Symmetry as braided tensor higher category

Xiao-Gang Wen (MIT)

Gravitational anomaly as braided tensor higher category:

Kong Wen arXiv:1405.5858

Kong Wen Zheng arXiv:1502.01690

2025/10/23, Topos Inst

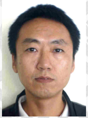
Symmetry as braided tensor higher category:

Ji Wen, arXiv:1912.13492

Kong Lan Wen Zhang Zheng, arXiv:2003.08898, 2005.14178

Chatterjee Wen, arXiv:2203.03596

Kong



Zheng

Ji

Lan

Zhang

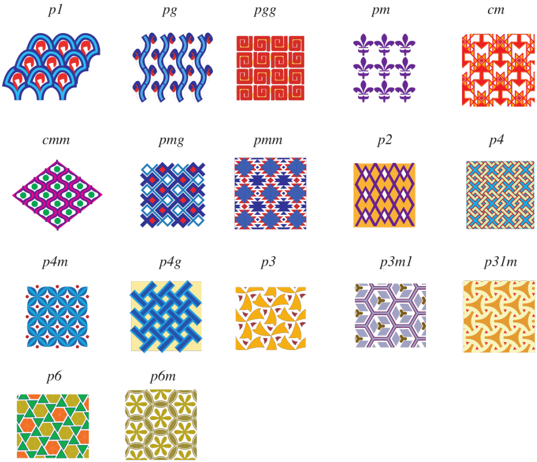
Chatterjee

Symmetry/group are important in physics/math

- **Classical symmetry** = transformations that leave a classical pattern **invariant**, which can be composed: **symmetry as group**

  Symmetries of wallpapers are described by 2-dimensional space groups (17 of them)

- Classification of finite groups = classification of finite symmetries, is a great achievement in mathematics

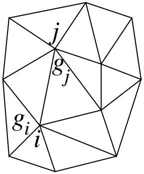


- **Quantum symmetry** is a certain "**structure**" of **quantum systems**, which is in general **beyond group**. Our world is a quantum world. It is quantum symmetry that govern our world

What is a quantum system (with no symmetry)?

A **quantum system** (also known as a lattice system) contains

- A graph with vertices (**sites**) labeled by $i, j, \cdots$
  $\to$ a sense of distance between two vertices i and j
- a total **Hilbert space** $\mathcal{V} = \bigotimes_i \mathcal{V}_i$ with a **tensor product decomposition** $\mathcal{V}_i = \mathbb{C}^k$ = finite Hilbert space on site-i



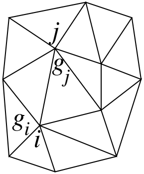
- The **algebra of all local operators**:
  $\mathcal{A} = \{O_i \mid \text{operators acting on } \otimes_{j \text{ near } i} \mathcal{V}_j\}$
- A **local Hamiltonian** (sum of locals): $H = \sum_i O_i$,
  that control dynamics (*ie* time evolution) $\mathrm{i}\partial_t|\psi\rangle = H|\psi\rangle, \ |\psi\rangle \in \mathcal{V}$

A quantum system $\sim$ a pair $(\mathcal{V} = \otimes_i \mathcal{V}_i, H = \sum_i O_i)$

The mathematical models for all quantum materials in physics

- **Remark**: The dimension of the graph is the dimension of the space, which is denoted as nd. We will use $n+1$D to indicate the dimension of space-time, whose space dimension is also nd.

A quantum system with $\mathbb{Z}_2$ symmetry – the old group theory point of view



- a total **Hilbert space** $\mathcal{V} = \bigotimes_i \mathcal{V}_i$ $\qquad \mathbb{Z}_2 = \{\uparrow, \downarrow\}$
  $\mathcal{V}_i = \mathbb{C}^2 = \text{span}\{|\uparrow\rangle, |\downarrow\rangle\}$ = Hilbert space on site-i
  Can be realized by a quantum spin system
  – a magnetic material
- A $\mathbb{Z}_2$ **global symmetry** is described by
  a symmetry transformation $W = \bigotimes_i X_i, \quad X_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
  $W^2 = 1$ generating the $\mathbb{Z}_2$ group (where the Pauli X_i-operator acts on $\mathcal{V}_i$ as $X_i|\uparrow\rangle = |\downarrow\rangle$, $X_i|\downarrow\rangle = |\uparrow\rangle$), *ie* flip spin up↔down
- The **algebra of symmetric local operators**:
  $\mathcal{A}^{\text{symm}} = \{O_i^{\text{symm}} \mid O_i^{\text{symm}} W = W O_i^{\text{symm}}\}$
- The Hamiltonian is a sum of local symmetric operators:
  $H = \sum_i O_i^{\text{symm}} = H^\dagger$, which has the $\mathbb{Z}_2$ symmetry.

A quantum system with a symmetry – the new operator algebra point of view

We do not need to use a transformation to pick **symmetric local operators**. We just need to pick a subset of local operators, which generate an algebra.

- A symmetry is actually defined by the algebra generated a subset of local operators $\mathcal{A} = \{O_i\}$, which is called **local operator sub-algebra (LOsA)**.

- Algebra of all local operators $\to$ trivial symmetry (*ie* no symmetry).
- Algebra of some local operators $\to$ non-trivial symmetry

- The Hamiltonian is a sum of local operators in the LOsA $\mathcal{A}$ $H = \sum_i O_i$ which has a **constrained dynamics** $\mathrm{i}\partial_t|\psi\rangle = H|\psi\rangle$, which means has a symmetry.

Symmetries are classified by braided tensor higher categories

Equivalent operator algebras $\leftrightarrow$ holo-equivalent symmetries

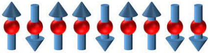
- **Conjecture**: holo-equivalent **symmetries in n-dimensional space are classified by braided tensor n-categories** which are centers of fusion n-categories. Those n-categories are called **symBTC**. They replace group to describe quantum symmetry.

- In 1-dimensional space, holo-equivalent symmetries are classified by braided tensor categories (*ie* MTCs) which are centers of fusion categories, or by **symMTC**s. They are described by the following data $(N, N_c^{ab}, \theta_a, F_d^{abc})$:

$N \in \mathbb{N} \to$ number of objects,
$N_c^{ab} \in \mathbb{N} \to$ fusion ring, where a, b, c label objects
$\theta_a \in U(1) \to$ the topological spin of object-a,
$F_d^{abc} \in \mathbb{C} \to$ the associator of the fusion, *etc*

Conservation law and fusion rule

- **Noether's theorem**: every continuous symmetry has a corresponding conservation law.
- Finite symmetry gives rise to a **fusion rule** (discrete conservation law)

  Consider an **Ising model** in 1-dimensional lattice (describing some magnetic materials) $H_{\text{Is}} = -\sum_{i\in\mathbb{Z}} JZ_iZ_{i+1} + hX_i$ 

  where $X_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $Z_i = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ in $\uparrow, \downarrow$ basis.

- X_i flips up-down spins: $X_i|\downarrow\rangle = |\uparrow\rangle, \quad X_i|\uparrow\rangle = |\downarrow\rangle$.
- Z_i preserve up-down spins: $Z_i|\uparrow\rangle = |\uparrow\rangle, \quad Z_i|\downarrow\rangle = -|\downarrow\rangle$.

- The LOsA $\mathcal{A}$ is generated by $Z_iZ_{i+1},\ X_i$, which contains all the allowed operations and encode the symmetry of the spin system.
- **A conservation law**: the actions of $Z_iZ_{i+1},\ X_i$ in the LOsA $\mathcal{A}$ have a **mod-2 conservation** for the number of **domain walls** (m's) $\quad \uparrow\uparrow\uparrow\uparrow \mid \downarrow\downarrow\downarrow\downarrow\downarrow\downarrow \mid \uparrow\uparrow\uparrow\uparrow$.

Another mod-2 conservation and fusion rule

- Choose the left-right spin basis for the local Hilbert space $\mathcal{V}_i$ on each site: $\leftarrow = \frac{|\uparrow\rangle+|\downarrow\rangle}{\sqrt{2}}, \quad \rightarrow = \frac{|\uparrow\rangle-|\downarrow\rangle}{\sqrt{2}}$.

- Z_i flips left-right spins: $Z_i|\leftarrow\rangle = |\rightarrow\rangle, \quad Z_i|\rightarrow\rangle = |\leftarrow\rangle$.
- X_i preserve left-right spins: $X_i|\rightarrow\rangle = |\rightarrow\rangle, \quad X_i|\leftarrow\rangle = -|\leftarrow\rangle$.

- Let $\cdots \rightarrow\rightarrow\rightarrow \cdots$ be the reference state (the vacuum).

  the actions of $Z_iZ_{i+1}, \ X_i$ in the LOsA $\mathcal{A}$ have a **mod-2 conservation** of the number of $\leftarrow$ spins (the $\mathbb{Z}_2$-**charge** e's)

  $$\rightarrow\rightarrow\rightarrow\rightarrow \leftarrow \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \leftarrow \rightarrow\rightarrow\rightarrow\rightarrow$$

- **The mod-2 conservations are encoded by a fusion rule.**

  $\mathbf{1}$ = no-excitations (no charges, no domain-walls)

  $e = \mathbb{Z}_2$-charge excitation

  m = domain-wall excitation

  $f = e \otimes m$ = charge/domain-wall bound state $\rightarrow$ $N = 4$ objects

$$e \otimes e = m \otimes m = f \otimes f = \mathbf{1}, \quad f = e \otimes m, \ e = f \otimes m, \ m = e \otimes m.$$

Compute $\theta_a,\ F^{abc}_d$ of symMTC via operator algebra

We see that 1-dimensional Ising model $H = -\sum_{i\in\mathbb{Z}} JZ_i Z_{i+1} + hX_i$ has a symmetry described by a symMTC with 4 objects $\mathbf{1}, e, m, f$ that form a $\mathbb{Z}_2 \times \mathbb{Z}_2$ fusion category. How to compute other data?

- The LOsA is generated by $X_i, Z_i Z_{i+1}$, which contains non local operators. The product of local operators can generate extended string operators, such as $X_1 Z_1 Z_2 X_3 X_4 Z_5 Z_6 X_7 X_8 \to$ **patch operator**

Compute θ_a, F_d^{abc} of symMTC via operator algebra

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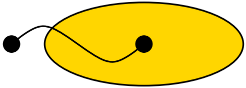
- **Transparent-patch operators**

$$O_{\text{patch}} O_{\text{LOsA}} = O_{\text{LOsA}} O_{\text{patch}}$$

for any local operator in LOsA $\mathcal{A}$, $O_{\text{LOsA}} \in \mathcal{A}$, that is far away from the boundary ∂patch.

- Patch can be string in 1d
- Patch can be string, disk in 2d
- Patch can be string, disk, ball in 3d



Equivalent transparent-patch operators and objects of symMTC

- Equivalent transparent-patch operators Ji Wen, arXiv:1912.13492

$$O_{\text{patch}} \sim O_{\text{patch}} O_{\text{LOsA}},$$

if the local operator $O_{\text{LOsA}} \in \mathcal{A}$ is near the boundary ∂patch.

- For our Ising model, we have 4 inequivalent classes of transparent-patch operators

$$O^{\mathbf{1}}_{\text{str}_{ij}} = \text{id}, \; O^{e}_{\text{str}_{ij}} = Z_i Z_j, \; O^{m}_{\text{str}_{ij}} = \prod_{k=i}^{j} X_k, \; O^{f}_{\text{str}_{ij}} = Z_{i-1}(\prod_{k=i}^{j} X_k) Z_j$$

Thus $N = 4$ **(the symMTC has 4 simple objects $\mathbf{1}, e, m, f$)**

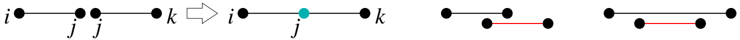
- $O^{m}_{\text{str}_{ij}} = \prod_{k=i}^{j} X_k$ (with non-empty bulk) generates the $\mathbb{Z}_2$ symmetry transformation on the patch and creates a pair of domain-walls m's.
- $O^{e}_{\text{str}_{ij}} = Z_i Z_j = (Z_i Z_{i+1})(Z_{i+1} Z_{i+2}) \cdots$ (with empty bulk) created a pair of $\mathbb{Z}_2$ charges e's.
- $O^{f}_{\text{str}_{ij}} \sim O^{e}_{\text{str}_{ij}} O^{m}_{\text{str}_{ij}}$ is the product of the above two patch operators.

Algebra of transparent-patch operators

$$O^e_{\text{str}_{ij}} O^e_{\text{str}_{ij}} = O^{\mathbf{1}}_{\text{str}_{ij}}, \quad O^m_{\text{str}_{ij}} O^m_{\text{str}_{ij}} = O^{\mathbf{1}}_{\text{str}_{ij}}, \quad O^e_{\text{str}_{ij}} O^m_{\text{str}_{ij}} = O^f_{\text{str}_{ij}}$$

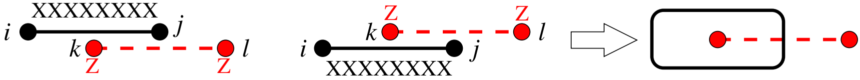
$\to$ Fusion ring N^{ab}_c (conservation) $e \otimes e = \mathbf{1}, \ m \otimes m = \mathbf{1}, \ e \otimes m = f$

$$O^e_{\text{str}_{ij}} O^e_{\text{str}_{jk}} = O^e_{\text{str}_{ik}}, \quad O^m_{\text{str}_{ij}} O^m_{\text{str}_{jk}} = O^m_{\text{str}_{ik}}, \quad O^m_{\text{str}_{ij}} O^e_{\text{str}_{kl}} = \pm O^e_{\text{str}_{kl}} O^m_{\text{str}_{ij}}$$



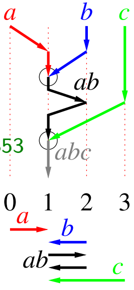
For example: Ji Wen, arXiv:1912.13492

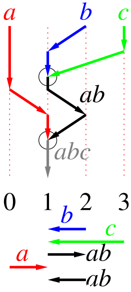
$$O^m_{\text{str}_{ij}} O^e_{\text{str}_{kl}} = -O^e_{\text{str}_{kl}} O^m_{\text{str}_{ij}}, \quad i \ll k \ll j \ll l$$



- Non-trivial braiding between boundaries of transparent-patch operators, $e, m \to$ additional data (mutual statistics $\theta_{em} = \pi$) to describe symmetry beyond conservation (fusion ring $e \otimes e = \mathbf{1}$)

Compute the associater F_d^{abc} from patch operators

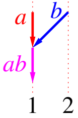




- Different orders of fusions differ by a phase
  $d = a \otimes b \otimes c \to (ab) \otimes c \to ((ab)c)$
  $d = a \otimes b \otimes c \to a \otimes (bc) \to (a(bc))$
  $((ab)c) = F_d^{abc}(a(bc))$ Kawagoe Levin arXiv:1910.11353

- The F-symbol can be computed from the transparent-patch operators $O^e_{\mathrm{str}_{ij}}$ and $O^m_{\mathrm{str}_{ij}}$, which can be viewed as hopping operators for the particles.

- We also need to choose and fix a way how a, b fuses:



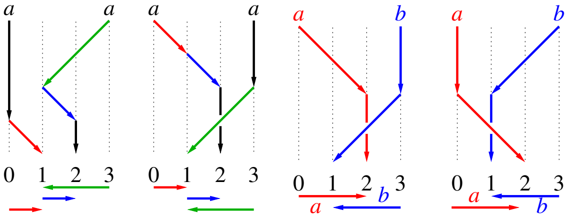
- We obtain the F-symbol via

$$O^a_{\mathrm{str}_{10}} O^b_{\mathrm{str}_{12}} O^{ab}_{\mathrm{str}_{21}} O^{ab}_{\mathrm{str}_{12}} O^c_{\mathrm{str}_{13}} = F_d^{abc} O^b_{\mathrm{str}_{12}} O^c_{\mathrm{str}_{13}} O^{ab}_{\mathrm{str}_{21}} O^a_{\mathrm{str}_{10}} O^{ab}_{\mathrm{str}_{12}}$$

The algebra of transparent-patch operators $O^e_{\mathrm{str}_{ij}}$ and $O^m_{\mathrm{str}_{ij}}$ $\to$ a fusion category $(\mathbf{1}, e, m, f; N^{ee}_{\mathbf{1}} = 1, \cdots; F^{eee}_e = 1, \cdots)$

Compute topological spin θ_a from patch operators

- The patch operator $O^e_{\text{str}_{ij}} = [e_i e_j]$ and $O^m_{\text{str}_{ij}} = [m_i m_j]$ can be viewed as a hopping operator of the e-particle and the m-particle. Using the statistics algebra of hopping operators, we can obtain the self/mutual statistics of the particles $\mathbf{1}, e$:



$O^a_{\text{str}_{13}} O^a_{\text{str}_{21}} O^a_{\text{str}_{10}} = \mathrm{e}^{\mathrm{i}\theta_a} O^a_{\text{str}_{10}} O^a_{\text{str}_{21}} O^a_{\text{str}_{13}}$ Levin-Wen cond-mat/0302460

- The algebra of all transparent-patch operators $(O^e_{\text{str}_{ij}}, O^m_{\text{str}_{ij}}, O^f_{\text{str}_{ij}})$ $\to$ a non-degenerate braided fusion 1-category (*ie* symMTC) $(\mathbf{1}, e, m, f; N^{ab}_c; F^{abc}_d; \mathrm{e}^{\mathrm{i}\theta_a}, \mathrm{e}^{\mathrm{i}\theta_{ab}})$

- $e \otimes e = m \otimes m = f \otimes f = \mathbf{1}$, $e \otimes m = f$, $m \otimes f = e$, $f \otimes e = m$
- $\mathbf{1}, e, m$ are bosons $\theta_{\mathbf{1},e,m} = 1$, and f is a fermion $\theta_f = -1$.
- e, m, f have π mutual statistics between them.

Classify 1d symmetries (up to holo-equivalence)

- Finite symmetry in 1-dimensional space → MTC. Finite symmetry n-dimensional space → non-degenerate braided fusion n-category
  But not every MTC describes a 1d symmetry.

- **Conjecture**: **1d symmetries (up to holo-equivalence) are 1-to-1 classified by MTCs in trivial Witt class**
  Kong Lan Wen Zhang Zheng 2003.08898, 2005.14178; Freed Telemen Moore 2209.07471

- A classification of MTCs (actually modular data) up to rank 12:

# of symm charge/defect (rank)	1	2	3	4	5	6	7	8	9	10	11	12
# of unitary MTCs	1	4	12	18	10	50	28	64	81	76	44	221
# of holo-classes of symm	1	0	0	3	0	0	0	6	6	3	0	3
# of (anomalous) group-symm	$1_{\mathbb{Z}_1}$	0	0	$2_{\mathbb{Z}_2^\omega}$	0	0	0	$6_{S_3^\omega}$	$3_{\mathbb{Z}_3^\omega}$	0	0	0

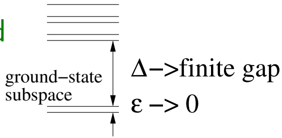
Ng Rowell Wen arXiv:2308.09670

- **Conjecture**: **nd symmetries (up to holo-equivalence) are 1-to-1 classified by braided fusion n-categories which is a center of a fusion n-category**
  Kong Lan Wen Zhang Zheng 2003.08898, 2005.14178; Freed Telemen Moore 2209.07471

Application of symMTC: classify gapped phases of matter

- We can classify all **gapped phases** of symmetric systems whose symmetry is described by a symMTC $\mathcal{M}$.

- A gapped Hamiltonian H is the one with gapped spectrum in large system-size limit.



- Two gapped Hamiltonians are equivalent if they can be connected by a path of gapped Hamiltonians.

- A **gapped phase** is an equivalence class of gapped Hamiltonians.

Gapped phases of symmetric systems with a symMTC $\mathcal{M}$ are 1-to-1 classified by the Lagrangian condensable algebras $\mathcal{A}$ of the symMTC $\mathcal{M}$.

- A **Lagrangian condensable algebra** $\mathcal{A} = \oplus A_a a, A_a \in \mathbb{Z}$
  $\sim$ a maximal set of objects with trivial braiding amoung them

Rank 4 holo-equivalent symmetries in 1d space

Three holo-equivalence classes of $1+1$D symmetries at rank-4:

- $\mathbb{Z}_2$**-symmetry**: symMTC $= \mathcal{D}_{\mathbb{Z}_2}$ (group-like)

  Two gapped phases:

- symmetric phase with a single ground state $\cdots \rightarrow\rightarrow\rightarrow\rightarrow \cdots$
- symmetry broken phase with two degenerate ground states:

  $\cdots \uparrow\uparrow\uparrow\uparrow \cdots$ and $\cdots \downarrow\downarrow\downarrow\downarrow \cdots$

- **Anomalous** $\mathbb{Z}_2$**-symmetry**: symMTC $= \mathcal{D}^{\omega}_{\mathbb{Z}_2}$ (group-like)

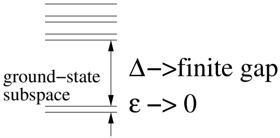
  One gapped phase:

- symmetry broken phase with two degenerate ground states.

- **double-Fibonacci symmetry**: symMTC $= \mathcal{M}_{\text{dFib}}$ (beyond group)

  One gapped phase:

- symmetry broken phase with two degenerate ground states.



Rank 12 holo-equivalent symmetries in 1d space

The rank-12 symmetries are all beyond group (*ie* the symMTCs are not group theoretical). The three symMTCs are Haagerup-Izumi MTC $\mathrm{HI}(1)_0$, $\mathrm{HI}(1)_1$, $\mathrm{HI}(1)_{-1}$.

- $\mathrm{HI}(1)_0$-**symmetry**: symMTC $= \mathrm{HI}(1)_0$
  Three gapped phases:

- symmetry broken phase with two degenerate ground states.
- symmetry broken phase with four degenerate ground states.
- symmetry broken phase with six degenerate ground states.

- $\mathrm{HI}(1)_1$-**symmetry**: symMTC $= \mathrm{HI}(1)_1$
  One gapped phase:

- symmetry broken phase with six degenerate ground states.

- $\mathrm{HI}(1)_{-1}$-**symmetry**: symMTC $= \mathrm{HI}(1)_{-1}$
  One gapped phase:

- symmetry broken phase with six degenerate ground states.

A rank-8 symmetry – S_3 symmetry in 1d space

- The S_3-symmetry is group like and is described by symMTC $\mathcal{D}_{S_3}$

d,s	$1,0$	$1,0$	$2,0$	$2,0$	$2,\frac{1}{3}$	$2,-\frac{1}{3}$	$3,0$	$3,\frac{1}{2}$
$\otimes$	$\mathbf{1}$	a_1	a_2	b	b_1	b_2	c	c_1
$\mathbf{1}$	$\mathbf{1}$	a_1	a_2	b	b_1	b_2	c	c_1
a_1	a_1	$\mathbf{1}$	a_2	b	b_1	b_2	c_1	c
a_2	a_2	a_2	$\mathbf{1}\oplus a_1\oplus a_2$	$b_1\oplus b_2$	$b\oplus b_2$	$b\oplus b_1$	$c\oplus c_1$	$c\oplus c_1$
b	b	b	$b_1\oplus b_2$	$\mathbf{1}\oplus a_1\oplus b$	$b_2\oplus a_2$	$b_1\oplus a_2$	$c\oplus c_1$	$c\oplus c_1$
b_1	b_1	b_1	$b\oplus b_2$	$b_2\oplus a_2$	$\mathbf{1}\oplus a_1\oplus b_1$	$b\oplus a_2$	$c\oplus c_1$	$c\oplus c_1$
b_2	b_2	b_2	$b\oplus b_1$	$b_1\oplus a_2$	$b\oplus a_2$	$\mathbf{1}\oplus a_1\oplus b_2$	$c\oplus c_1$	$c\oplus c_1$
c	c	c_1	$c\oplus c_1$	$c\oplus c_1$	$c\oplus c_1$	$c\oplus c_1$	$\mathbf{1}\oplus a_2\oplus b\oplus b_1\oplus b_2$	$a_1\oplus a_2\oplus b\oplus b_1\oplus b_2$
c_1	c_1	c	$c\oplus c_1$	$c\oplus c_1$	$c\oplus c_1$	$c\oplus c_1$	$a_1\oplus a_2\oplus b\oplus b_1\oplus b_2$	$\mathbf{1}\oplus a_2\oplus b\oplus b_1\oplus b_2$

- From the Lagrangian condensable algebras of $\mathcal{D}_{S_3}$, we find that the symMTC has four gapped phases with ground state degeneracy 1 (symmetric) and $2,3,6$ (symmetry breaking).
  Phase-1 ($\mathcal{A}_1=\mathbf{1}\oplus b\oplus c$), Phase-2 ($\mathcal{A}_2=\mathbf{1}\oplus a_1\oplus 2b$)
  Phase-3 ($\mathcal{A}_3=\mathbf{1}\oplus a_2\oplus c$), Phase-6 ($\mathcal{A}_6=\mathbf{1}\oplus a_1\oplus 2a_2$)
- The symMTC $\mathcal{D}_{S_3}$ has a $\mathbb{Z}_2$ automorphism $a_2\leftrightarrow b$ which gives rise to an automorphism: Phase-1 $\leftrightarrow$ Phase-3, Phase-2 $\leftrightarrow$ Phase-6
- The two phase transitions, (Phase-2 $\leftrightarrow$ Phase-3) and (Phase-1 $\leftrightarrow$ Phase-6), are described by the same critical theory.

Symmetry, group, and beyond

- **Symmetry** is a natural phenomon
- To sovle **polynomial equations**, **Lagrange** (1770) and **Galois** (1829) developed **group theory**.
- **Fedorov** and **Schönflies** used group theory in classical physics to classify 230 **crystals** (1880s).
- **Wigner** and **Weyl** used group theory in quantum physics to describe **quantum symmetries** (1920s).

100 years later, we find that **quantum symmetries** (which is the same as non-invertible gravitational anomalies) are actually described by **unitary braided tensor higher categories** which are centers of fusion higher categories Kong Wen Zheng 1502.01690

Kong Lan Wen Zhang Zheng 2003.08898, 2005.14178; Freed Telemen Moore 2209.07471

Symmetry is so important in physics and the landscaped of physics has changed