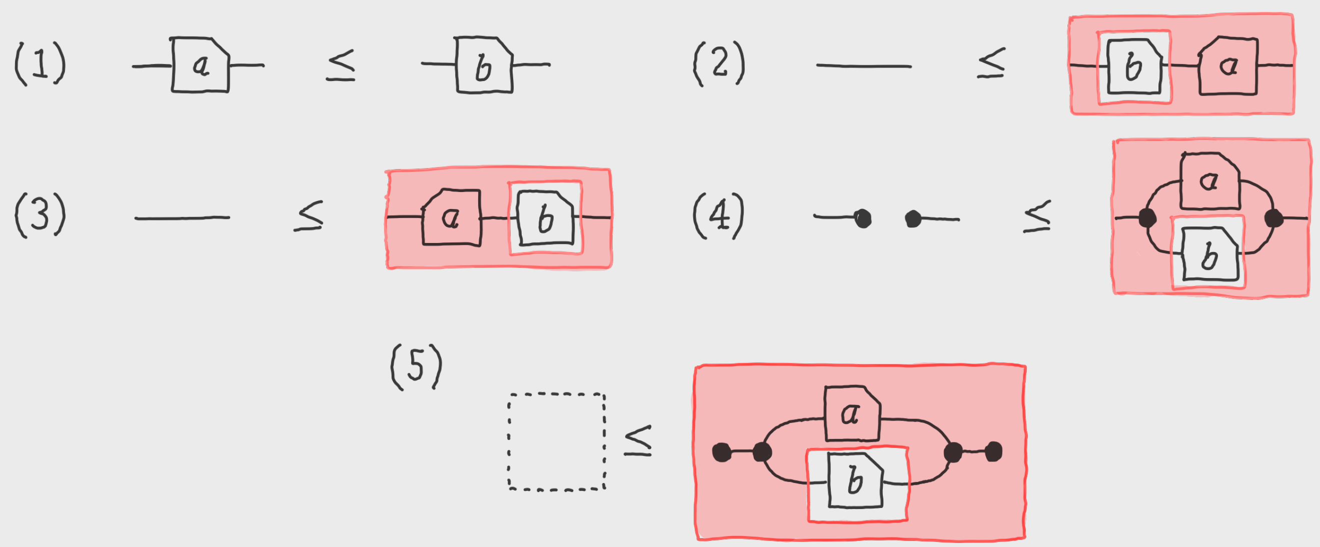

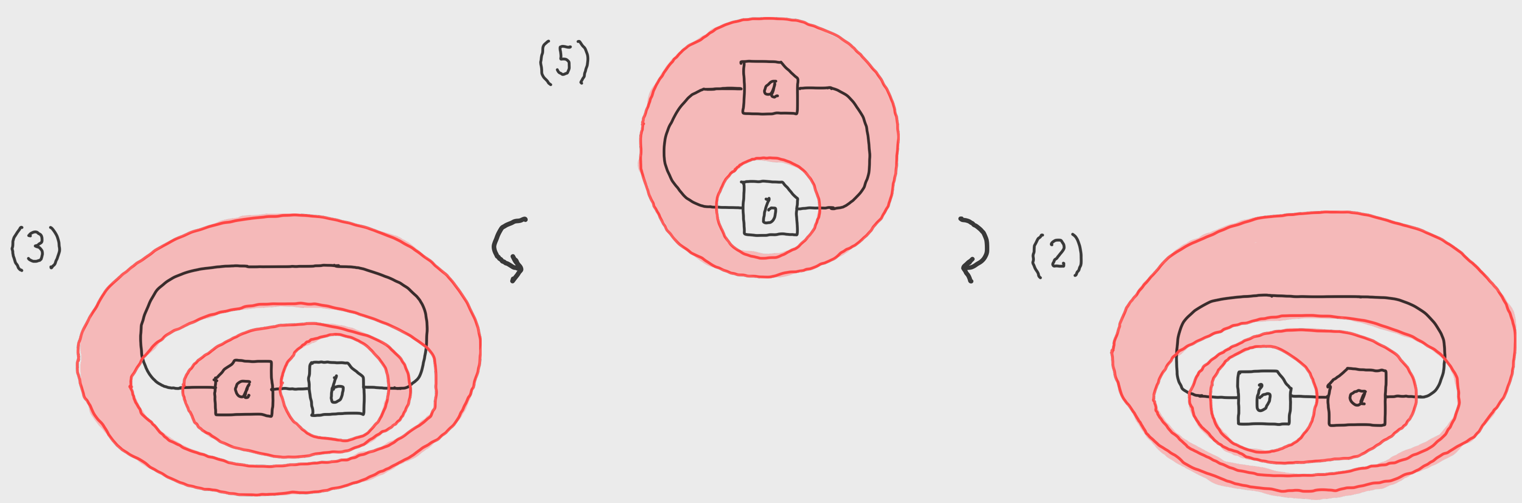

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… then scribe the graph (and completeness does the rest).

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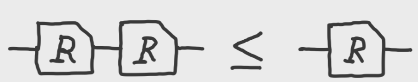
[Reflexive] 

[Transitive] 

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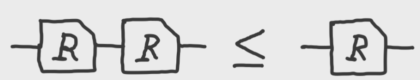
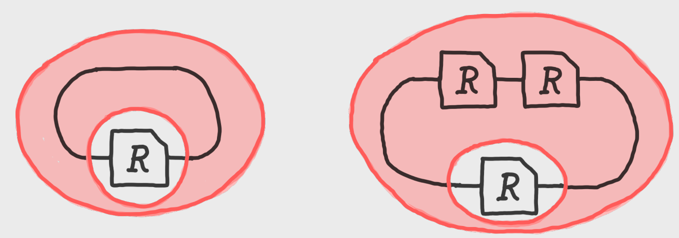
[Transitive] 



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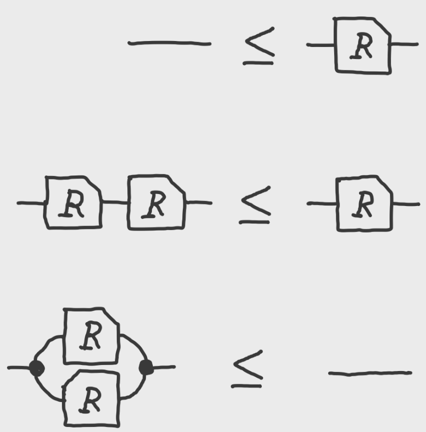

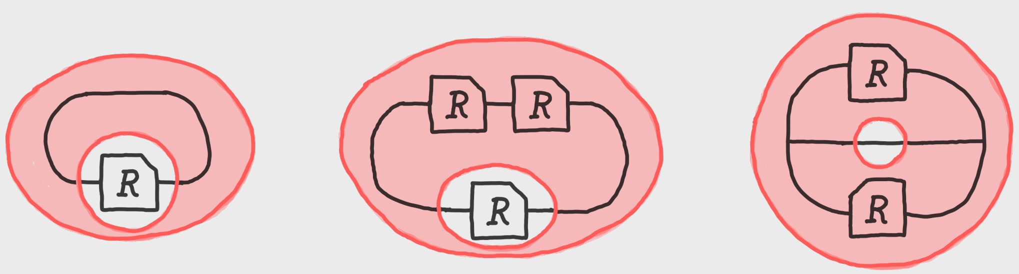

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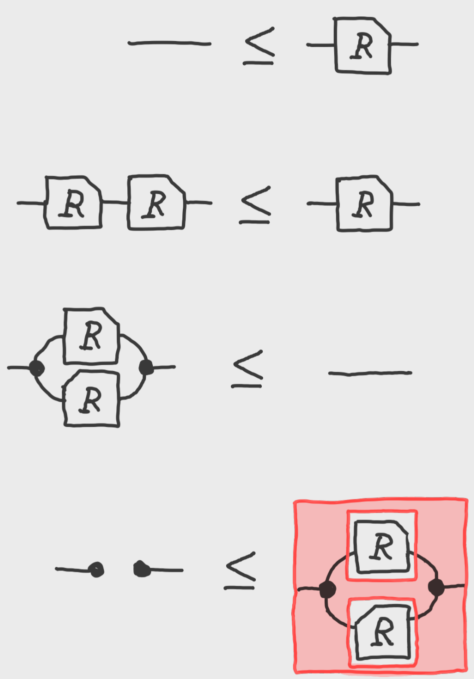

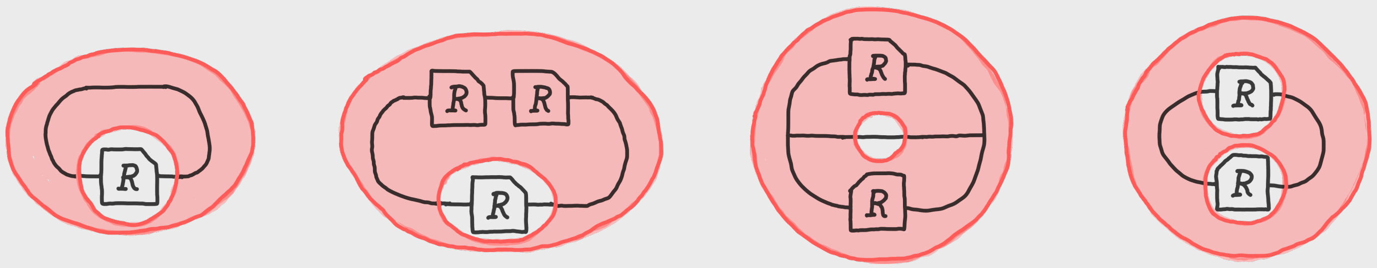

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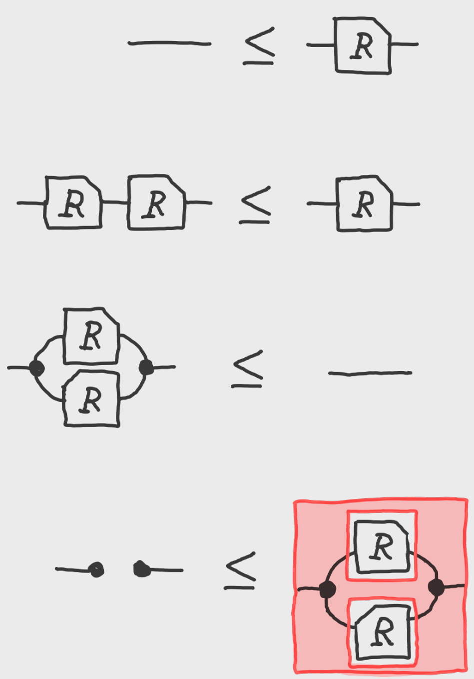
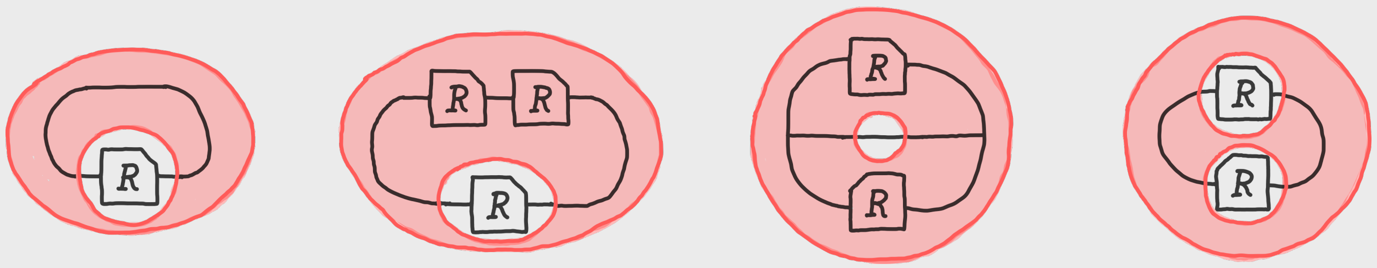
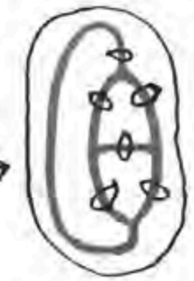


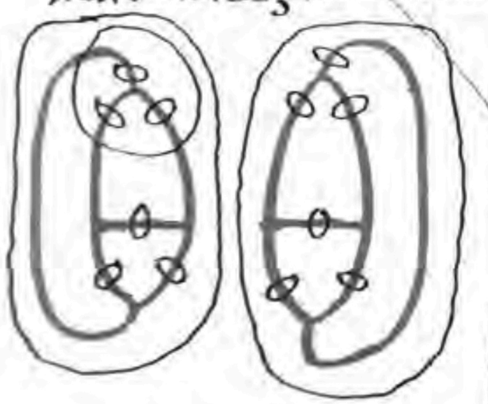


Vindicates Pierce’s use of the ‘scroll’!

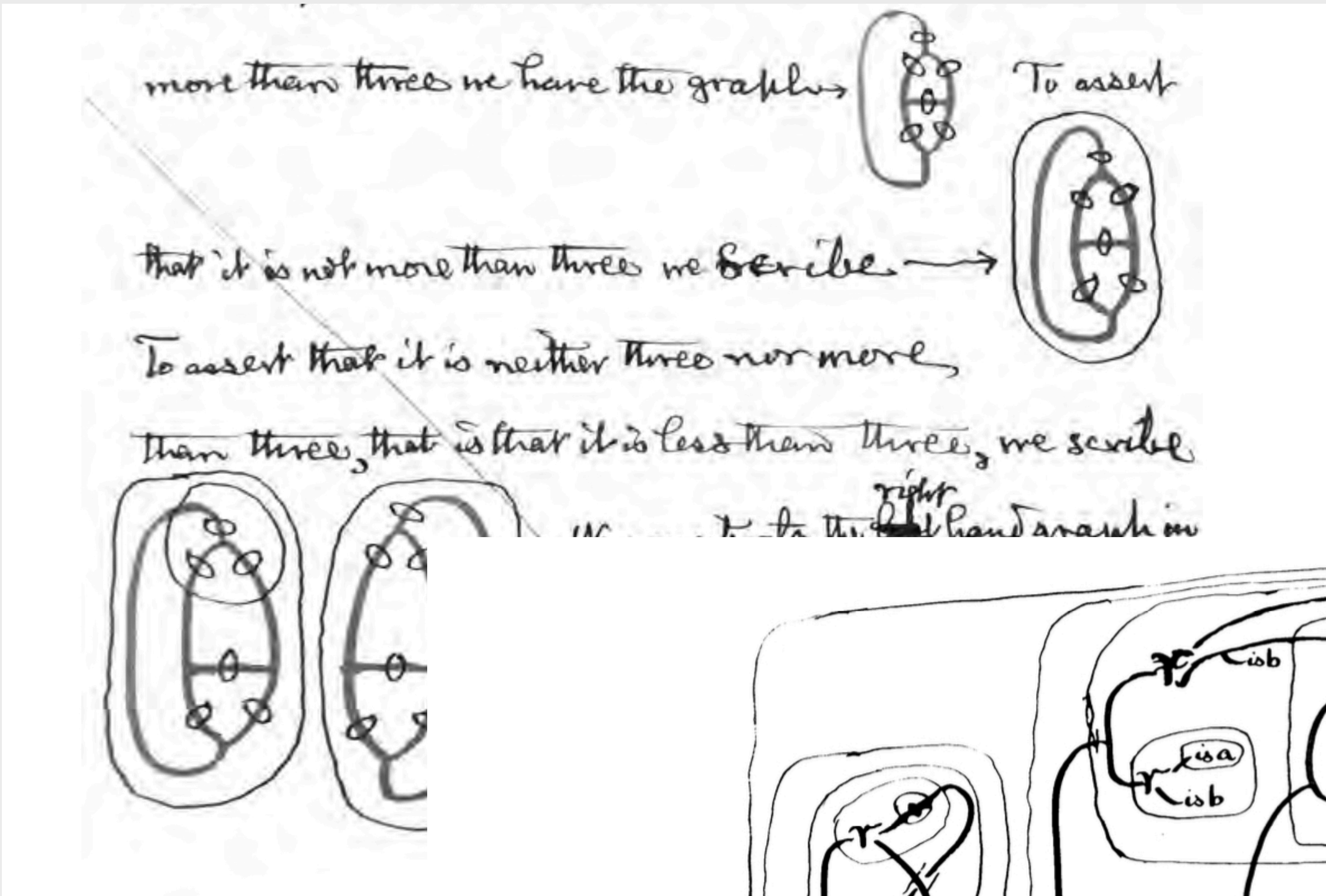
more than three we have the graph  To assert

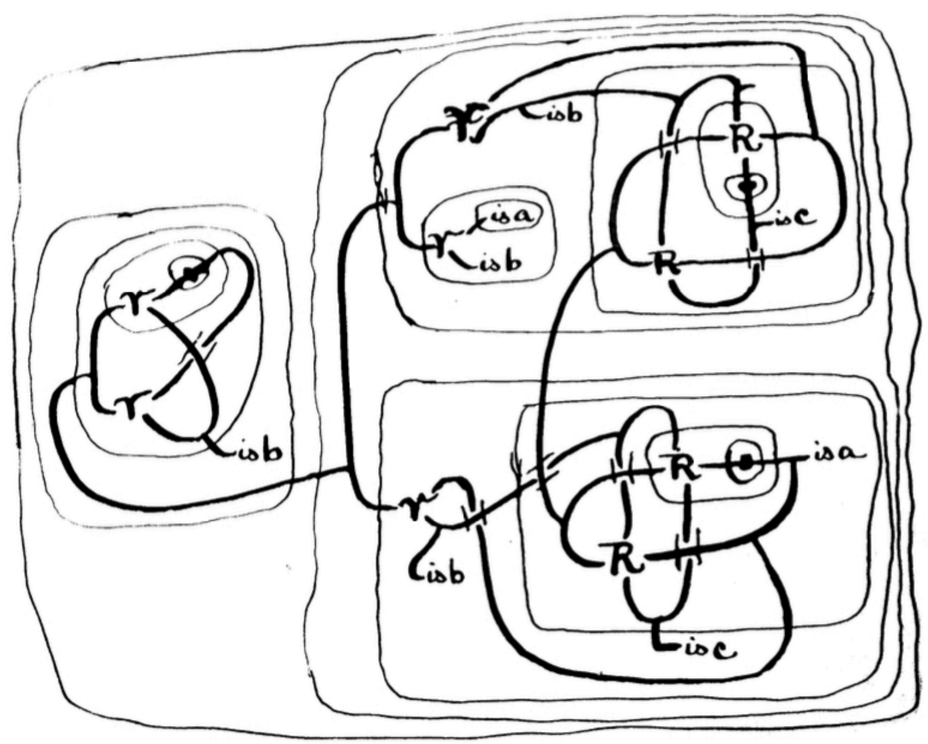
that it is not more than three we scribe → 

To assert that it is neither three nor more than three, that is that it is less than three, we scribe

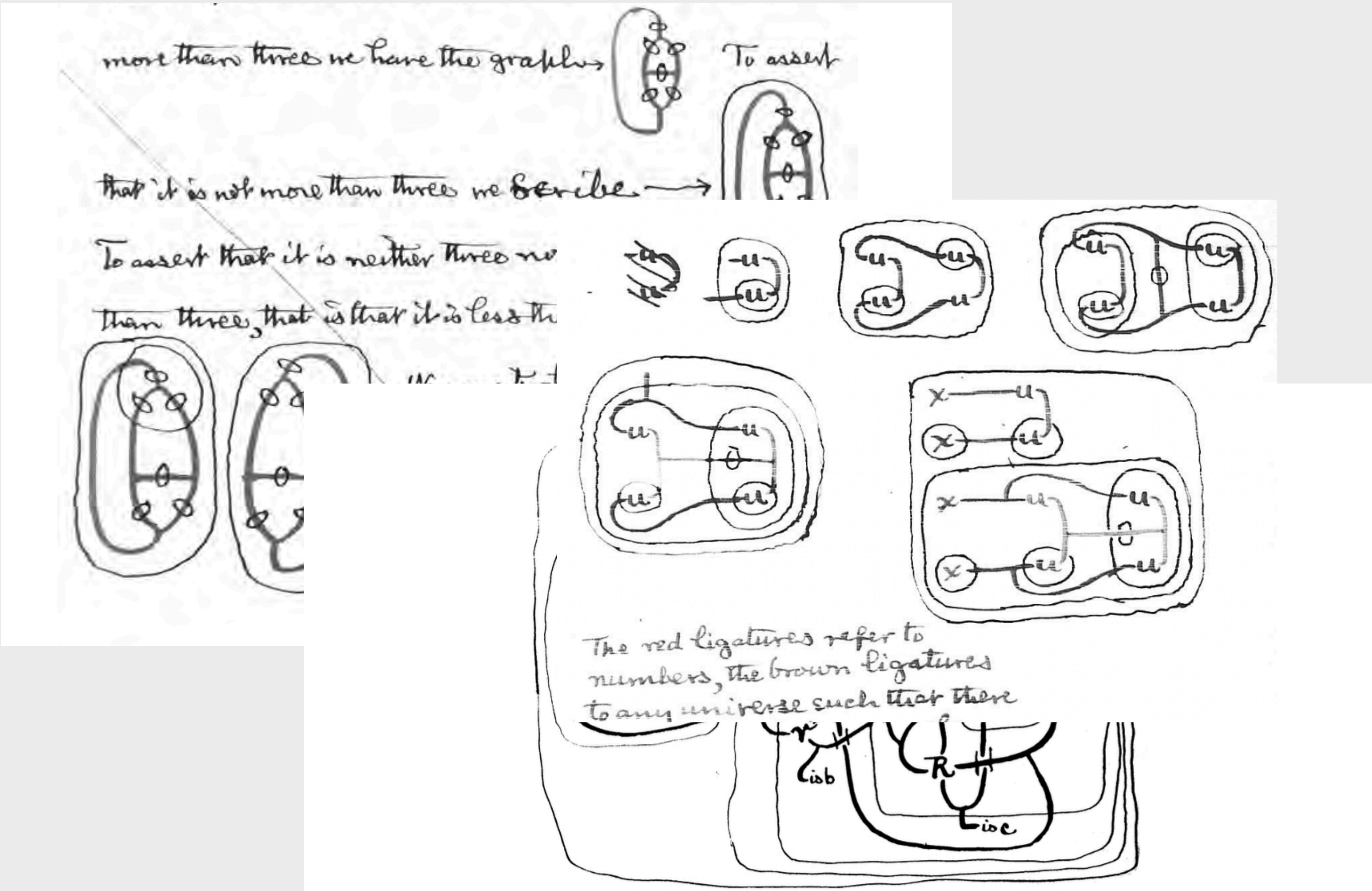


We now iterate the ~~left~~ right hand graph in the second cut of the left hand one thus





Such are the very simplest of the relations which mathematicians are in the habit of handling! The practical advantage of writing the above in the form $[c] = [a]^{[b]}$ is obvious. Yet for logical purposes the analyzed expression is necessary.



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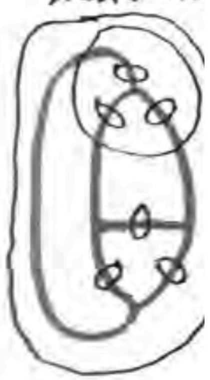


Peripatetic Talks. No. 7

[R 505] *The Fundamental Principles of Existential Graphs restated in a new form.*

Rule I. Any graph, A, that can be illatively transformed into a graph, B, which can be illatively transformed into a third graph, C, can itself be illatively transformed into C. Write t as an abbreviation for "can be illatively transformed into". Then, in a universe of graphs, this rule is represented by Fig. 1.

Fig. 1

(This rule is Aristotle's maxim known as the *Nota notae*, "Nota notae est nota rei ipsius", as applied to graphs. De Morgan's *principle of the transitiveness of the copula* and Aristotle's *Dictum de omni et de nullo* are substantially this.)

Rule II. Every graph, B, such that every graph, A, into which it, B, cannot be illatively transformed, but which, A, can be illatively transformed into it, B, can be illatively transformed in a second graph, C, can itself, B, be be illatively transformed into C. This is represented by Fig. 2.

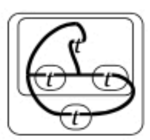
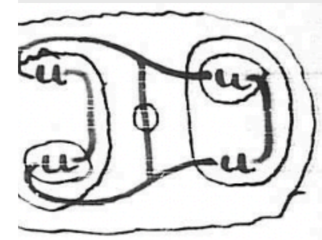


Fig. 2

(This may be called the inductive principle of graphical transformation.)

More clearly stated: Take any two graphs, B and C. Suppose that every graph, A, which can be transformed into B although B cannot be transformed into it, can be transformed into C. Then B can itself be illatively transformed into C.



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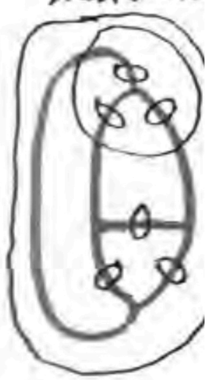

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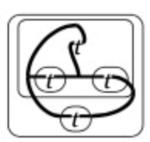
Rule II. Every graph, B, such that every graph, A, tively transformed, but which, A, can be illat be illatively transformed in a second graph, transformed into C. This is represented by Fig. _.



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Rule III. Into any graph, B, such that into every graph, C, that cannot be illatively transformed into it, B, but into which, C, it, B, can be illatively transformed, a given graph, A, can be illatively transformed, this graph, A, can be illatively transformed. Fig. 3 represents this rule.

Fig. 3

(This may be called the hypothetical principle of graphical transformation.)

It may be more clearly stated as follows: Take any two graphs, A and B; and suppose that A can be illatively transformed into every graph, C, which cannot be illatively transformed into B but into which B can be illatively transformed. Then A can be illatively transformed in B also.

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Outline

- ~~introduce Peirce's Existential Graphs (à la regular logic and cartesian bicategories)~~
- ~~move to the Neo-Peircean Calculus of Relations (à la residuation and cyclic bilinear logic)~~
- ~~demonstrate topological advantages~~

Aim

Introduce Peirce's *Existential Graphs*…

… as a precursor to string diagrams …

... and as the inspiration for recent
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Introduce Peirce's *Existential Graphs*…

… as a precursor to string diagrams …

... and as the inspiration for recent developments in categorical logic.

… Peirce understood the relational setting for cartesian bicategories and for classical (cyclic) bilinear logic!

"… my Existential Graphs, by which all deduction is reduced to insertions and erasures, and in which there are no connecting signs except the writing of terms on the same area enclosed in an oval … and also heavy lines to express the identity of the individual objects… This ought to be the Logic of the Future."

- Letter to William James, 1909

Thanks!