Learning weighted automata

Alexandra Silva

Cornell Bowers CIS
Computer Science

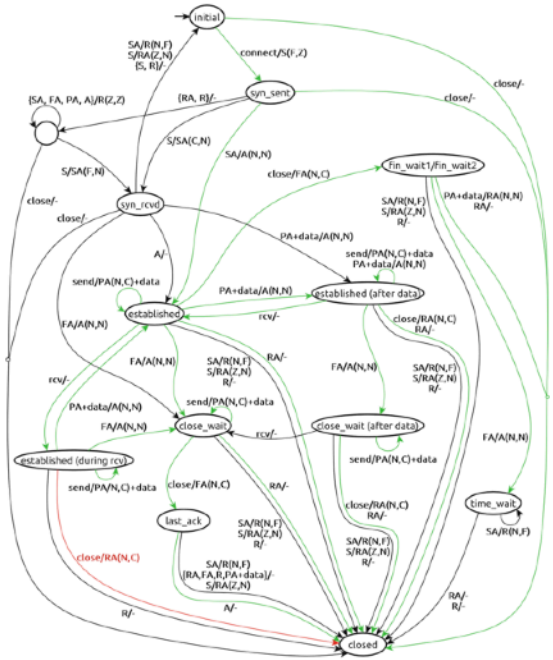
Active Automata Learning

Black-box interactions

Zoom-in specific part of code

Incrementally build a model

Refine with properties of interest



Active Automata Learning

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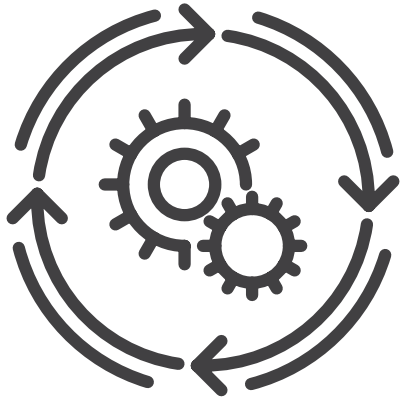

ncrementally uild a model

Refine with roperties of interest

Automated

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DFA Learning (L*, Angluin'87)

Regular Language

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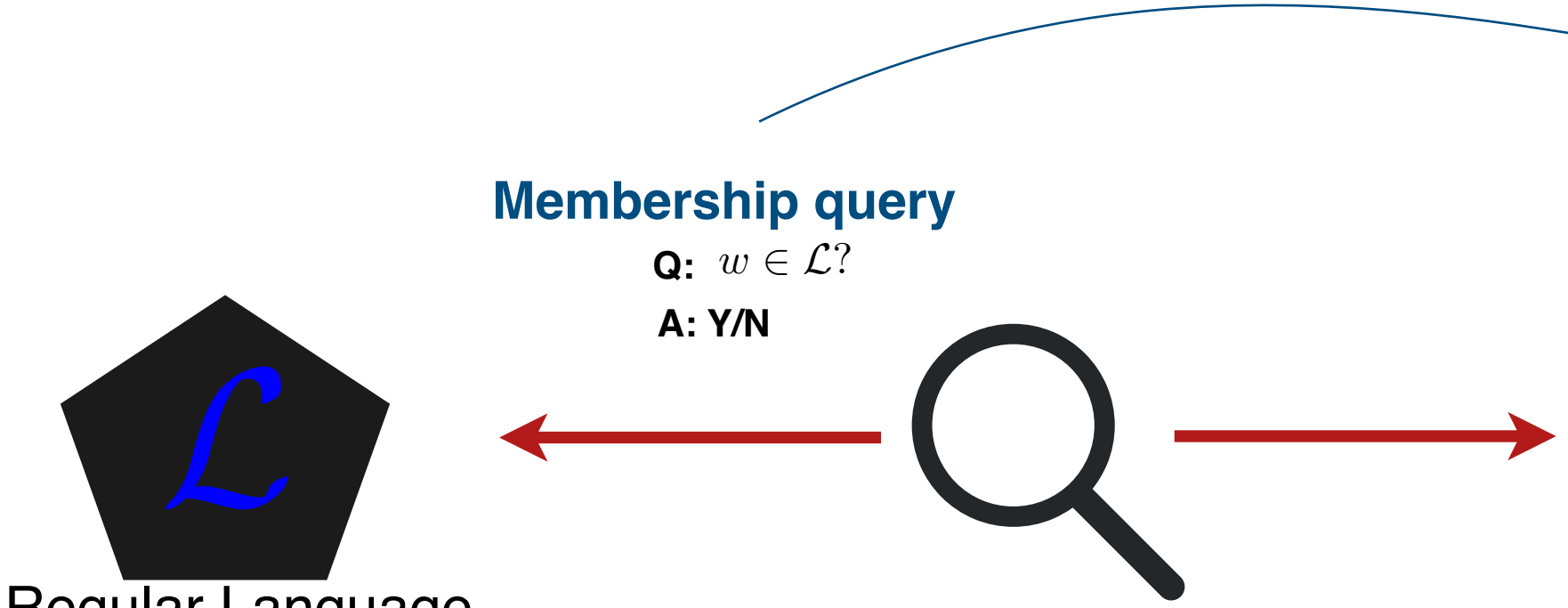
DFA Learning (L*, Angluin'87)



Observation table

	ϵ	a
ϵ	0	0
a	0	1
aa	1	0
b	0	0
ab	0	0
aaa	0	0
aab	0	0

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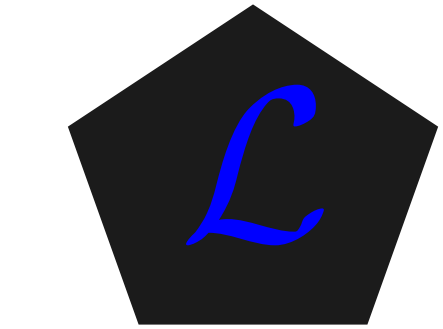
DFA Learning (L*, Angluin'87)

Membership query

Q: $w \in \mathcal{L}?$

A: Y/N



Regular Language

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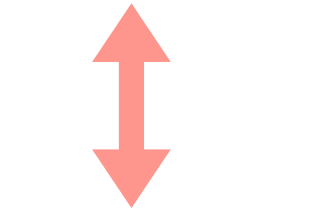
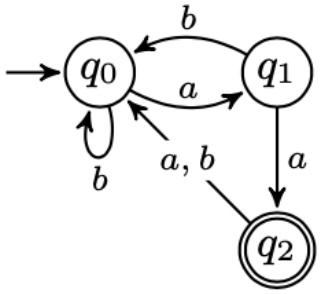
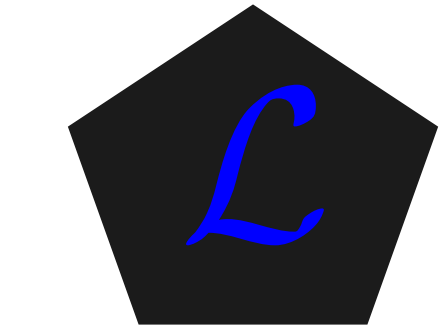




DFA Learning (L*, Angluin'87)



Regular Language

Membership query

Q: $w \in \mathcal{L}?$

A: Y/N

Equivalence query

Q: $\mathcal{L}(H) = \mathcal{L}?$

A: Y / N
+ counterexample

Observation table

	ϵ	a
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aa	1	0
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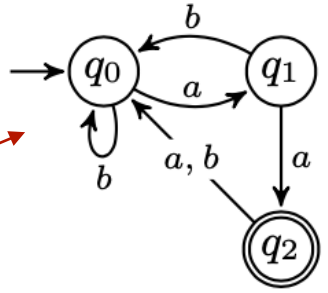

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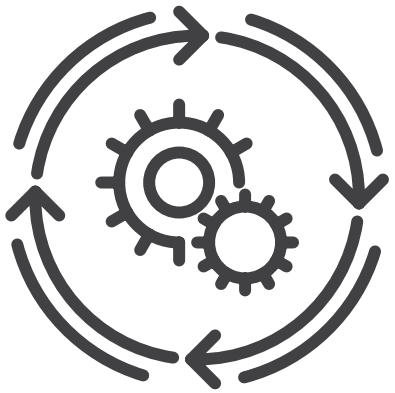
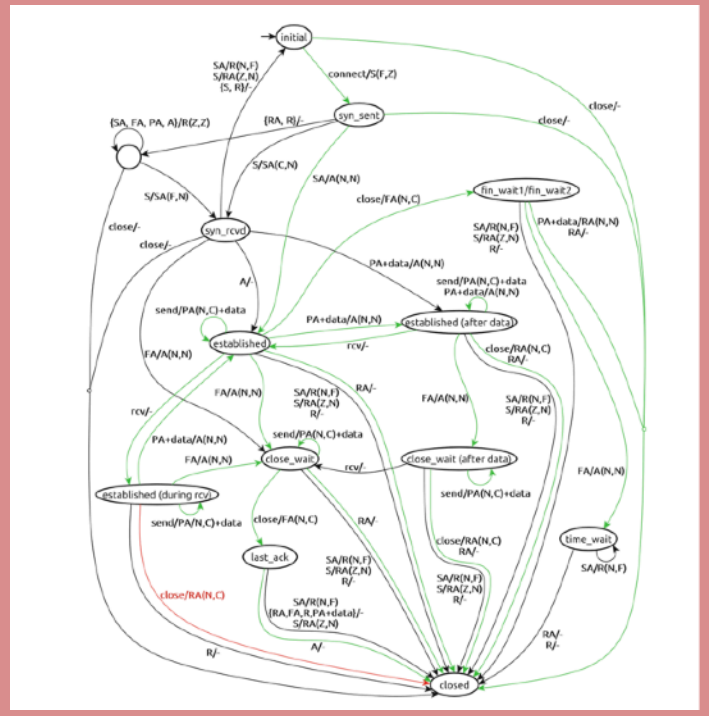


Learns the Minimal DFA

L* at work

L* at work

Model of Windows 8 TCP implementation

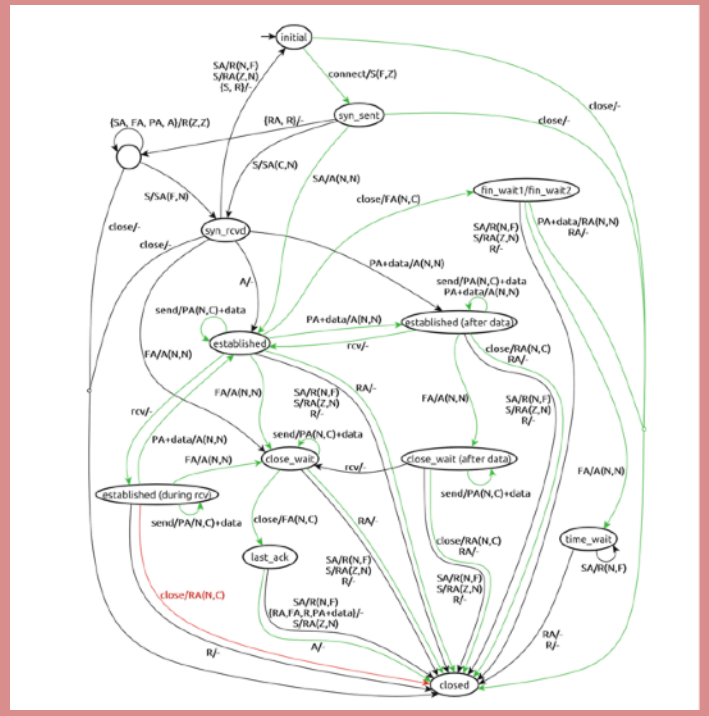


Combining Model Learning and Model Checking to Analyze TCP Implementations

Paul Fiterău-Broştean, Ramon Janssen, and Frits Vaandrager(✉)

L* at work

Model of Windows 8 TCP implementation



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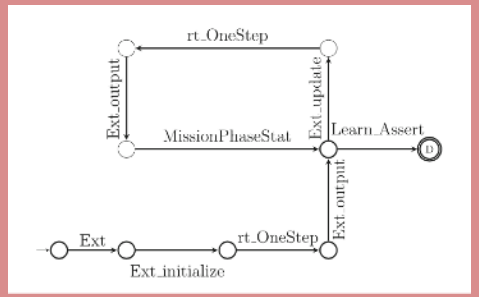
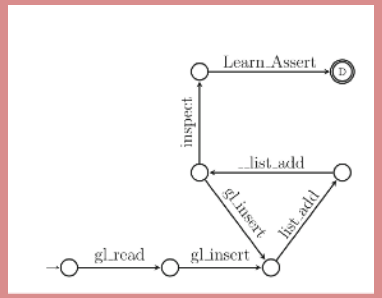
Learning Error Traces (function calls)



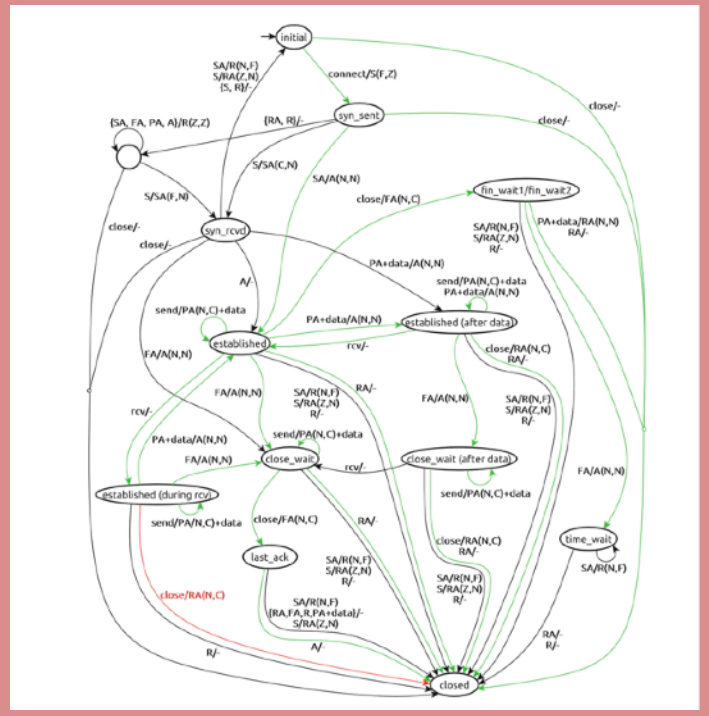


Learning the Language of Error

Martin Chapman[1], Hana Chockler[1](✉), Pascal Kesseli[2], Daniel Kroening[2], Ofer Strichman[3], and Michael Tautschnig[4]

L* at work

Model of Windows 8 TCP implementation



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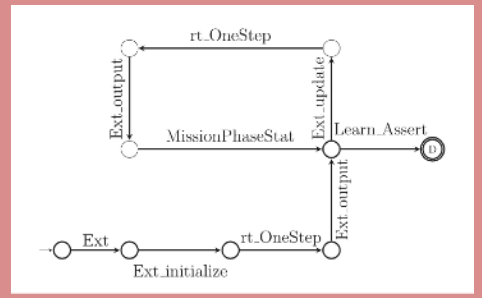

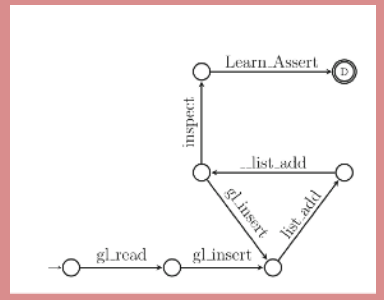

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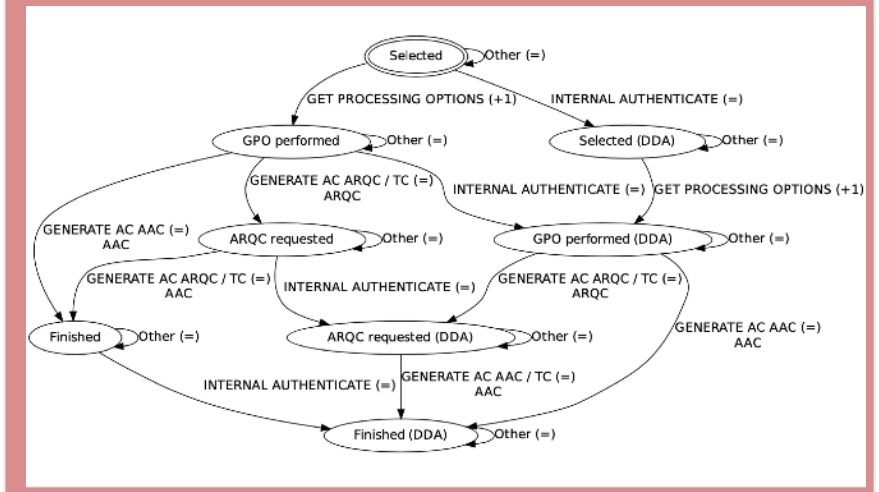
Model of Visa Debit on Barclays card

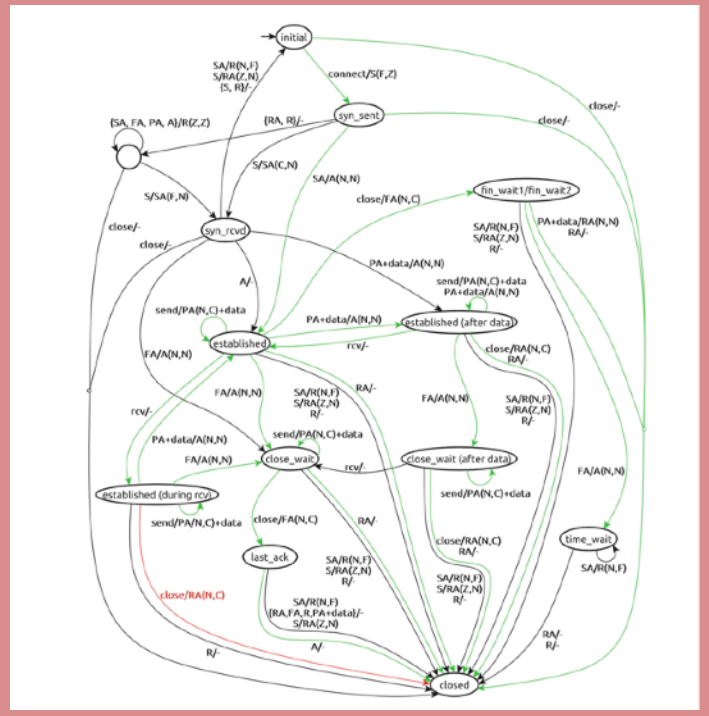


Formal models of bank cards for free

Fides Aarts, Joeri de Ruiter, and Erik Poll

L* at work

Model of Windows 8 TCP implementation



Combining Model Learning and Model Checking to Analyze TCP Implementations

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Active learning

Infer automaton through oracle (membership and equivalence queries)

Deterministic finite automata

Gold, Angluin 80's

Weighted automata

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Active learning

Infer automaton through oracle (membership and equivalence queries)

Deterministic finite automata

Gold, Angluin 80's

Weighted automata

$$L \in 2^{A^*}$$

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Active learning

Infer automaton through oracle (membership and equivalence queries)

Deterministic finite automata

Gold, Angluin 80's

$$L \in 2^{A^*}$$

Weighted automata

$$L \in \mathbb{S}^{A^*}$$

Semiring

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Active learning

Infer automaton through oracle (membership and equivalence queries)

$$L \in 2^{A^*}$$

$$L \in \mathbb{S}^{A^*}$$

Semiring

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L* for DFAs

L* learns minimal DFA for a *regular language* assuming an oracle that answers:

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Membership queries

$$w \in \mathcal{L}?$$

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Equivalence queries

$$\mathcal{L}(H) = \mathcal{L}?$$

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L* for DFAs

L* learns minimal DFA for a *regular language* assuming an oracle that answers:

Membership queries

$$w \in \mathcal{L}?$$

Equivalence queries

$$\mathcal{L}(H) = \mathcal{L}?$$

No = counter-example

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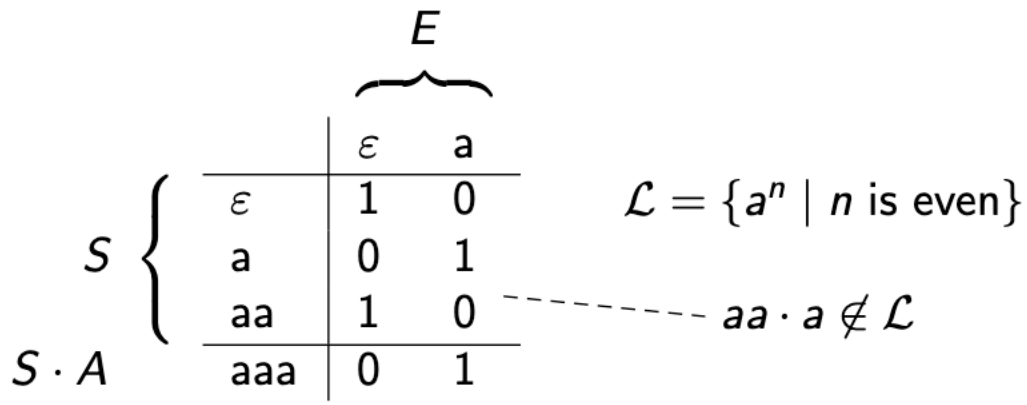
L* for DFAs

$S, E \subseteq A^*$



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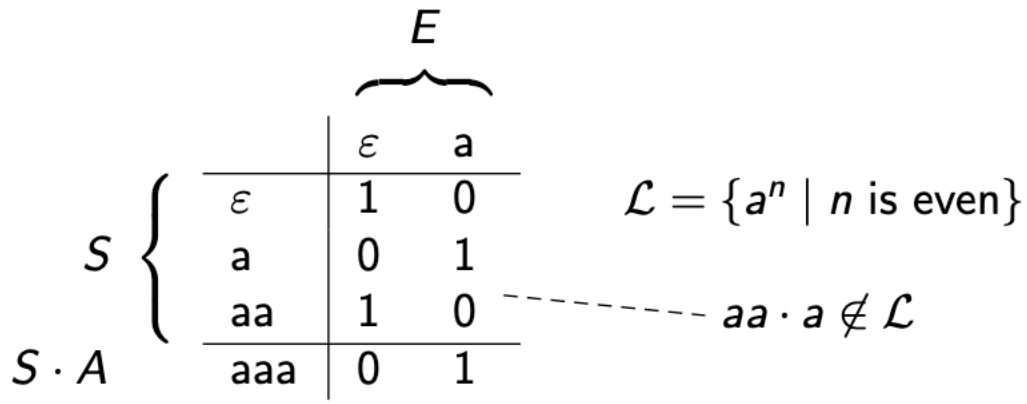

$S, E \leftarrow \{\varepsilon\}$

Repeat until no more counterexamples:

1. Close table (changing S)
2. Equivalence query
3. Add suffixes of counterexample to E

L* for DFAs, example

$$A = \{a\}$$

$$a^n \in \mathcal{L} \iff n \equiv 0 \mod 3$$

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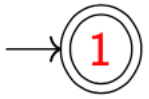
	ε
ε	1
a	0



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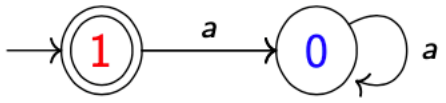

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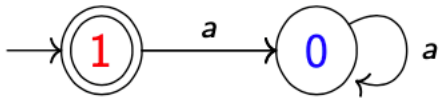

Counterexample: aaa

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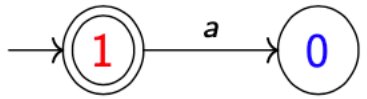
$$A = \{a\}$$

$$a^n \in \mathcal{L} \iff n \equiv 0 \mod 3$$

	ε	*a*	*aa*	*aaa*
ε	1	0	0	1
a	0	0	1	0
aa	0	1	0	0



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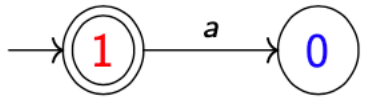

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	ε	a	aa	aaa
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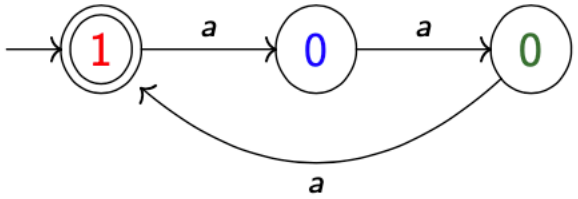
a	0	0	1	0
aa	0	1	0	0
aaa	1	0	0	1



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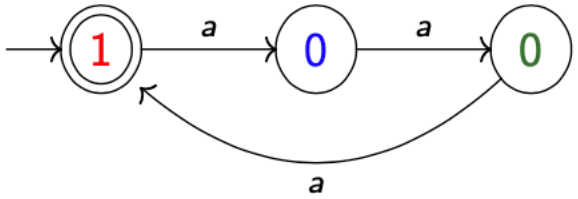
	ε	a	aa	aaa
ε	1	0	0	1
a	0	0	1	0
aa	0	1	0	0
aaa	1	0	0	1



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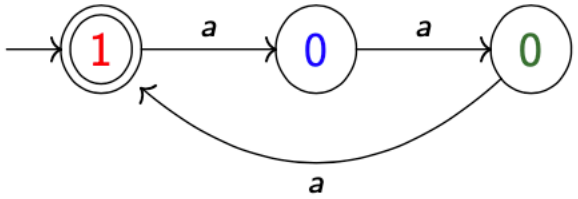
L* for DFAs, example

$$A = \{a\}$$

$$a^n \in \mathcal{L} \iff n \equiv 0 \mod 3$$

	ε	a	aa	aaa
ε	1	0	0	1
a	0	0	1	0
aa	0	1	0	0
aaa	1	0	0	1



Terminates!

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DFAs vs WFAs

$\mathbb{S}$ semiring (e.g. $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}, \mathbb{N}, 2$), FQ free semimodule over Q

DFA	WFA
initial state in Q	initial state in FQ
$Q \downarrow 2 \times Q^A$	$Q \downarrow \mathbb{S} \times (FQ)^A$

DFAs vs WFAs

$\mathbb{S}$ semiring (e.g. $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}, 2$), FQ free semimodule over Q

DFA	WFA
initial state in Q	initial state in FQ
$Q \to 2 \times Q^A$	$Q \to \mathbb{S} \times (FQ)^A$

Accepts weighted language $A^* \to \mathbb{S}$

Multiplies along the path with final output

Sums over paths

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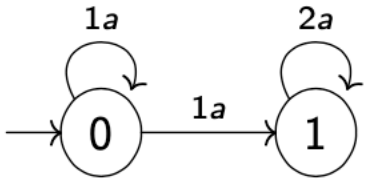
DFAs vs WFAs



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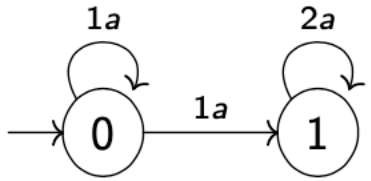
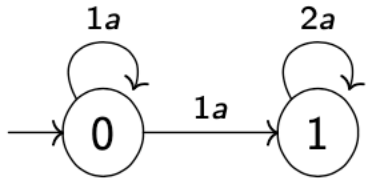


$$\mathcal{L}(\varepsilon) = 0$$
$$\mathcal{L}(a) = 1 \cdot 0 + 1 \cdot 1 = 1$$
$$\mathcal{L}(aa) = 1 \cdot 1 \cdot 0 + 1 \cdot 1 \cdot 1 + 1 \cdot 2 \cdot 1 = 3$$
$$\mathcal{L}(aaa) = 1 \cdot 1 \cdot 1 \cdot 0 + 1 \cdot 1 \cdot 1 \cdot 1 + 1 \cdot 1 \cdot 2 \cdot 1 + 1 \cdot 2 \cdot 2 \cdot 1 = 7$$

DFAs vs WFAs



$$\mathcal{L}(\varepsilon) = 0$$
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$$\mathcal{L}(aa) = 1 \cdot 1 \cdot 0 + 1 \cdot 1 \cdot 1 + 1 \cdot 2 \cdot 1 = 3$$
$$\mathcal{L}(aaa) = 1 \cdot 1 \cdot 1 \cdot 0 + 1 \cdot 1 \cdot 1 \cdot 1 + 1 \cdot 1 \cdot 2 \cdot 1 + 1 \cdot 2 \cdot 2 \cdot 1 = 7$$

$$\mathcal{L}(a^n) = 2^n - 1$$

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Learning WFAs

Table:

Cells have values from semiring rather than 0,1

Membership Queries:

Return weight associated with word

Equivalence Queries:

Submit hypothesis WFA

Counterexample = work on which weight differs

Learning WFAs

$S, E \leftarrow \{\varepsilon\}$

Repeat until no more counterexamples:

1. Close table (changing S)
2. Equivalence query
3. Add suffixes of counterexample to E

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Learning WFAs

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each lower row a linear combination of upper rows

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each lower row a linear combination of upper rows

Requirement for semiring: solving linear systems of equations should be computable.

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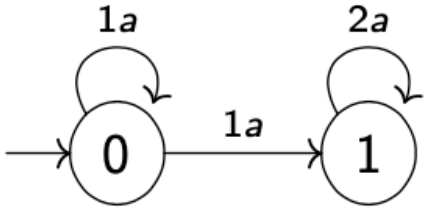
Learning WFAs, example

$$\mathcal{L}(a^n) = 2^n - 1$$



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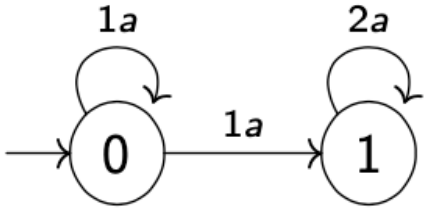
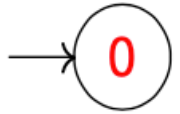
Learning WFAs, example

$$\mathcal{L}(a^n) = 2^n - 1$$

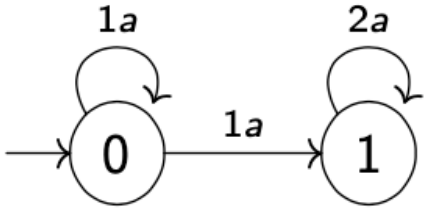


	ε
ε	0
a	1

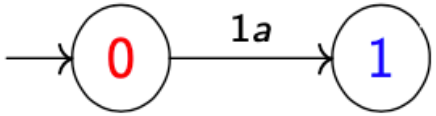


Learning WFAs, example

$$\mathcal{L}(a^n) = 2^n - 1$$



	ε
ε	0
a	1
aa	3



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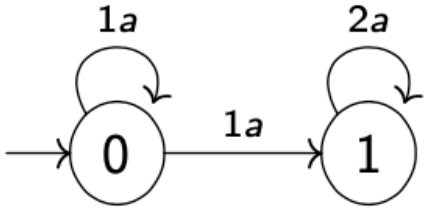

	ε
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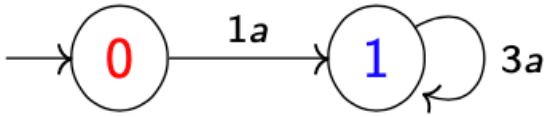

Counterexample: aaa

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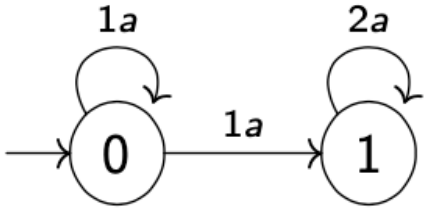
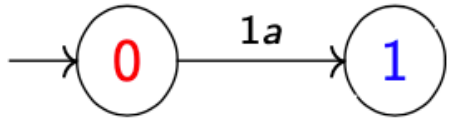
Learning WFAs, example

$$\mathcal{L}(a^n) = 2^n - 1$$



	ε	a	aa	aaa
ε	0	1	3	7
a	1	3	7	15
aa	3	7	15	31



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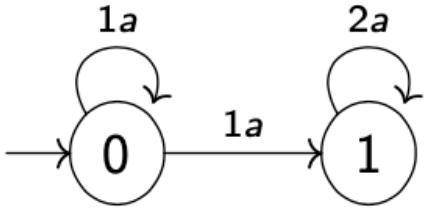
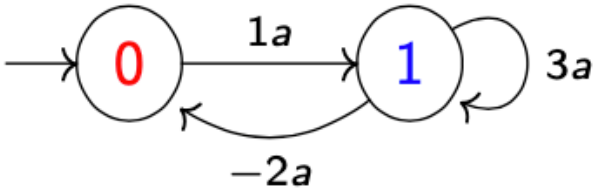
$$\mathcal{L}(a^n) = 2^n - 1$$



	ε	a	aa	aaa
ε	0	1	3	7
a	1	3	7	15
aa	3	7	15	31



Does it terminate?

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L* for DFAs: termination consequence of regular languages having finitely many Myhill-Nerode classes = minimal DFA

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L* for WFAs: depends on the semiring

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Field: *terminates as a consequence of basis existence, our algorithm instantiates one of Bergadano and Varricchio (1996)*

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Boolean semiring: *WFAs = NFAs our algorithm instantiates one of Bollig et al (2009)*

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L* for DFAs: termination consequence of regular languages having finitely many Myhill-Nerode classes = minimal DFA

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Does it terminate?

L* for DFAs: termination consequence of regular languages having finitely many Myhill-Nerode classes = minimal DFA

L* for WFAs: depends on the semiring

Field: *terminates as a consequence of basis existence, our algorithm instantiates one of Bergadano and Varricchio (1996)*

Boolean semiring: *WFAs = NFAs our algorithm instantiates one of Bollig et al (2009)*

NFAs have no minimal representative!

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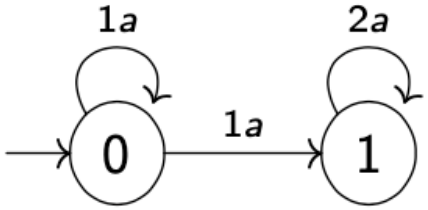
Learning WFAs

Does it terminate for any semiring?

No.

The bad semiring

$$\mathcal{L}(a^n) = 2^n - 1$$



Learning over $\mathbb{Q}$

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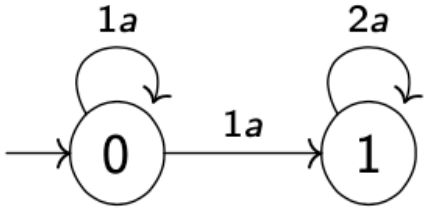
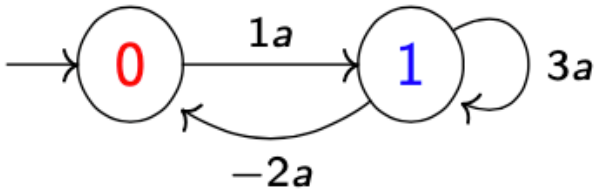
The bad semiring

$$\mathcal{L}(a^n) = 2^n - 1$$



Learning over $\mathbb{Q}$



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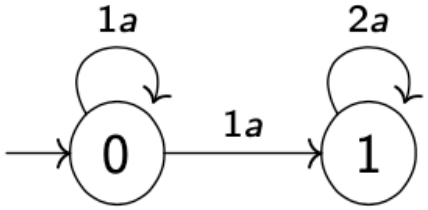
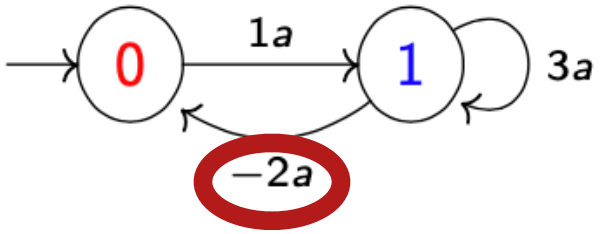
The bad semiring

$$\mathcal{L}(a^n) = 2^n - 1$$



Learning over $\mathbb{Q}$



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The bad semiring

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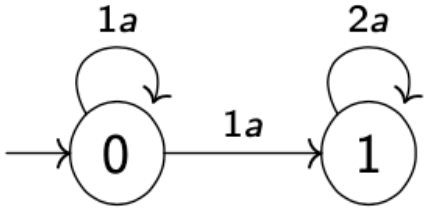
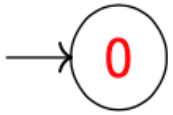


	ε	a
ε	0	1
a	1	3



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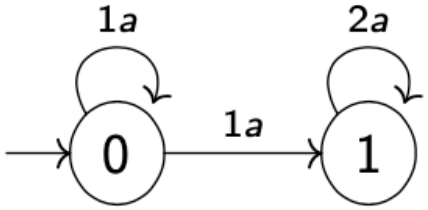
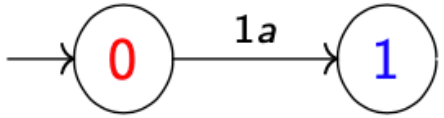


	ε	a
ε	0	1
a	1	3
aa	3	7



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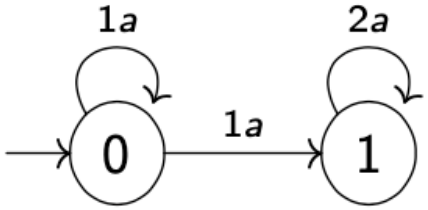
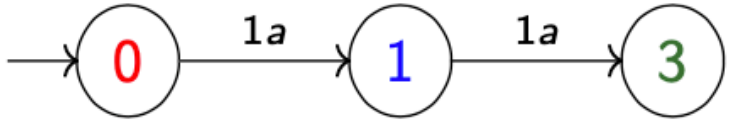


	ε	a
ε	0	1
a	1	3
aa	3	7
aaa	7	15



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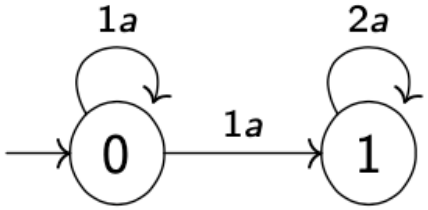
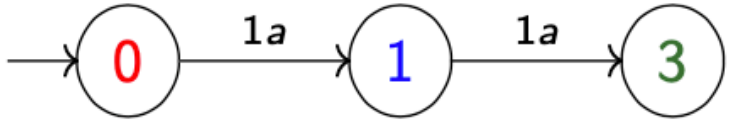


	ε	a
ε	0	1
a	1	3
aa	3	7
aaa	7	15



...

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Termination issue

The algorithm approximates the Hankel matrix of the language. Linear combinations of rows in:

	ε	a	aa	aaa	...
ε	0	1	3	7	
a	1	3	7	15	
aa	3	7	15	31	...
aaa	7	15	31	63	
...			...		

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	ε	a	aa	aaa	...
ε	0	1	3	7	
a	1	3	7	15	
aa	3	7	15	31	...
aaa	7	15	31	63	
...			...		

This is not finitely generated (over commutative monoids)

Termination Conditions

Algorithm terminates assuming:

Termination Conditions

Algorithm terminates assuming:

Progress measure with bound

number, increases when rows separate via extra column

Ascending chain condition on Hankel matrix

Subsemimodule chains converge: if $S_1 \subseteq S_2 \subseteq \cdots \subseteq H$ are subsemimodules, then $\exists n :$

$$S_n = S_{n+1} = S_{n+2} = \cdots$$

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Termination Argument

Termination Argument

Assume:

Progress measure with bound

Ascending chain condition on Hankel Matrix

Termination Argument

Assume:

Progress measure with bound

Ascending chain condition on Hankel Matrix

Then:

Modules generated by (S_n, A*) form chain below Hankel matrix

Converges, from that point on closedness guaranteed

Bounded progress measure ⇒ finitely many counterexamples

Counter example always leads to closedness defect or rows distinguished by new column

Summary

Main ingredients for effective terminating algorithm:

1. Progress measure with bound
2. Ascending chain condition on Hankel matrix
3. Procedure to determine/fix closedness: solvability of finite system of linear equations

Learning over a field

Learning over a field

1. Progress measure with bound
   - Dimension of vector space spanned by table
   - ≤ minimal WFA size

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   - Dimension of vector space spanned by table
   - ≤ minimal WFA size
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   - Vector space dimension increases with strict inclusion
   - Minimal WFA size = Hankel matrix dimension
3. Procedure for closedness: solvability of finite system of linear equations
   - Gaussian elimination

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Learning over a finite semiring

Learning over a finite semiring

1. Progress measure with bound
   - Set size of semimodule spanned by table
   - ≤ determinization of correct automaton

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3. Procedure for closedness: solvability of finite system of linear equations
   - Try all linear combinations of rows

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Fields and finite semirings OK, can we do more?

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Principal ideal domains

Principal ideal domain = ***integral domain*** with ***all ideals principal***

Integral domain: commutative ring, $ab=0 \Rightarrow a=0 \vee b=0$

All ideals principal: generated by one element

Examples: Z, Z[i], K[x] for K a field

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Modules over PID

A module is free if and only if it is torsion free: $pm=0 \Rightarrow p=0 \vee m=0$

A submodule of a free and finitely generated module is

- Free and **finitely** generated
- With **smaller** (or equal) rank

If a finitely generated free module is a quotient of another, its rank is smaller or equal

Learning over PIDs

Learning over PIDs

1. Progress measure with bound
   - Rank of the module spanned by the table (which is free and f.g!)
   - ≤ rank of Hankel matrix

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   - Solve equations via Smith normal form (exists for PIDs), some further assumptions on computability (hold for integers)

Learning over PIDs

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   - ≤ rank of Hankel matrix
2. Ascending chain condition on Hankel matrix
   - yes
3. Procedure for closedness: solvability of finite system of linear equations
   - Solve equations via Smith normal form (exists for PIDs), some further assumptions on computability (hold for integers)

Learning algorithm terminates for the integers!

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Key takeaway

Active learning algorithm for WFAs:

1. Termination depends on properties of the semiring: works for fields, finite semirings, integers, not naturals.
2. Solvability of finite systems of linear equations essential.

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Active learning algorithm for WFAs:

1. Termination depends on properties of the semiring: works for fields, finite semirings, integers, not naturals.
2. Solvability of finite systems of linear equations essential.

How about probabilistic automata?

$$A^* \to [0, 1]$$

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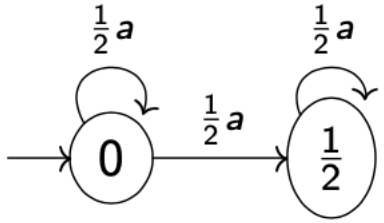
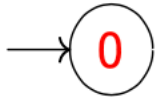
Probabilistic automata



$$\mathcal{L}(a^n) = \frac{n}{2^{n+1}}$$

	ε
ε	0
a	0.25



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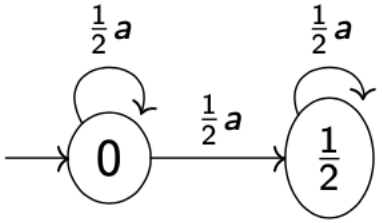
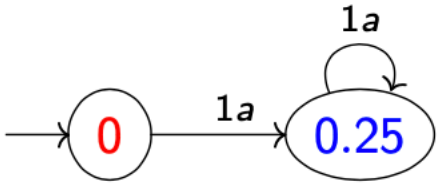
Probabilistic automata

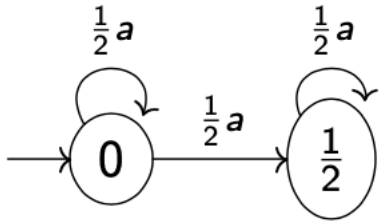


$$\mathcal{L}(a^n) = \frac{n}{2^{n+1}}$$

	ε
ε	0
a	0.25
aa	0.25



Probabilistic automata



$$\mathcal{L}(a^n) = \frac{n}{2^{n+1}}$$

	ε	a
ε	0	0.25
a	0.25	0.25
aa	0.25	0.1875

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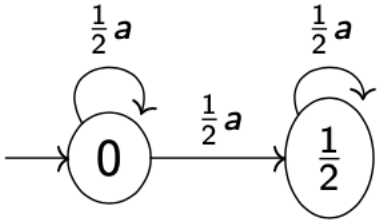
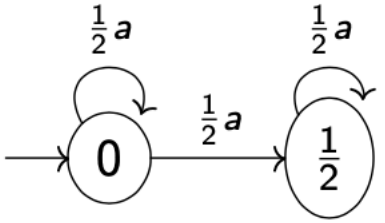
Probabilistic automata



$$\mathcal{L}(a^n) = \frac{n}{2^{n+1}}$$

	ε	a
ε	0	0.25
a	0.25	0.25
aa	0.25	0.1875
aaa	0.1875	0.125

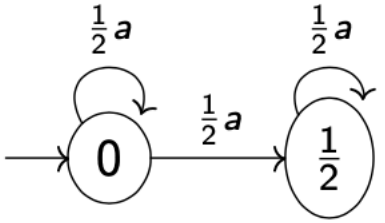
Probabilistic automata



$$\mathcal{L}(a^n) = \frac{n}{2^{n+1}}$$

	ε	a	Ratio
ε	0	0.25	0
a	0.25	0.25	1
aa	0.25	0.1875	1.333...
aaa	0.1875	0.125	1.5

Probabilistic automata



$$\mathcal{L}(a^n) = \frac{n}{2^{n+1}}$$

	ε	a	Ratio
ε	0	0.25	0
a	0.25	0.25	1
aa	0.25	0.1875	1.333...
aaa	0.1875	0.125	1.5

Problem: rows of the table are not finitely generated

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Probabilistic automata, partial solutions

Ongoing work with Leon Witzman

- If we know the number of states of target automaton, there exists an active learning algorithm (that uses at most exponential space);
- Even if the real automaton has rational coefficients the systems we solve might lack rational solutions (real algebraic numbers);
- Use a quadratic program to approximate solution;
- Counterexample handling (to avoid assuming number of states) unclear;
- Learning happens in convex sets, can we use this algebraic structure?

???

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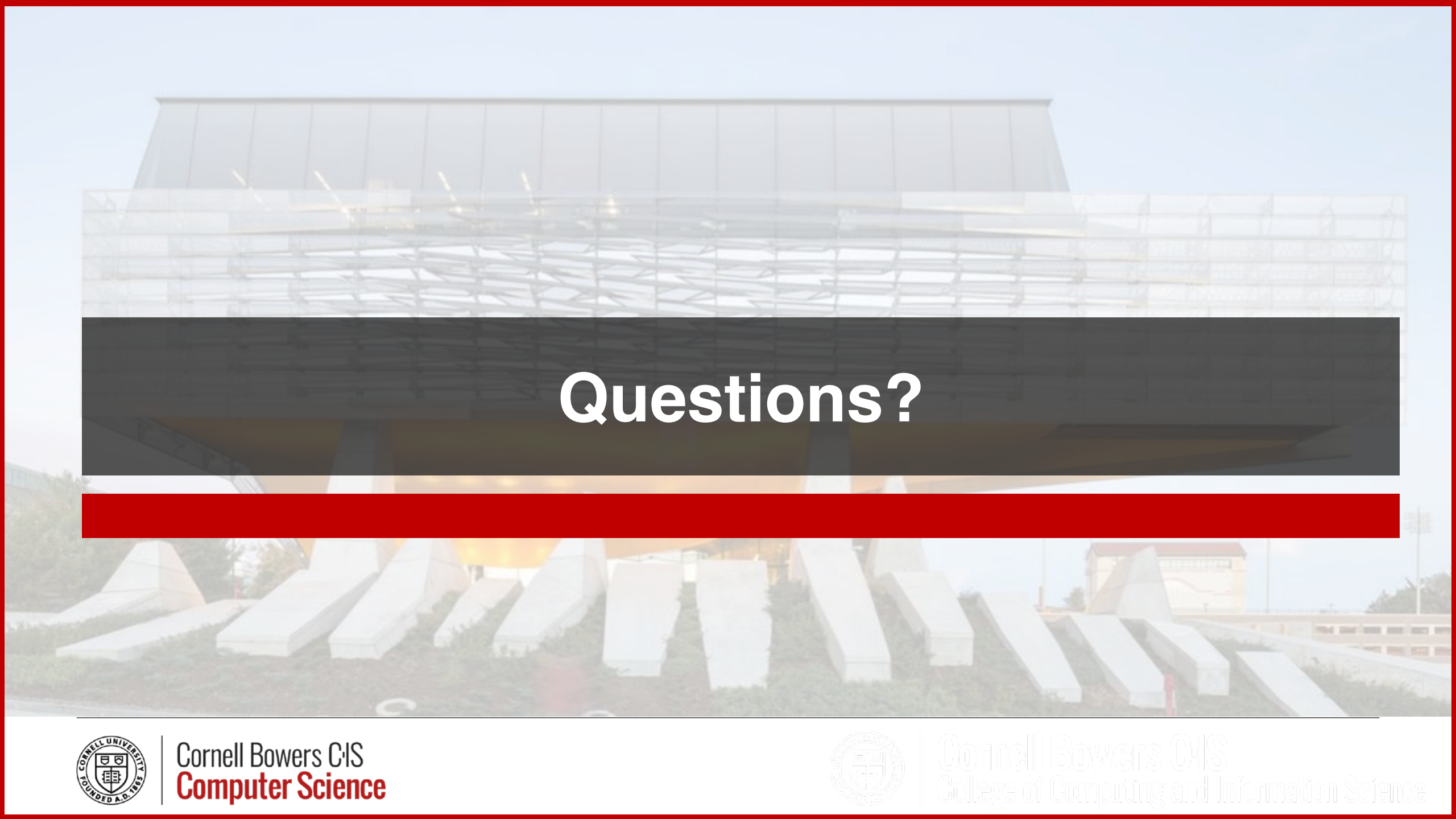

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