A DOUBLE-CATEGORICAL APPROACH TO LENSES VIA ALGEBRAIC WEAK FACTORISATION SYSTEMS

Bryce Clarke

Inria Saclay, Palaiseau, France

bryceclarke.github.io

Topos Institute Colloquium, 3 November 2022

MOTIVATION

Double categories

- horizontal
- vertical

?

Lenses

- forwards
- backwards

?

AWFS

- left coalgebras
- right algebras

Bourke & Garner ('16)

Johnson, Rosebrugh, & Wood ('10, '12, '13)

OVERVIEW OF THE TALK

1. Examples of lenses (with laws)
2. Basics of AWFS
3. Lenses are the R-algebras, or right class, of an AWFS
4. Basics of double categories
5. The R-algebras of an AWFS are vertical morphisms of a double cat.
6. Lenses are vert. morphisms in the right-connected completion of a double cat.

SPLIT EPIMORPHISMS

- A split epimorphism is a morphism with a chosen section.

$$\begin{tikzcd} & B \arrow[dl, "\varphi"'] \arrow[dr, equal] & \\ A \arrow[rr, "f"'] & & B \end{tikzcd}$$

- The free split epimorphism is constructed using coproducts.

$$\begin{tikzcd} A \arrow[d, "f"'] & & A+B \arrow[d, shift right, "{[f,1_B]}"'] \\ B \arrow[r, phantom, "\rightsquigarrow"] & & B \arrow[u, shift right, "\imath_B"'] \end{tikzcd}$$

CLASSICAL LENSES

- A **classical lens** is a GET morphism together with a PUT morphism

$$\begin{tikzcd} & A\times B \arrow[dl, "p"'] \arrow[dr, "\pi_1"] & \\ A \arrow[rr, "f"'] & & B \end{tikzcd}$$

such that the following commute.

$$\begin{tikzcd} A \arrow[r, "\langle 1_A, f\rangle"] \arrow[dr, equal] & A\times B \arrow[d, "p"] \\ & A \end{tikzcd} \qquad \begin{tikzcd} A\times B\times B \arrow[r, "p\times 1_B"] \arrow[d, "\pi_{0,2}"'] & A\times B \arrow[d, "p"] \\ A\times B \arrow[r, "p"] & B \end{tikzcd}$$

- The **free classical lens** is given by the product projection.

$$\begin{tikzcd} A \arrow[r, "f"] & B \end{tikzcd}$$

$$\rightsquigarrow$$

$$\begin{tikzcd} & A\times B\times B \arrow[dl, "\pi_{0,2}"'] \arrow[dr, "\pi_2"] & \\ A\times B \arrow[rr, "\pi_1"'] & & B \end{tikzcd}$$

SPLIT OPFIBRATIONS

- A split opfibration is a functor with a splitting, i.e. a choice of lifts

$$\begin{tikzcd} A \arrow[d, "f"'] & a \arrow[r, "\varphi(a,u)"] & a' \\ B & fa \arrow[r, "u"] & b \end{tikzcd}$$

such that the following axioms hold.

1. $\varphi(a, 1_{fa}) = 1_a$
2. $\varphi(a, v \circ u) = \varphi(a', v) \circ \varphi(a, u)$
3. $\varphi(a,u)$ is opcartesian.

- The free split opfibration on a functor $f : A \to B$ is given by the comma category projection.

$$\begin{tikzcd} f/1_B \arrow[d, "\pi_B"'] & fa \arrow[r, "f(1_a)"'] \arrow[d, "w"'] & fa \arrow[d, "u \circ w"] \\ & b \arrow[r, "u"] & b' \\ B & b \arrow[r, "u"] & b' \end{tikzcd}$$

DELTA LENSES

- A delta lens is a functor equipped with a lifting operation

$$\begin{array}{ccc} A & & a \xrightarrow{\varphi(a,u)} a' \\ {\scriptstyle f}\downarrow & & \vdots \qquad \vdots \qquad \vdots \\ B & & fa \xrightarrow{u} b \end{array}$$

such that the following axioms hold.

1. $\varphi(a, 1_{fa}) = 1_a$
2. $\varphi(a, v \circ u) = \varphi(a', v) \circ \varphi(a, u)$
3. ~~$\varphi(a,u)$ is opcartesian.~~

- The free delta lens on a functor $f : A \longrightarrow B$ is given by... ???

DELTA LENSES (2ND ATTEMPT)

- A **delta lens** is a compatible **functor** and **cofunctor**, i.e. a diagram

$$\begin{tikzcd} & X \arrow[dl, "\varphi"'] \arrow[dr, "f\varphi"] & \\ A \arrow[rr, "f"'] & & B \end{tikzcd}$$

such that φ is **bijective-on-objects** and $f\varphi$ is a **discrete opfibration**.

- The **free delta lens** on a functor $f: A \to B$ is constructed using the comprehensive factorisation and pushouts along b.o.o. functors

$$\begin{tikzcd} A_0 \arrow[r, "\text{initial}"] \arrow[d, "\text{b.o.o.}"'] & J_f \arrow[d, "\text{b.o.o.}"] \arrow[ddr, bend left, "\text{discrete opfibration}"] & \\ A \arrow[r, "\iota_A"'] \arrow[drr, bend right, "f"'] \arrow[ur, phantom, "\ulcorner" very near end] & A +_{A_0} J \arrow[dr, dashed] & \\ & & B \end{tikzcd}$$

LENSES SUMMARY

- A "lens" has a forwards component and a backwards component.
- Several examples including:
  - split epimorphisms;
  - classical lenses;
  - split opfibrations;
  - delta lenses.
- Lenses are morphisms equipped with algebraic structure.

- Each kind of lens is an algebra of a monad R on $Sq(\mathcal{C}) = \mathcal{C}^2$ over the codomain functor $cod : Sq(\mathcal{C}) \to \mathcal{C}$.
- Each morphism factors through a free lens.

$$\begin{tikzcd} A \arrow[r, "\eta_f"] \arrow[d, "f"'] & \bullet \arrow[d, "Rf"] \\ B \arrow[r, equal] & B \end{tikzcd}$$

- How do we compose lenses?

A BRIEF INTRODUCTION TO AWFS

- An algebraic weak factorisation system on $\mathcal{C}$ consists of:
  - A functorial factorisation (L,E,R):

$$\begin{tikzcd} A \arrow[r, "Lf"'] \arrow[d, "h"'] \arrow[rr, bend left, "f"] & Ef \arrow[r, "Rf"'] \arrow[d, "E\langle h,k\rangle"] & B \arrow[d, "k"] \\ C \arrow[r, "Lg"] \arrow[rr, bend right, "g"'] & Eg \arrow[r, "Rg"] & D \end{tikzcd}$$

  - Comonad (L,ε,Δ) & monad (R,η,μ)
  - Distributive law $\delta : LR \Rightarrow RL$.

- Given an L-coalgebra (f,p) and an R-algebra (g,q) and a square

$$\begin{tikzcd} A \arrow[rrr, "h"] \arrow[dr, "Lf"] \arrow[ddd, "f"'] & & & C \arrow[dl, "Rf"] \arrow[ddd, "g"] \\ & Ef \arrow[r, "E\langle h,k\rangle"] \arrow[dl, "Rf"] & Eg \arrow[ur, "q"] \arrow[dr, "Rg"] & \\ & & & \\ B \arrow[ur, "p"] \arrow[rrr, "k"'] & & & D \end{tikzcd}$$

there is a canonical diagonal filler.

$$\varphi_{f,g}(h,k) = q \circ E\langle h,k\rangle \circ p : B \longrightarrow C$$

COMPOSING R-ALGEBRAS VIA LIFTING

- We may compose R-algebras:

$$\begin{tikzcd}
A \arrow[r, equal] \arrow[dd, "{(Lh,\Delta_h)}"'] \arrow[dr, "f"] & A \arrow[d, "{(f,p)}"] \\
 & B \arrow[d, "{(g,q)}"] \\
Eh \arrow[r, "Rh"'] \arrow[ur, "{\varphi(f,Rh)}"] \arrow[uur, "{\varphi(1_A,\varphi(f,Rh))}"] & C
\end{tikzcd}$$

- The composite R-algebra on $h=gf$ is:

$$\varphi(1_A,\varphi(f,Rh)) : Eh \longrightarrow A$$

- Example: composition of classical lenses as R-algebras.

$$\begin{tikzcd}
A \arrow[r, equal] \arrow[dddd, "{\langle 1_A, gf\rangle}"'] \arrow[ddr, "f"] & A \arrow[dd, "f"] & \\
 & & A\times B \arrow[ul, "p"'] \arrow[dl, "\pi_B"] \\
 & B \arrow[dd, "g"] & \\
 & & B\times C \arrow[ul, "q"'] \arrow[dl, "\pi_C"] \\
A\times C \arrow[r, "\pi_C"'] \arrow[uur, "{q\circ(f\times 1_C)}"] \arrow[uuuur, "{p\circ\langle \pi_A, q\circ(f\times 1_C)\rangle}"] & C &
\end{tikzcd}$$

LENSES & LIFTING

- Classical lenses admit lifts against split monomorphisms:

$$\begin{tikzcd}
A \arrow[rrr, "h"] \arrow[dr, "1_A"] \arrow[dd, "f"'] & & & C \arrow[dd, "g"] \\
& A \arrow[r, "\langle h, kf \rangle"] & C \times D \arrow[ur, "q"] \arrow[dr, "\pi_D"'] & \\
B \arrow[ur, "p"] \arrow[rrr, "k"'] & & & D
\end{tikzcd}$$

- Split opfibrations admit lifts against LARI functors. For example:

$$\begin{tikzcd}
\{0\} \arrow[r, "a"] \arrow[d] & A \arrow[d, "f"] \\
\{0 \to 1\} \arrow[r, "u"'] \arrow[ur, "\varphi(a,u)"] & B
\end{tikzcd}$$

LENSES & AWFS

Lenses

AWFS

backwards component = algebra structure

$$\begin{tikzcd} & Ef \arrow[dl, "p"'] \arrow[dr, "Rf"] & \\ A \arrow[rr, "f"'] & & B \end{tikzcd}$$

forwards component = underlying morphism

- BIG IDEA: Lenses are R-algebras for an AWFS.
- We have morphisms of lenses, sequential composition of lenses, and chosen lifts against L-coalgebras.

DOUBLE CATEGORIES

A **double category** $\mathbb{D}$ consists of:

- objects $A, B, C, \dots$
- **horizontal** morphisms $\bullet \longrightarrow \bullet$
- **vertical** morphisms $\bullet \xrightarrow{\ \mid\ } \bullet$
- cells

$$\begin{tikzcd} A \arrow[r, "h"] \arrow[d, "f"', "|" marking] \arrow[dr, phantom, "\theta"] & C \arrow[d, "g", "|" marking] \\ B \arrow[r, "k"'] & D \end{tikzcd}$$

which compose horizontally & vertically.

- A **double category** is a category object in $\mathcal{CAT}$:

$$\begin{tikzcd} \mathbb{D}_0 \arrow[r, "id" description] & \mathbb{D}_1 \arrow[l, shift right=2, "dom"'] \arrow[l, shift left=2, "cod"] & \mathbb{D}_1 \times_{\mathbb{D}_0} \mathbb{D}_1 \arrow[l, "comp"'] \end{tikzcd}$$

- For a category $\mathcal{C}$, there is a double category $\mathbb{S}q(\mathcal{C})$ with cells:

$$\begin{tikzcd} A \arrow[r, "h"] \arrow[d, "f"'] \arrow[dr, phantom, "\circlearrowright"] & C \arrow[d, "g"] \\ B \arrow[r, "k"'] & D \end{tikzcd}$$

TWO IMPORTANT PROPERTIES

- $\mathbb{D}$ is **right-connected** if $\mathrm{cod} \dashv \mathrm{id}$.

$$\begin{tikzcd} A \arrow[r, "h"] \arrow[d, "f"'] \arrow[dr, phantom, "\theta"] & C \arrow[r, "\bar{g}"] \arrow[d, "g"'] \arrow[dr, phantom, "\rho_g"] & D \arrow[d, "1_D"] \\ B \arrow[r, "k"'] & D \arrow[r, "1_D"'] & D \end{tikzcd}$$

$$=$$

$$\begin{tikzcd} A \arrow[r, "\bar{f}"] \arrow[d, "f"'] \arrow[dr, phantom, "\rho_f"] & B \arrow[r, "k"] \arrow[d, "1_B"] \arrow[dr, phantom, "1_k"] & D \arrow[d, "1_D"] \\ B \arrow[r, "1_B"'] & B \arrow[r, "k"'] & D \end{tikzcd}$$

$$\mathbb{D} \xrightarrow{\;U\;} \mathbb{S}q(\mathcal{D}_0)$$

$$\begin{tikzcd} A \arrow[r, "h"] \arrow[d, "f"'] \arrow[dr, phantom, "\theta"] & C \arrow[d, "g"] \\ B \arrow[r, "k"'] & D \end{tikzcd} \longmapsto \begin{tikzcd} A \arrow[r, "h"] \arrow[d, "\bar{f}"'] \arrow[dr, phantom, "\circlearrowright"] & C \arrow[d, "\bar{g}"] \\ B \arrow[r, "k"'] & D \end{tikzcd}$$

- A right-connected double cat. $\mathbb{D}$ is **monadic** if

$$\mathcal{D}_1 \xrightarrow{\;U_1\;} Sq(\mathcal{D}_0)$$

is strictly monadic.

AWFS & DOUBLE CATEGORIES

AWFS

Double cats.

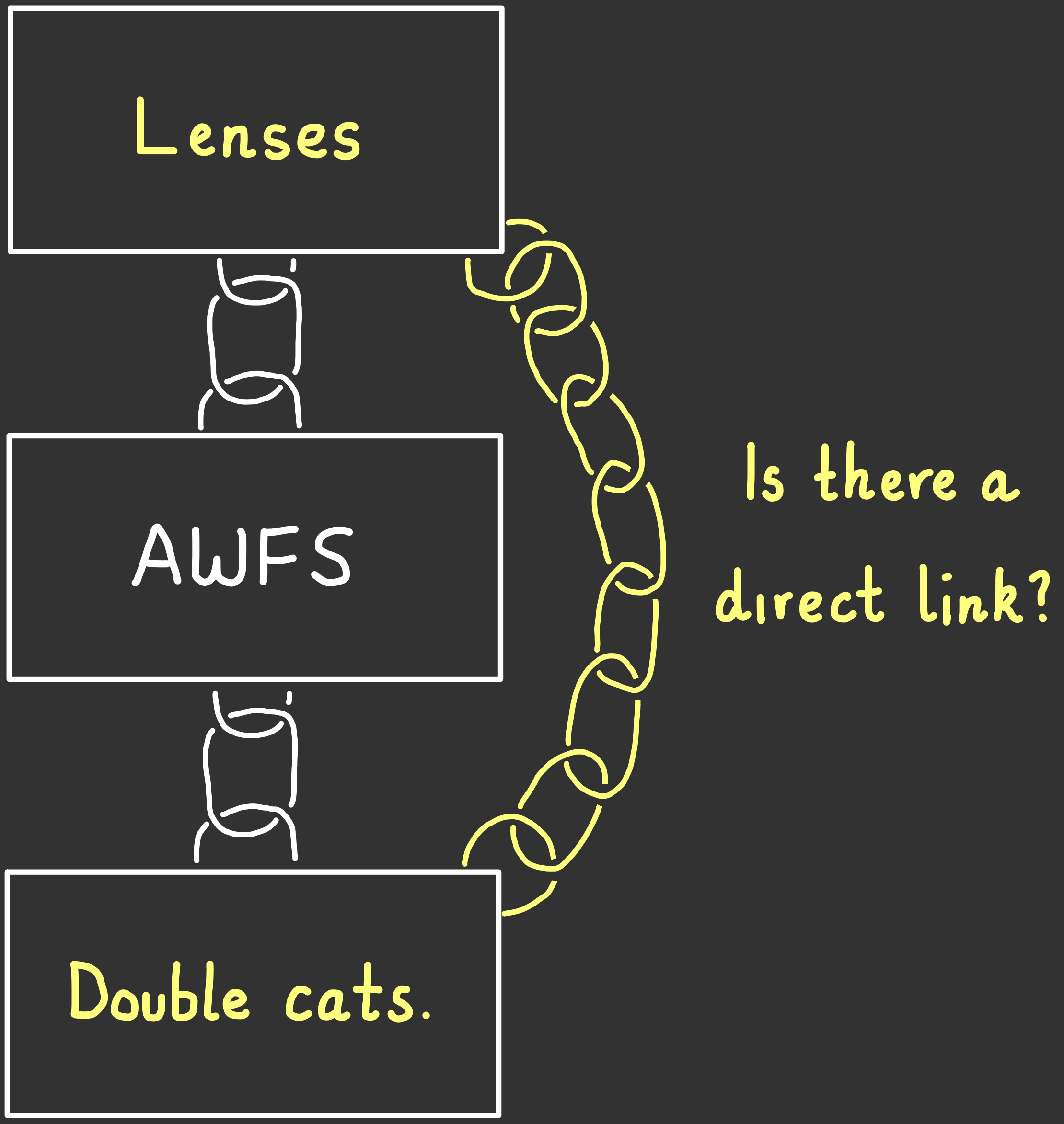
horizontal arrows = all morphisms

$$\begin{tikzcd} A \arrow[r, "h"] \arrow[d, "{(f,p)}"', "|" marking] & C \arrow[d, "{(g,q)}", "|" marking] \\ B \arrow[r, "k"'] & D \end{tikzcd}$$

vertical arrows = R-algebras

- BIG IDEA: Each AWFS yields a double category of algebras R-$\mathbb{A}$lg.
- Each monadic right-connected double category yields an AWFS.

ANOTHER APPROACH TO LENSES?



- For each kind of lens, there is a double category with cells:

$$\begin{tikzcd} A \arrow[r, "\text{morphism}"] \arrow[d, "\text{lens}"'] & C \arrow[d, "\text{lens}"] \\ B \arrow[r, "\text{morphism}"'] & D \end{tikzcd}$$

- What if the backwards component was an independent morphism rather than algebraic structure?

THE RIGHT-CONNECTED COMPLETION

The **right-connected completion** $\mathbb{\Gamma}(\mathbb{D})$ of a double category $\mathbb{D}$ has:

- the same objects and horizontal morphisms as $\mathbb{D}$;
- vertical morphisms $(f,\alpha,f') : A \nrightarrow B$ given by cells in $\mathbb{D}$ of the form:

"lens"

$$
\begin{tikzcd}
A \arrow[r, "f'"] \arrow[d, "f"', "|" marking] \arrow[dr, phantom, "\alpha"] & B \arrow[d, "1_B", "|" marking] \\
B \arrow[r, "1_B"'] & B
\end{tikzcd}
$$

- cells given by cells θ in $\mathbb{D}$ satisfying the condition:

$$
\begin{tikzcd}
A \arrow[r, "h"] \arrow[d, "f"', "|" marking] \arrow[dr, phantom, "\theta"] & C \arrow[r, "g'"] \arrow[d, "g"', "|" marking] \arrow[dr, phantom, "\beta"] & D \arrow[d, "1_D", "|" marking] \\
B \arrow[r, "k"'] & D \arrow[r, "1_D"'] & D
\end{tikzcd}
=
\begin{tikzcd}
A \arrow[r, "f'"] \arrow[d, "f"', "|" marking] \arrow[dr, phantom, "\alpha"] & B \arrow[r, "k"] \arrow[d, "1_B", "|" marking] \arrow[dr, phantom, "1_k"] & D \arrow[d, "1_D", "|" marking] \\
B \arrow[r, "1_B"'] & B \arrow[r, "k"'] & D
\end{tikzcd}
$$

THE RIGHT-CONNECTED COMPLETION

- **Composition** of vertical morphisms in $\mathbb{\Gamma}(\mathbb{D})$ is defined using the composition of cells in $\mathbb{D}$:

$$
\begin{tikzcd}
A \arrow[d, "{(f,\alpha,f')}"'] \\
B \arrow[d, "{(g,\beta,g')}"'] \\
C
\end{tikzcd}
\quad \rightsquigarrow \quad
\begin{tikzcd}
A \arrow[r, "f'"] \arrow[d, "f"'] \arrow[dr, phantom, "\alpha"] & B \arrow[r, "g'"] \arrow[d, "1_B"] \arrow[dr, phantom, "1_{g'}"] & C \arrow[d, "1_C"] \\
B \arrow[r, "1_B"] \arrow[d, "g"'] \arrow[dr, phantom, "1_g"] & B \arrow[r, "g'"] \arrow[d, "g"'] \arrow[dr, phantom, "\beta"] & C \arrow[d, "1_C"] \\
C \arrow[r, "1_C"'] & C \arrow[r, "1_C"'] & C
\end{tikzcd}
$$

EXAMPLES

- Let $\mathbb{S}\mathsf{q}(\mathcal{C})^{v}$ be the vertical opposite of the double category of squares.

$$\begin{tikzcd} A \arrow[r, "h"] & C \\ B \arrow[u, "f"] \arrow[r, "k"'] \arrow[ur, phantom, "\circlearrowright"] & D \arrow[u, "g"'] \end{tikzcd}$$

Then $\mathbb{\Gamma}(\mathbb{S}\mathsf{q}(\mathcal{C})^{v})$ is $\mathbb{S}\mathsf{pEpi}(\mathcal{C})$.

$$\begin{tikzcd} A \arrow[r, "f"] & B \\ B \arrow[u, "\varphi"] \arrow[r, equal] \arrow[ur, phantom, "\circlearrowright"] & B \arrow[u, equal] \end{tikzcd}$$

- Let $\mathbb{C}\mathsf{of}$ be the double category of categories, functors, cofunctors, and compatible squares.

Vertical morphisms in $\mathbb{\Gamma}(\mathbb{C}\mathsf{of})$ are delta lenses.

functor

cofunctor

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\varphi"'] \arrow[dr, phantom, "\circlearrowright"] & B \arrow[d, "1_B"] \\ B \arrow[r, "1_B"'] & B \end{tikzcd}$$

COMPARING COMPOSITION

- Split epimorphisms as R-algebras for an AWFS:

$$\begin{tikzcd}
A \arrow[r, equal] \arrow[ddr, "f"] \arrow[dddd, "\iota_A"'] & A \arrow[dd, "f"] & \\
& & A+B \arrow[ul, "{[1_A,\varphi]}"'] \arrow[dl, "{[f,1_B]}"] \\
& B \arrow[dd, "g"] & \\
& & B+C \arrow[ul, "{[1_B,\psi]}"'] \arrow[dl, "{[g,1_C]}"] \\
A+C \arrow[uuuur, "{[1_A,\varphi\psi]}"] \arrow[uur, "{[f,\psi]}"'] \arrow[r, "{[fg,1_C]}"'] & C &
\end{tikzcd}$$

- Split epimorphisms as vertical morphisms in $\mathbb{\Gamma}(\mathbb{S}\mathrm{q}(\mathcal{C})^{\mathrm{v}})$:

$$\begin{tikzcd}
A \arrow[r, "f"] & B \arrow[r, "g"] & C \\
B \arrow[u, "\varphi"] \arrow[r, "1_B"] & B \arrow[u, "1_B"'] \arrow[r, "g"] & C \arrow[u, "1_C"'] \\
C \arrow[u, "\psi"] \arrow[r, "1_C"'] & C \arrow[u, "\psi"] \arrow[r, "1_C"'] & C \arrow[u, "1_C"']
\end{tikzcd}$$

COMONADICITY

- There is a double functor

$$\mathbb{\Gamma}(\mathbb{D}) \xrightarrow{V} \mathbb{D}$$

$$\begin{tikzcd} A \arrow[r, "h"] \arrow[d, "{(f,\alpha,f')}"'] \arrow[dr, phantom, "\theta"] & C \arrow[d, "{(g,\beta,g')}"] \\ B \arrow[r, "k"'] & D \end{tikzcd} \longmapsto \begin{tikzcd} A \arrow[r, "h"] \arrow[d, "f"'] \arrow[dr, phantom, "\theta"] & C \arrow[d, "g"] \\ B \arrow[r, "k"'] & D \end{tikzcd}$$

with underlying functor:

$$\Gamma(\mathbb{D})_1 \xrightarrow{V_1} \mathcal{D}_1$$

When is it **comonadic**?

- V_1 is **comonadic** $\Leftrightarrow$ each fibre $\mathrm{cod}^{-1}\{B\}$ admits products with the vertical identity $1_B : B \twoheadrightarrow B$.

- The functor $V_1 : \mathrm{SpEpi}(\mathcal{C}) \longrightarrow \mathrm{Sq}(\mathcal{C})^{v}$ is comonadic if $\mathcal{C}$ has products.

$$\begin{array}{c} A \\ f\uparrow \\ B \end{array} \rightsquigarrow \begin{array}{c} A\times B \\ \langle f,1_B\rangle \uparrow \downarrow \pi_B \\ B \end{array}$$

WHEN DO THESE APPROACHES AGREE?

INFORMALLY

$$\begin{tikzcd} & Ef \arrow[dl, "p"'] \arrow[dr, "Rf"] & \\ A \arrow[rr, "f"'] & & B \end{tikzcd}$$

morphisms equipped with algebraic structure

$\updownarrow$???

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\varphi"'] \arrow[dr, phantom, "\alpha"] & B \arrow[d, "1_B"] \\ B \arrow[r, "1_B"'] & B \end{tikzcd}$$

horizontal morphisms equipped with vertical morphisms (coalgebraically)

FORMALLY

$$\Gamma(\mathbb{D})_1 \xrightarrow{\text{monadic?}} \mathrm{Sq}(\mathcal{D}_0)$$

- What conditions to ask of $\mathbb{D}$?
- SUFFICIENT (for left adjoint):
  - $\mathrm{dom}: \mathcal{D}_1 \longrightarrow \mathcal{D}_0$ has a lari.
  - $\mathrm{cod}: \mathcal{D}_1 \longrightarrow \mathcal{D}_0$ is an opfibration
- Examples include split epimorphisms and delta lenses. Are there others?

FUTURE DIRECTIONS

- Are <u>all</u> lenses satisfying "laws" the right class of an AWFS?
- What are further interesting examples of $\mathbb{L}\Gamma(\mathbb{D})$?
- Interaction with companions in $\mathbb{D}$:

$$\begin{tikzcd} & \mathbb{L}\Gamma(\mathbb{D}) \arrow[dl, "U"'] \arrow[dr, "V"] & \\ \mathbb{S}q(\mathcal{D}_0) \arrow[rr, "(-)_*"'] \arrow[rr, phantom, shift left=3, "\overset{\text{globular}}{\Longrightarrow}"] & & \mathbb{D} \end{tikzcd}$$

- How can we expand this picture to include other double categories of "lawless" lenses and optics?

$$\begin{tikzcd} A^- & A^+ \times B^- \arrow[l, "f^\#"'] \\ A^+ \arrow[r, "f"] & B^+ \end{tikzcd}$$

- For which $\mathcal{C}$ does a monadic right-connected $\mathbb{D}$ embed fully faithfully into $\mathbb{L}\Gamma(\mathcal{C})$? e.g. $\mathbb{S}p\mathbb{O}pf \hookrightarrow \mathbb{L}\Gamma(\mathcal{C}of)$.

SUMMARY OF THE TALK

INDIRECT APPROACH

$$
\begin{tikzcd}
 & Ef \arrow[dl, "P"'] \arrow[dr, "Rf"] & \\
A \arrow[rr, "f"'] & & B
\end{tikzcd}
$$

- Lenses are morphisms equipped with an R-algebra structure from an AWFS.

- PROS: Nice properties, captures lifting.

DIRECT APPROACH

$$
\begin{tikzcd}
A \arrow[r, "f"] \arrow[d, "\varphi"'] \arrow[dr, phantom, "\alpha"] & B \arrow[d, "1_B"] \\
B \arrow[r, "1_B"'] & B
\end{tikzcd}
$$

- Lenses are horizontal morphisms equipped with a vertical morphism via a cell in a double category.

- PROS: Easy composition, very general.

VIRTUAL DOUBLE CATEGORIES WORKSHOP

28 November to 2 December 2022

Held virtually on Zoom

- Nicolas Behr
- John Bourke
- Matteo Capucci
- Matthew Di Meglio
- Bojana Femić
- Seerp Roald Koudenburg
- Michael Lambert
- Jade Master
- Lyne Moser
- Chad Nester
- Susan Niefield
- Juan Orendain
- Simona Paoli
- Robert Paré
- Claudio Pisani
- Dorette Pronk
- Brandon Shapiro
- Christina Vasilakopoulou
- Paula Verdugo

bryceclarke.github.io/virtual-double-categories-workshop