EXAMPLES of (∞,∞)-CATEGORIES

joint with

A. Hadzihasanovic
F. Loubaton
V. Ozornova
T. Walde

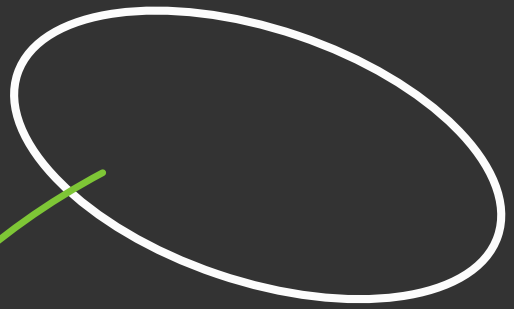
[HLOR]
+
[ORW]

S. Silver
A. Dutta
S. Luneia
V. Zapata-Castro

[DLRS]
+
[DLRZ]

Plan

Plan

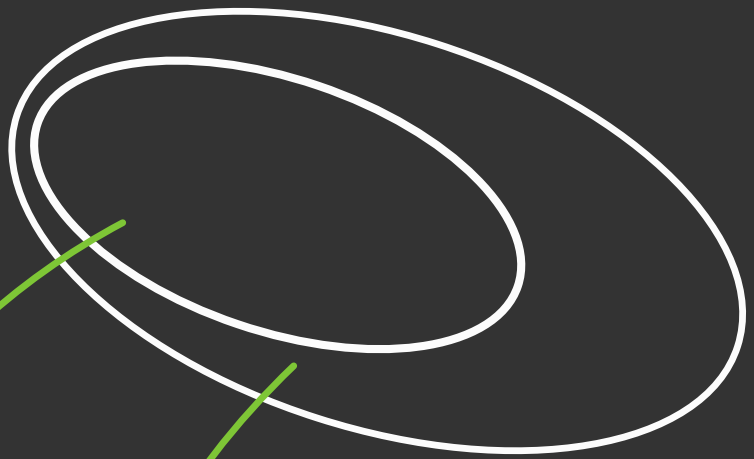


$\mathrm{Cat}_{(\infty,0)}$

$\simeq$

Spaces

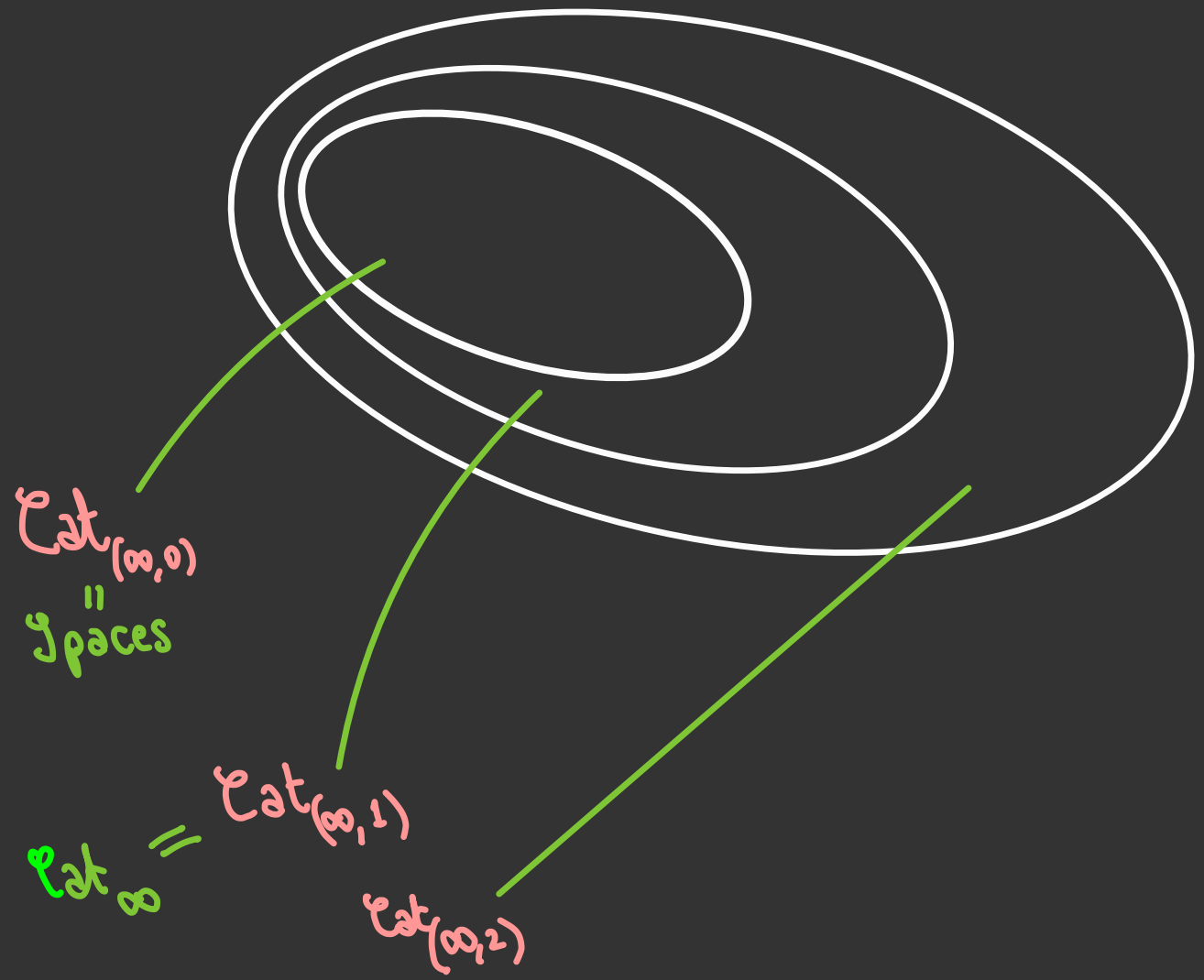
Plan



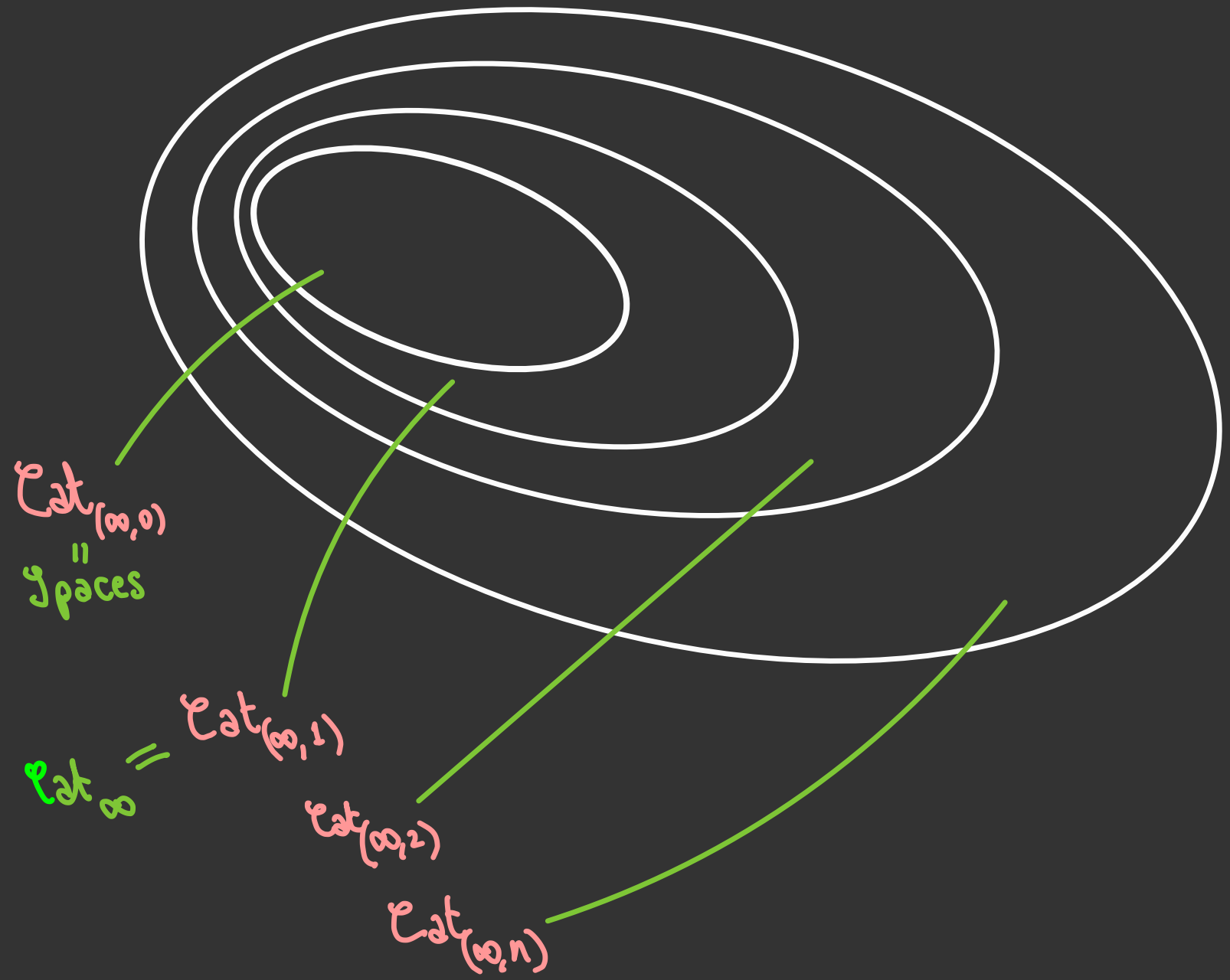
$\mathcal{C}at_{(\infty,0)} = \mathcal{S}paces$

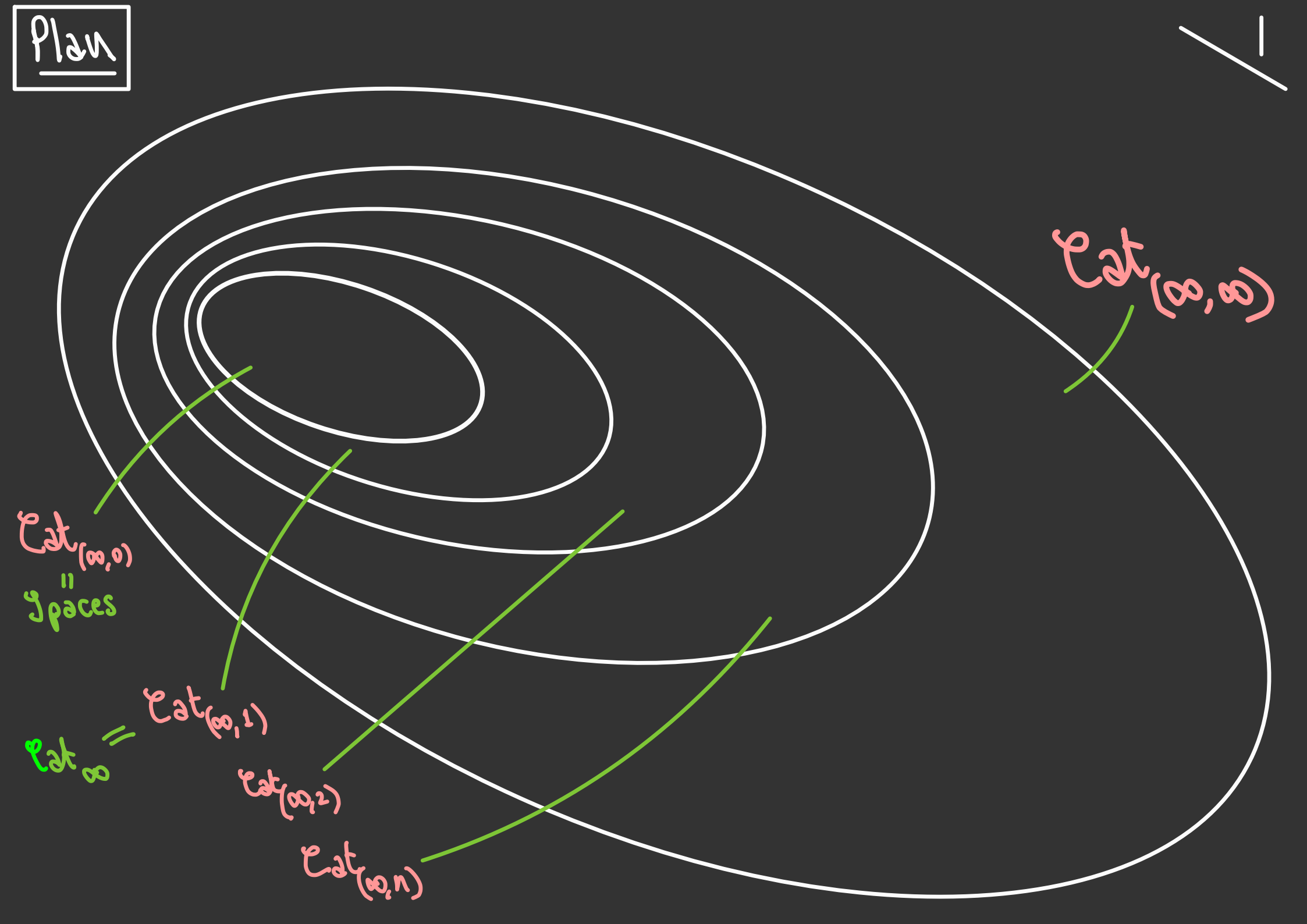
$\mathcal{C}at_\infty = \mathcal{C}at_{(\infty,1)}$

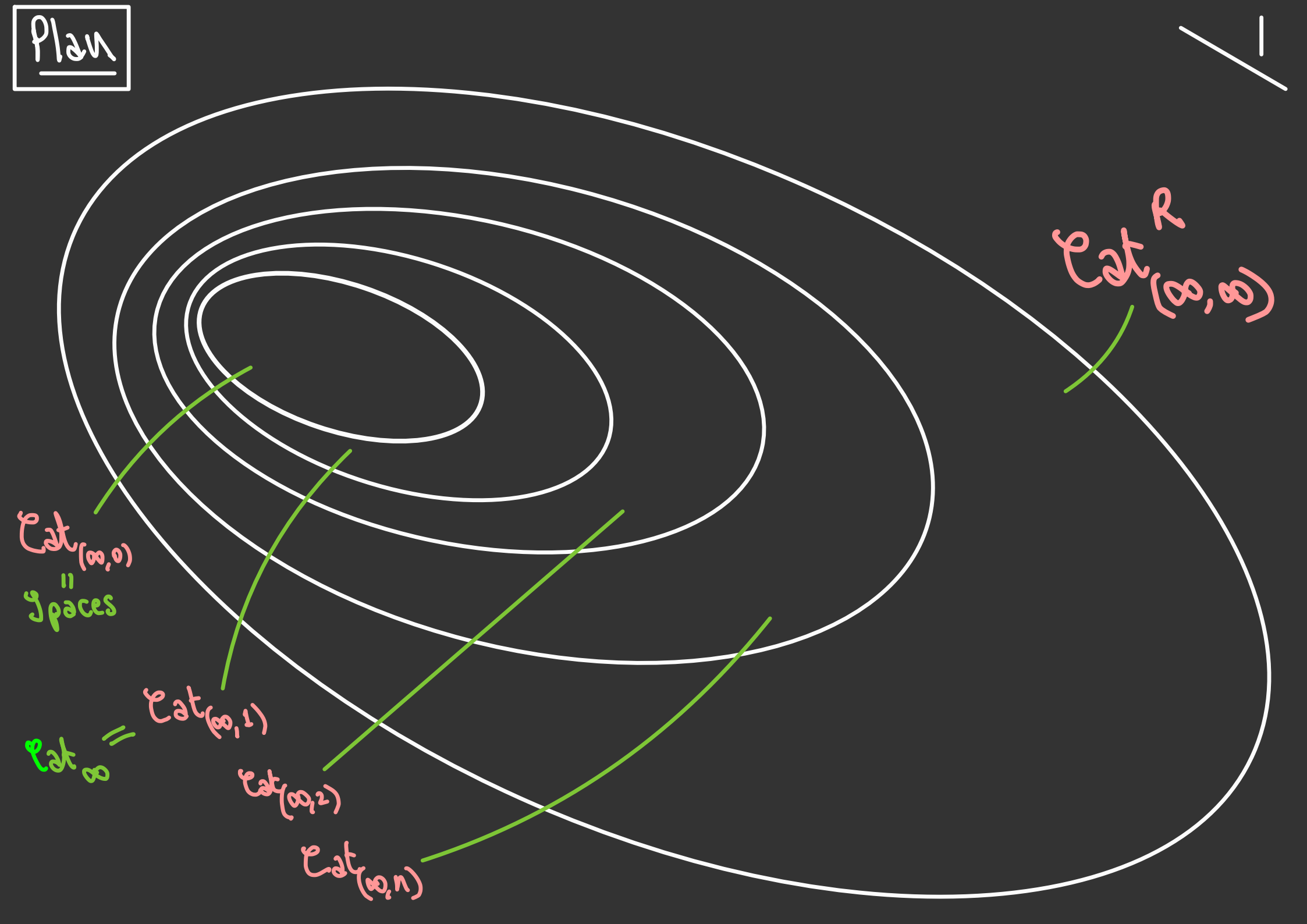
Plan

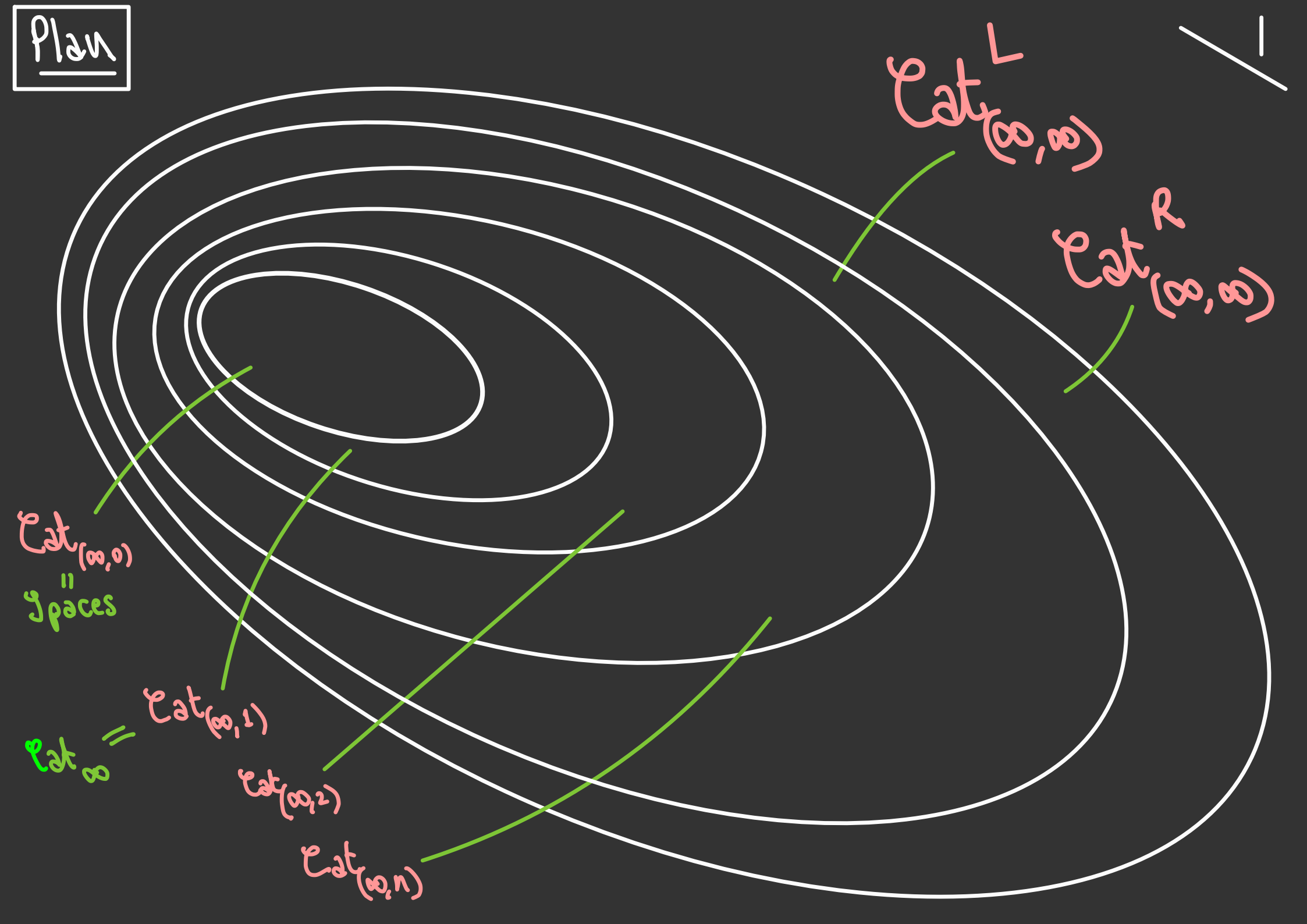

$\mathcal{C}at_{(\infty,0)}$
$=$
$\mathcal{S}paces$
$\mathcal{C}at_{(\infty,1)}$
$\mathcal{C}at_{\infty}$
$=$
$\mathcal{C}at_{(\infty,2)}$

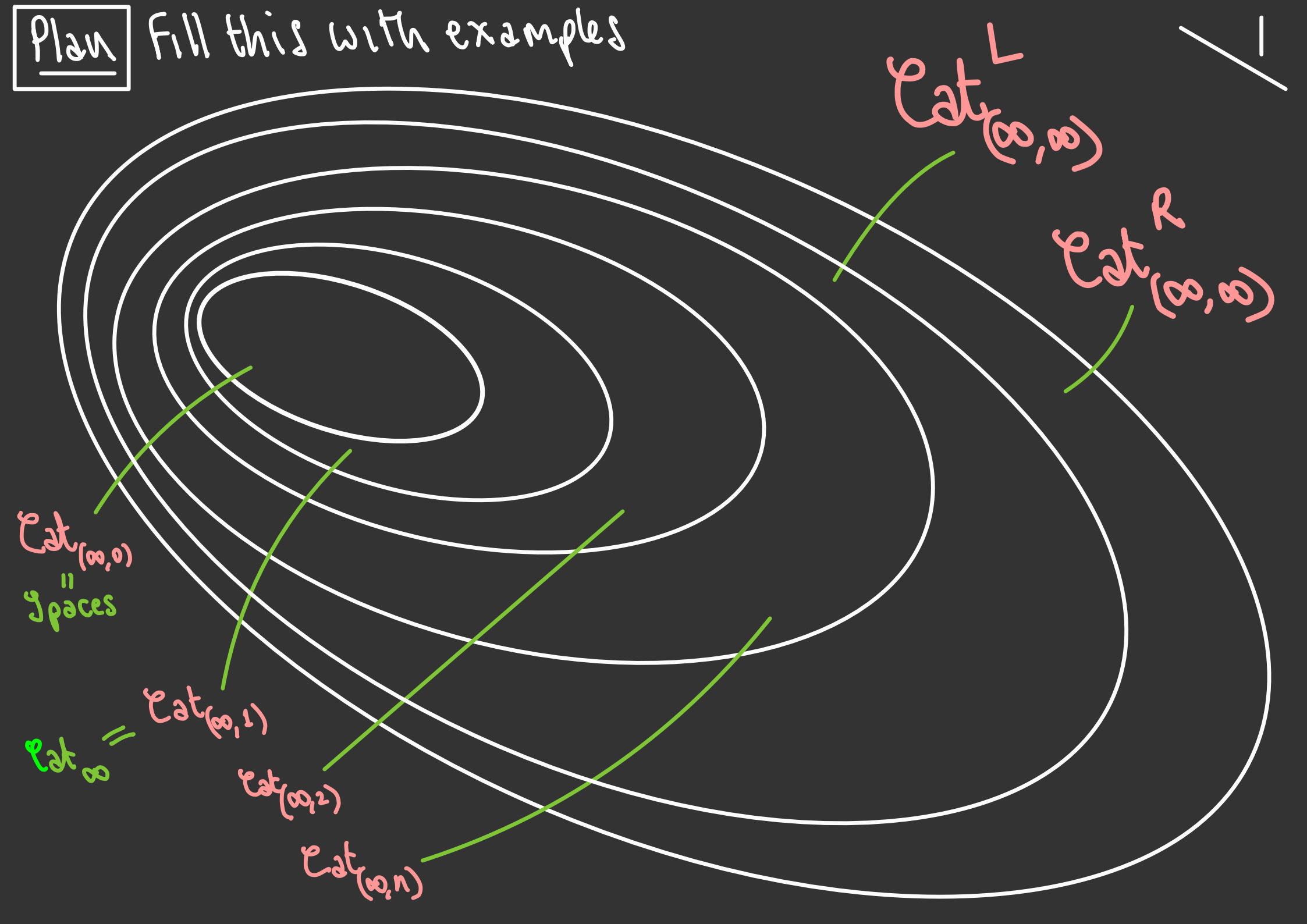
Plan


$\mathcal{C}at_{(\infty,0)}$
$=$
Spaces
$\mathcal{C}at_\infty$
$=$
$\mathcal{C}at_{(\infty,1)}$
$\mathcal{C}at_{(\infty,2)}$
$\mathcal{C}at_{(\infty,n)}$


Plan
$\mathcal{C}at_{(\infty,\infty)}$
$\mathcal{C}at_{(\infty,0)}$
$=$
Spaces
$\mathcal{C}at_{(\infty,1)}$
$\mathcal{C}at_{\infty}$ $=$
$\mathcal{C}at_{(\infty,2)}$
$\mathcal{C}at_{(\infty,n)}$


Plan
$\mathcal{C}at^{R}_{(\infty,\infty)}$
$\mathcal{C}at_{(\infty,0)}$
$=$
Spaces
$\mathcal{C}at_{\infty} = \mathcal{C}at_{(\infty,1)}$
$\mathcal{C}at_{(\infty,2)}$
$\mathcal{C}at_{(\infty,n)}$


Plan
$\mathcal{C}at^{L}_{(\infty,\infty)}$
$\mathcal{C}at^{R}_{(\infty,\infty)}$
$\mathcal{C}at_{(\infty,0)}$
$=$
$\mathcal{S}paces$
$\mathcal{C}at_{(\infty,1)}$
$\mathcal{C}at_{\infty} =$
$\mathcal{C}at_{(\infty,2)}$
$\mathcal{C}at_{(\infty,n)}$


Plan
Fill this with examples
$\mathcal{C}at^{L}_{(\infty,\infty)}$
$\mathcal{C}at^{R}_{(\infty,\infty)}$
$\mathcal{C}at_{(\infty,0)}$
$=$
Spaces
$\mathcal{C}at_{(\infty,1)}$
$\mathcal{C}at_{\infty} =$
$\mathcal{C}at_{(\infty,2)}$
$\mathcal{C}at_{(\infty,n)}$

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)}$

$\mathrm{Cat}^{L}_{(\infty,\infty)}$

$\mathrm{Cat}^{R}_{(\infty,\infty)}$

(∞,n)- and (∞,∞)-categories

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

$\mathrm{Cat}_{(\infty,0)} = \mathrm{Space}$

(∞,n)- and (∞,∞)-categories

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

$\mathrm{Cat}_{(\infty,0)} = \mathrm{Space}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_{\infty}$

(∞,n)- and (∞,∞)-categories

$\mathcal{D} \in Cat_{(\infty,n)}$ consists of

- $Ob\,\mathcal{D} \in Space$

$Cat_{(\infty,0)} = Space$

$Cat_{(\infty,1)} = Cat_\infty$

$\mathcal{D} \in Cat^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in Cat^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathrm{Spc}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

$\mathrm{Cat}_{(\infty,0)} = \mathrm{Spc}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathcal{C}at_{(\infty,n)} \hookrightarrow \mathcal{C}at_{(\infty,n+1)}$

$\mathcal{D} \in \mathcal{C}at_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}pace$
- $\mathcal{D}(c,d) \in \mathcal{C}at_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathcal{C}at_{(\infty,0)} = \mathcal{S}pace$

$\mathcal{C}at_{(\infty,1)} = \mathcal{C}at_\infty$

$\mathcal{D} \in \mathcal{C}at^L_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathcal{C}at^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \underset{1}{\overset{\mathrm{tr}_n}{\rightleftarrows}} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \underset{\iota}{\overset{\mathrm{tr}_n}{\rightleftarrows}} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{\langle 0 \rangle} \in \mathrm{Cat}_{(\infty,0)}$

$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathcal{C}at_{(\infty,n)} \underset{1}{\overset{tr_n}{\rightleftarrows}} \mathcal{C}at_{(\infty,n+1)}$

$\mathcal{D} \in \mathcal{C}at_{(\infty,n)}$ consists of

- $Ob\,\mathcal{D} \in \mathcal{S}pace$
- $\mathcal{D}(c,d) \in \mathcal{C}at_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathcal{C}at_{(\infty,0)} = \mathcal{S}pace$

$\mathcal{C}at_{(\infty,1)} = \mathcal{C}at_\infty$

$\mathcal{D} \in \mathcal{C}at^L_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{\langle 0 \rangle} \in \mathcal{C}at_{(\infty,0)}$

$\mathcal{D}^{\langle 1 \rangle} \in \mathcal{C}at_{(\infty,1)}$

$\mathcal{D} \in \mathcal{C}at^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathcal{C}at_{(\infty,n)} \underset{1}{\overset{tr_n}{\rightleftarrows}} \mathcal{C}at_{(\infty,n+1)}$

$\mathcal{D} \in \mathcal{C}at_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}pace$
- $\mathcal{D}(c,d) \in \mathcal{C}at_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathcal{C}at_{(\infty,0)} = \mathcal{S}pace$

$\mathcal{C}at_{(\infty,1)} = \mathcal{C}at_\infty$

$\mathcal{D} \in \mathcal{C}at^L_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{\langle 0\rangle} \in \mathcal{C}at_{(\infty,0)}$

$\mathcal{D}^{\langle 1\rangle} \in \mathcal{C}at_{(\infty,1)}$

...

$\mathcal{D}^{\langle n\rangle} \in \mathcal{C}at_{(\infty,n)}$

$\mathcal{D}^{\langle n+1\rangle} \in \mathcal{C}at_{(\infty,n+1)}$

...

$\mathcal{D} \in \mathcal{C}at^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

tr_n: $\mathrm{Cat}_{(\infty,n)} \rightleftarrows \mathrm{Cat}_{(\infty,n+1)}$: ι

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{\langle 0\rangle} \in \mathrm{Cat}_{(\infty,0)}$

$\Uparrow$

$\mathcal{D}^{\langle 1\rangle} \in \mathrm{Cat}_{(\infty,1)}$

$\Uparrow$

$\vdots$

$\Uparrow$

$\mathcal{D}^{\langle n\rangle} \in \mathrm{Cat}_{(\infty,n)}$

$\Uparrow$

$\mathcal{D}^{\langle n+1\rangle} \in \mathrm{Cat}_{(\infty,n+1)}$

$\Uparrow$

$\vdots$

$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \underset{\iota}{\overset{\mathrm{tr}_n}{\rightleftarrows}} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

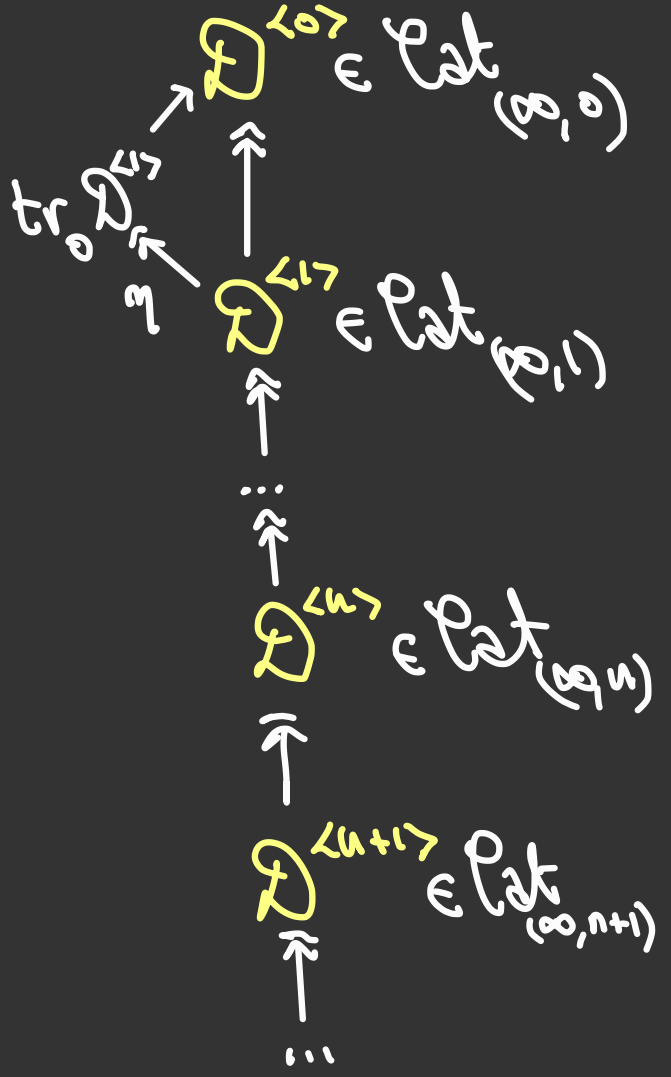
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^{L}_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \underset{\iota}{\overset{\mathrm{tr}_n}{\leftrightarrows}} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,1)}$ consists of

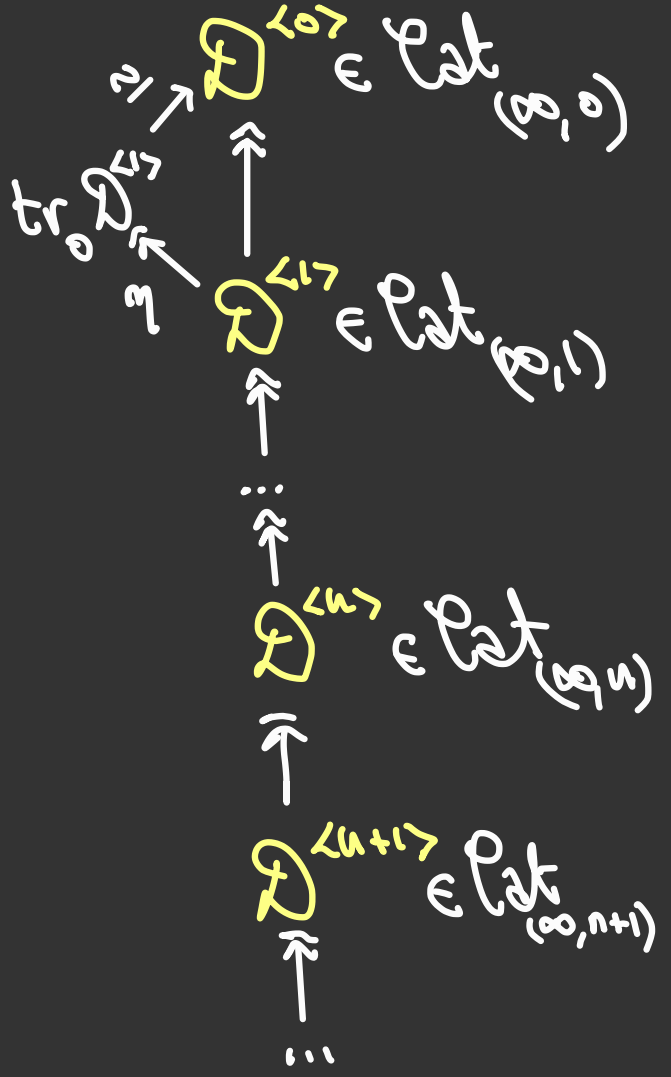
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \underset{1}{\overset{\mathrm{tr}_n}{\leftarrows}} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

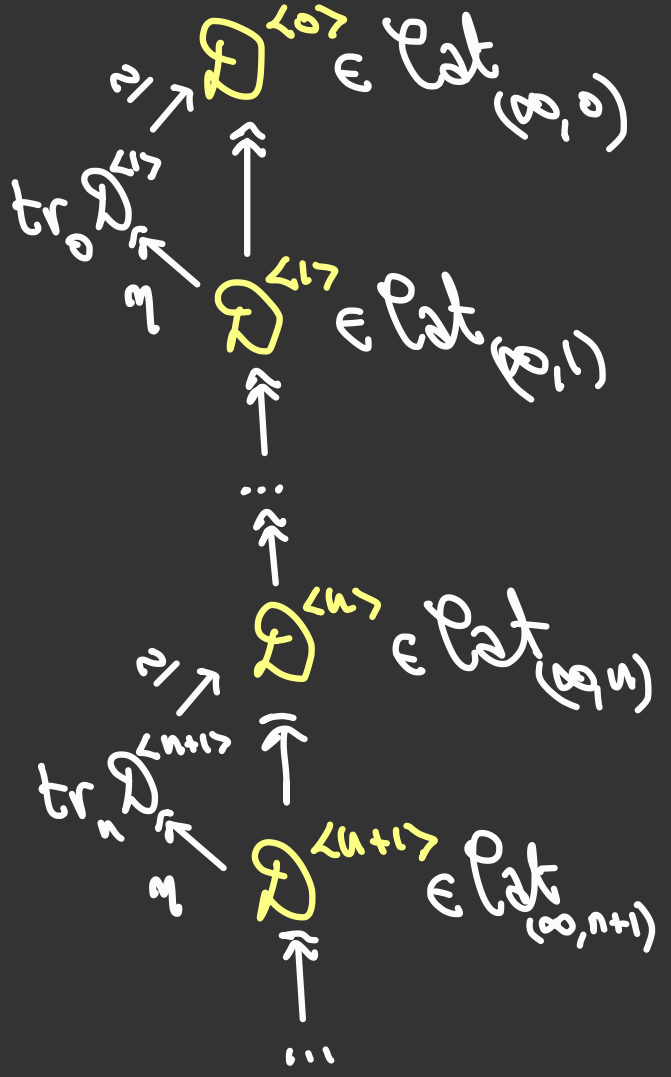
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \rightleftarrows \mathrm{Cat}_{(\infty,n+1)}$ (inclusion $\dashv$ tr_n, $\mathrm{tr}_n \dashv \mathrm{core}_n$)

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

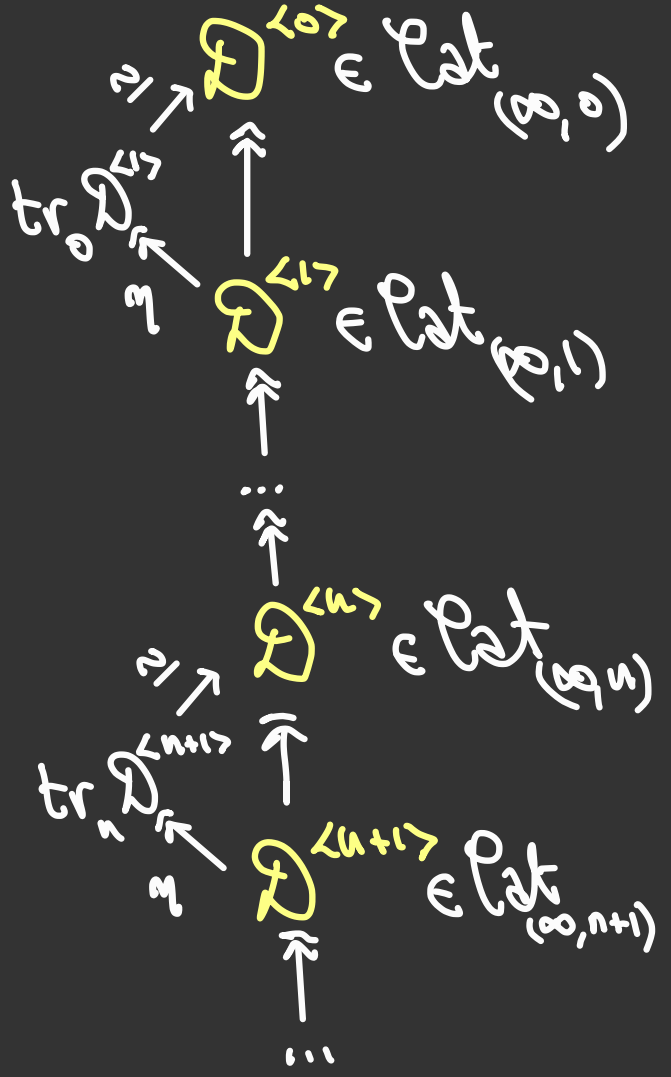
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \rightleftarrows \mathrm{Cat}_{(\infty,n+1)}$ (inclusion $\dashv$ tr_n, $\mathrm{tr}_n \dashv \mathrm{core}_n$)

$\mathcal{D} \in \mathrm{Cat}_{(\infty,1)}$ consists of

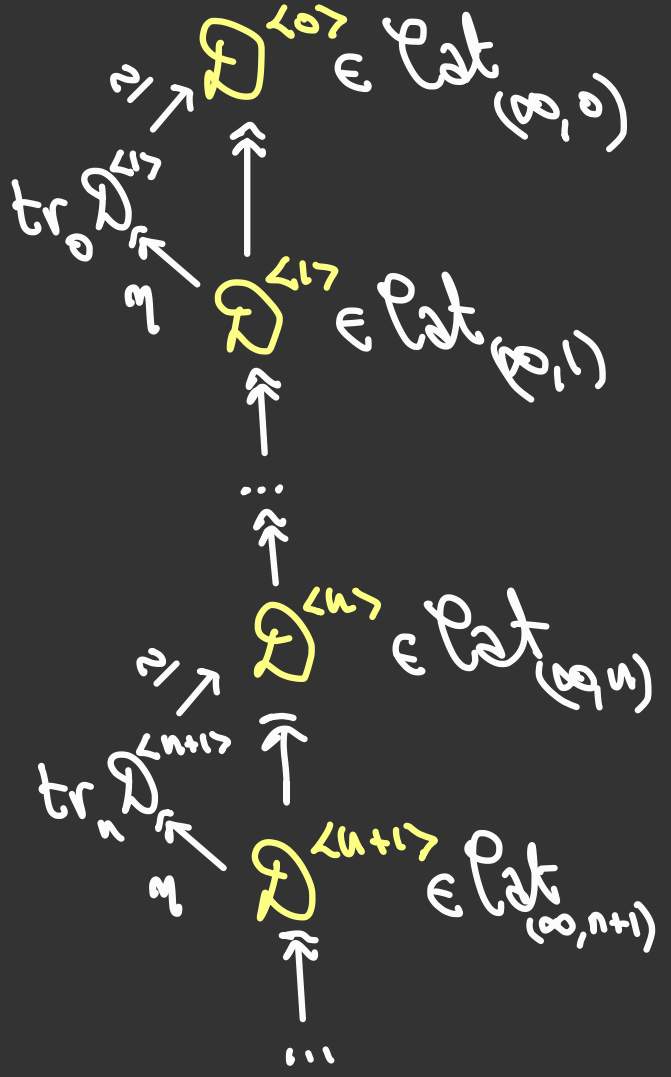
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{(0)} \in \mathrm{Cat}_{(\infty,0)}$

(∞,n)- and (∞,∞)-categories

$\mathcal{C}at_{(\infty,n)} \rightleftarrows \mathcal{C}at_{(\infty,n+1)}$ (inclusion $\dashv$ tr_n, $\mathrm{tr}_n \dashv \mathrm{core}_n$)

$\mathcal{D} \in \mathcal{C}at_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}pace$
- $\mathcal{D}(c,d) \in \mathcal{C}at_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathcal{C}at_{(\infty,0)} = \mathcal{S}pace$

$\mathcal{C}at_{(\infty,1)} = \mathcal{C}at_\infty$

$\mathcal{D} \in \mathcal{C}at^L_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{\langle 0\rangle} \in \mathcal{C}at_{(\infty,0)}$

$\simeq$ / $\mathrm{tr}_0 \mathcal{D}^{\langle 1\rangle}$ / η

$\mathcal{D}^{\langle 1\rangle} \in \mathcal{C}at_{(\infty,1)}$

$\vdots$

$\mathcal{D}^{\langle n\rangle} \in \mathcal{C}at_{(\infty,n)}$

$\simeq$ / $\mathrm{tr}_n \mathcal{D}^{\langle n+1\rangle}$ / η

$\mathcal{D}^{\langle n+1\rangle} \in \mathcal{C}at_{(\infty,n+1)}$

$\vdots$

$\mathcal{D} \in \mathcal{C}at^R_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{(0)} \in \mathcal{C}at_{(\infty,0)}$

$\mathcal{D}^{(1)} \in \mathcal{C}at_{(\infty,1)}$

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \rightleftarrows \mathrm{Cat}_{(\infty,n+1)}$ (core$_n$, ι, ι, tr$_n$)

$\mathcal{D} \in \mathrm{Cat}_{(\infty,1)}$ consists of

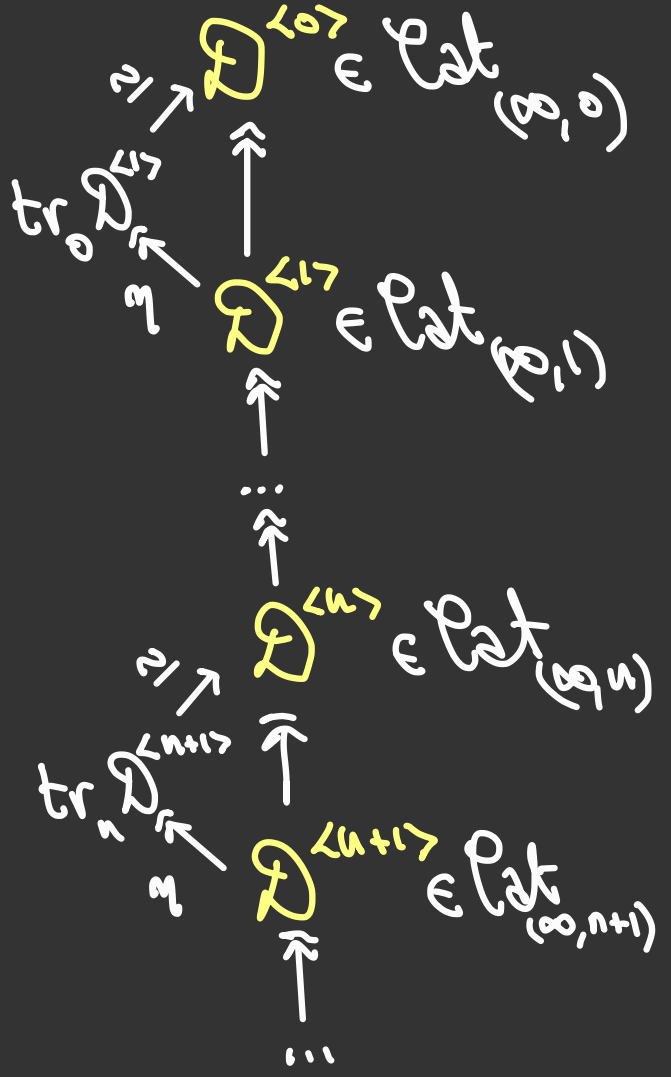
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

+ weak composition
+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:

$\mathcal{D}^{(0)} \in \mathrm{Cat}_{(\infty,0)}$

$\mathcal{D}^{(1)} \in \mathrm{Cat}_{(\infty,1)}$

$\mathcal{D}^{(n)} \in \mathrm{Cat}_{(\infty,n)}$

$\mathcal{D}^{(n+1)} \in \mathrm{Cat}_{(\infty,n+1)}$

(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \overset{\mathrm{tr}_n \dashv \iota \dashv \mathrm{core}_n}{\rightleftarrows} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

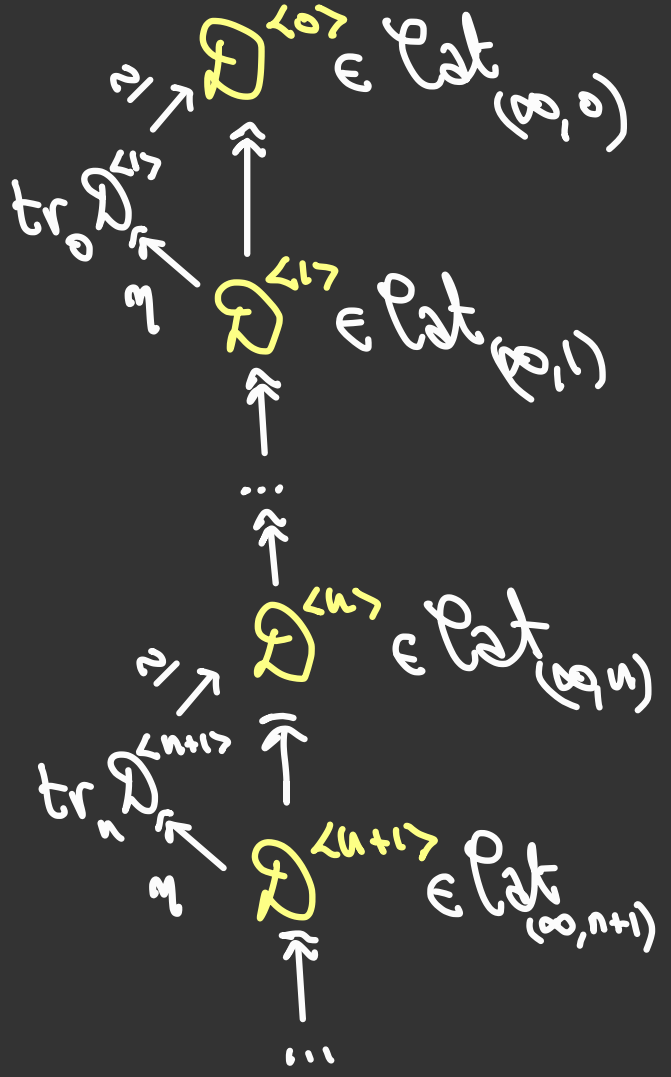
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

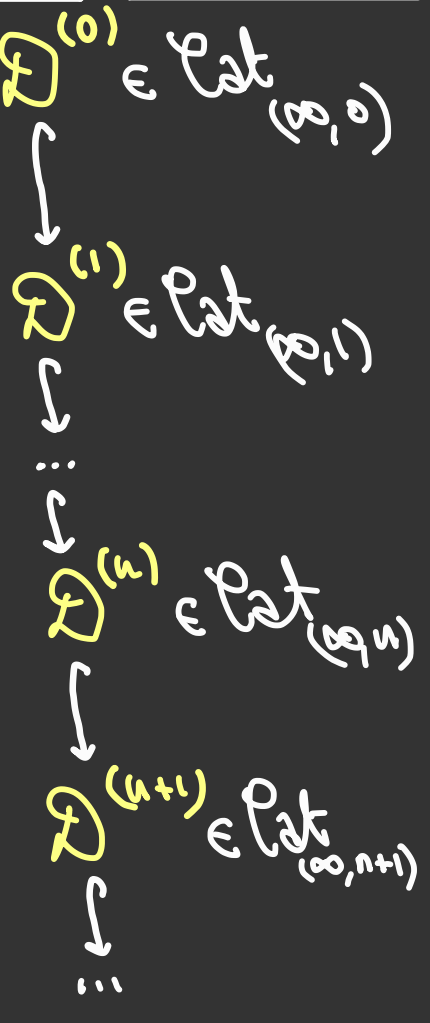
$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:



(∞,n)- and (∞,∞)-categories

$\mathcal{C}at_{(\infty,n)} \rightleftarrows \mathcal{C}at_{(\infty,n+1)}$ (adjunction $tr_n \dashv \dashv core_n$)

$\mathcal{D} \in \mathcal{C}at_{(\infty,n)}$ consists of

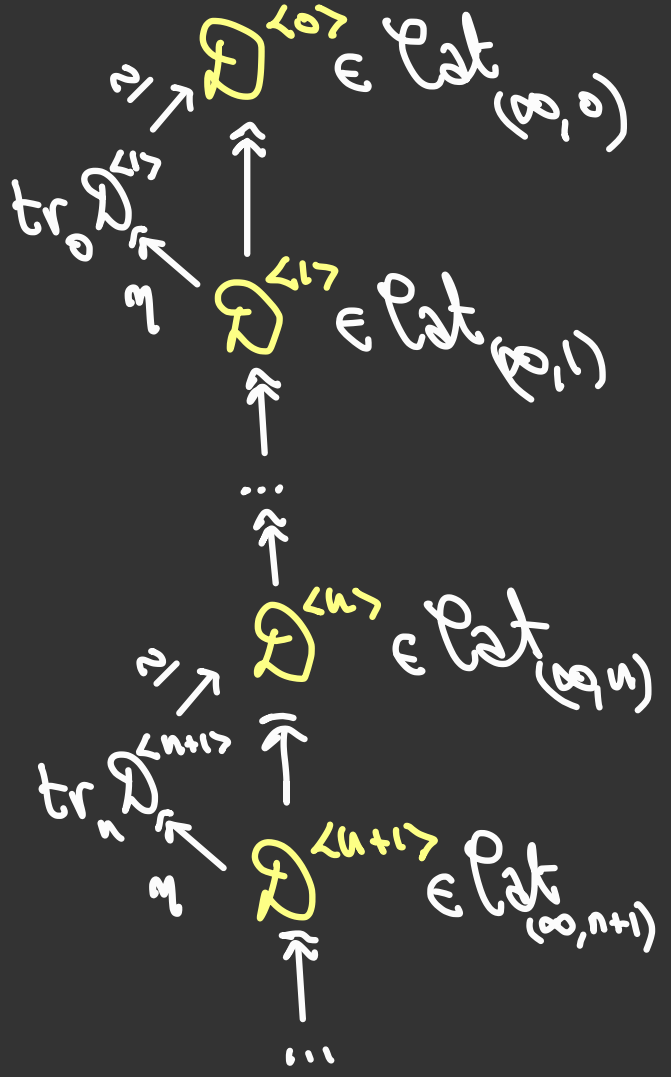
- $Ob\,\mathcal{D} \in \mathcal{S}pace$
- $\mathcal{D}(c,d) \in \mathcal{C}at_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

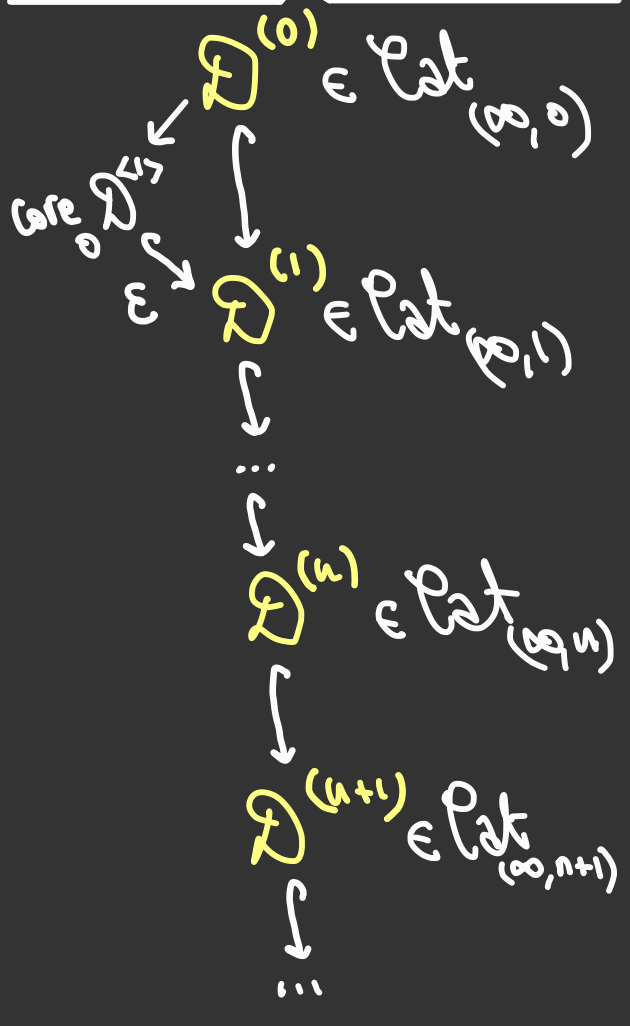
$\mathcal{C}at_{(\infty,0)} = \mathcal{S}pace$

$\mathcal{C}at_{(\infty,1)} = \mathcal{C}at_\infty$

$\mathcal{D} \in \mathcal{C}at^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathcal{C}at^R_{(\infty,\infty)}$ consists of:



(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \rightleftarrows \mathrm{Cat}_{(\infty,n+1)}$ (inclusion $\dashv$ tr_n $\dashv$ core_n)

$\mathcal{D} \in \mathrm{Cat}_{(\infty,1)}$ consists of

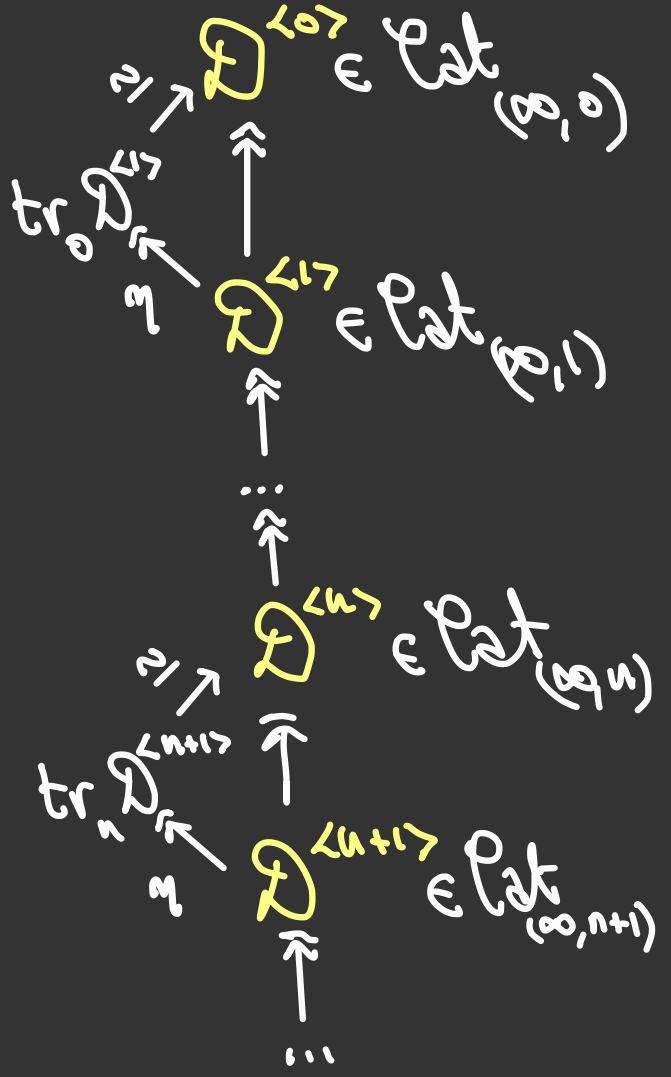
- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

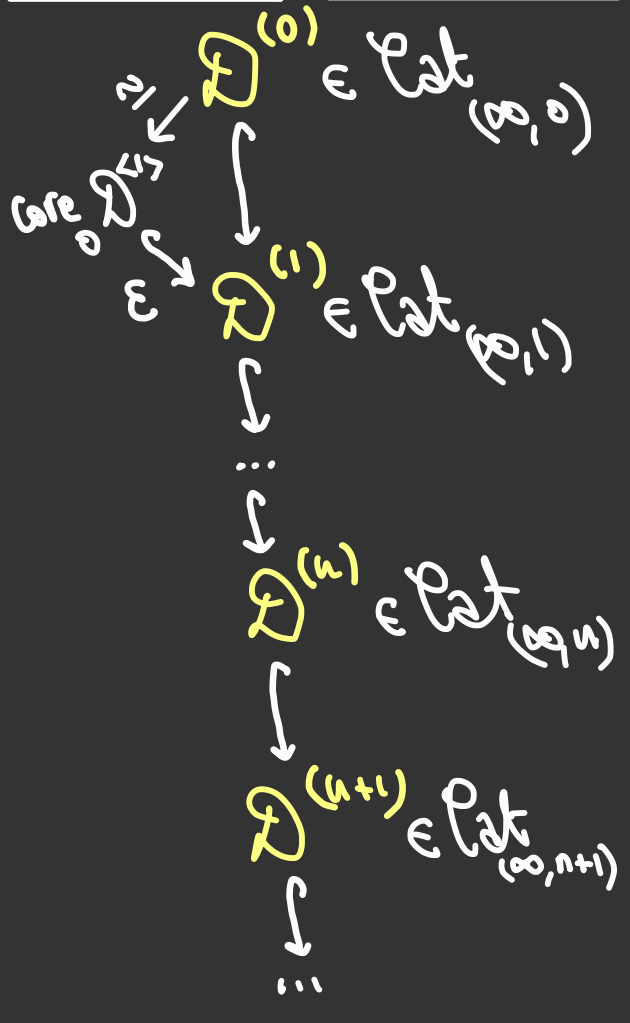
$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

$\mathrm{Cat}_{(\infty,1)} = \mathrm{Cat}_\infty$

$\mathcal{D} \in \mathrm{Cat}^L_{(\infty,\infty)}$ consists of:



$\mathcal{D} \in \mathrm{Cat}^R_{(\infty,\infty)}$ consists of:



(∞,n)- and (∞,∞)-categories

$\mathrm{Cat}_{(\infty,n)} \overset{\mathrm{tr}_n \dashv \mathrm{core}_n}{\rightleftarrows} \mathrm{Cat}_{(\infty,n+1)}$

$\mathcal{D} \in \mathrm{Cat}_{(\infty,n)}$ consists of

- $\mathrm{Ob}\,\mathcal{D} \in \mathcal{S}\mathrm{pace}$
- $\mathcal{D}(c,d) \in \mathrm{Cat}_{(\infty,n-1)}$

\+ weak composition
\+ weak identity

$\mathrm{Cat}_{(\infty,0)} = \mathcal{S}\mathrm{pace}$

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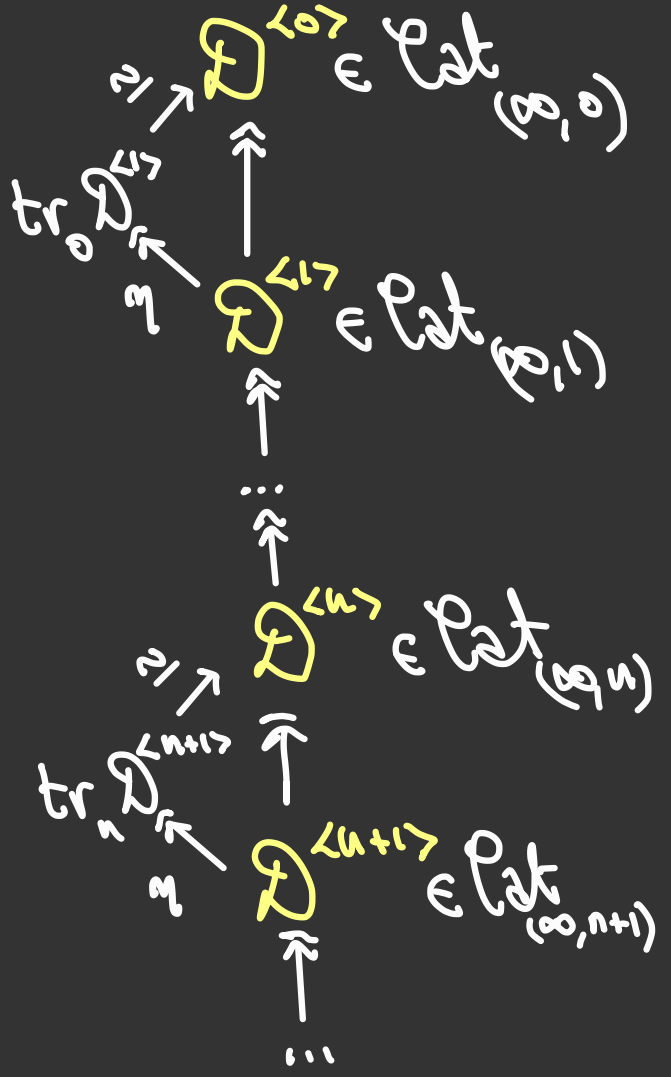

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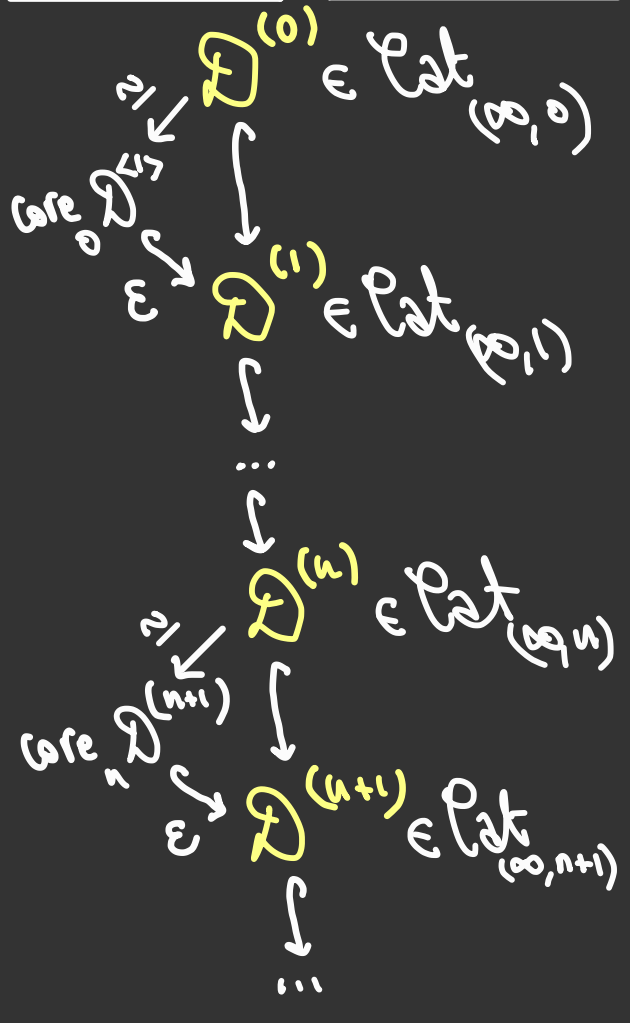
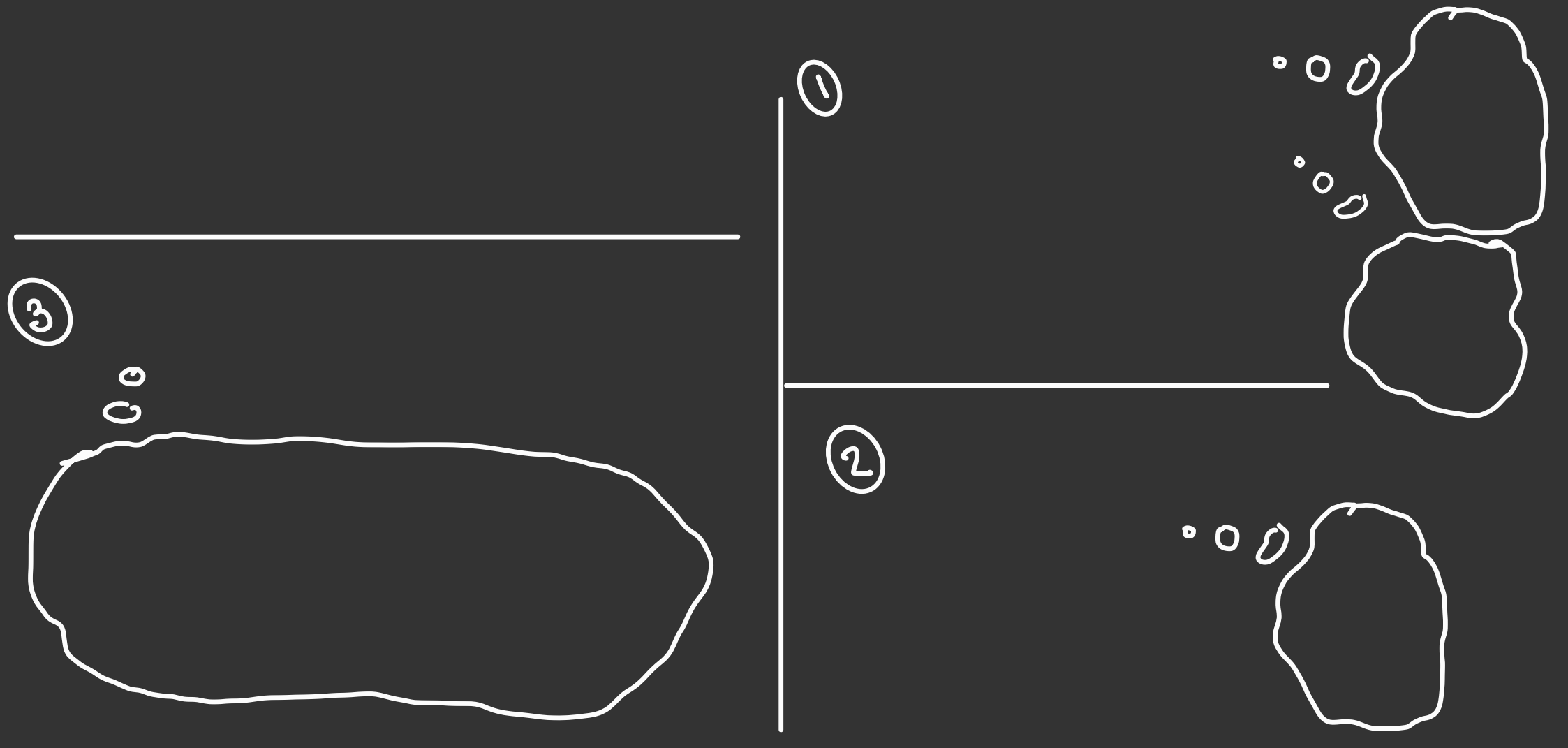


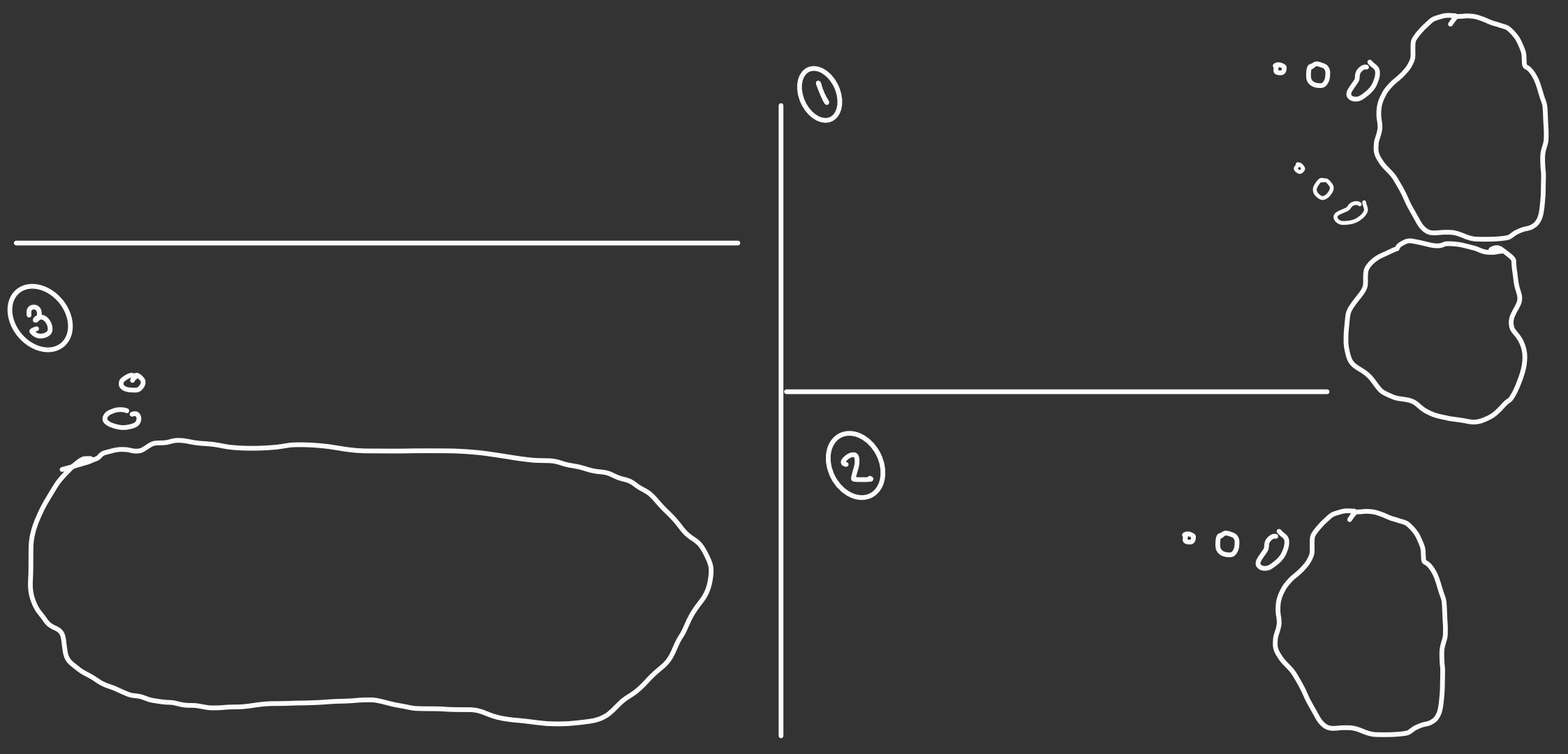
Complicial sets [Verity]

Def



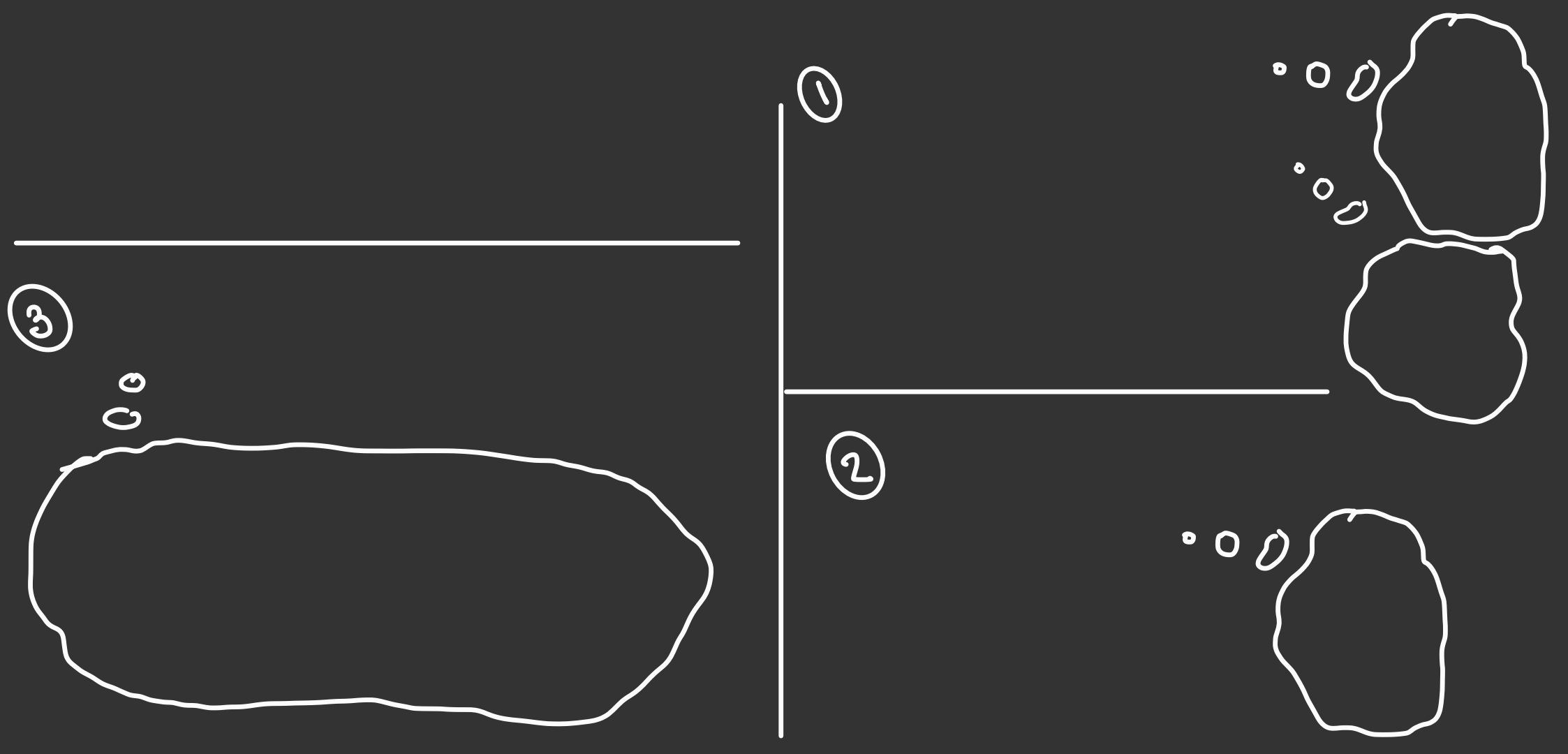
Complicial sets [Verity]

Def A complicial set



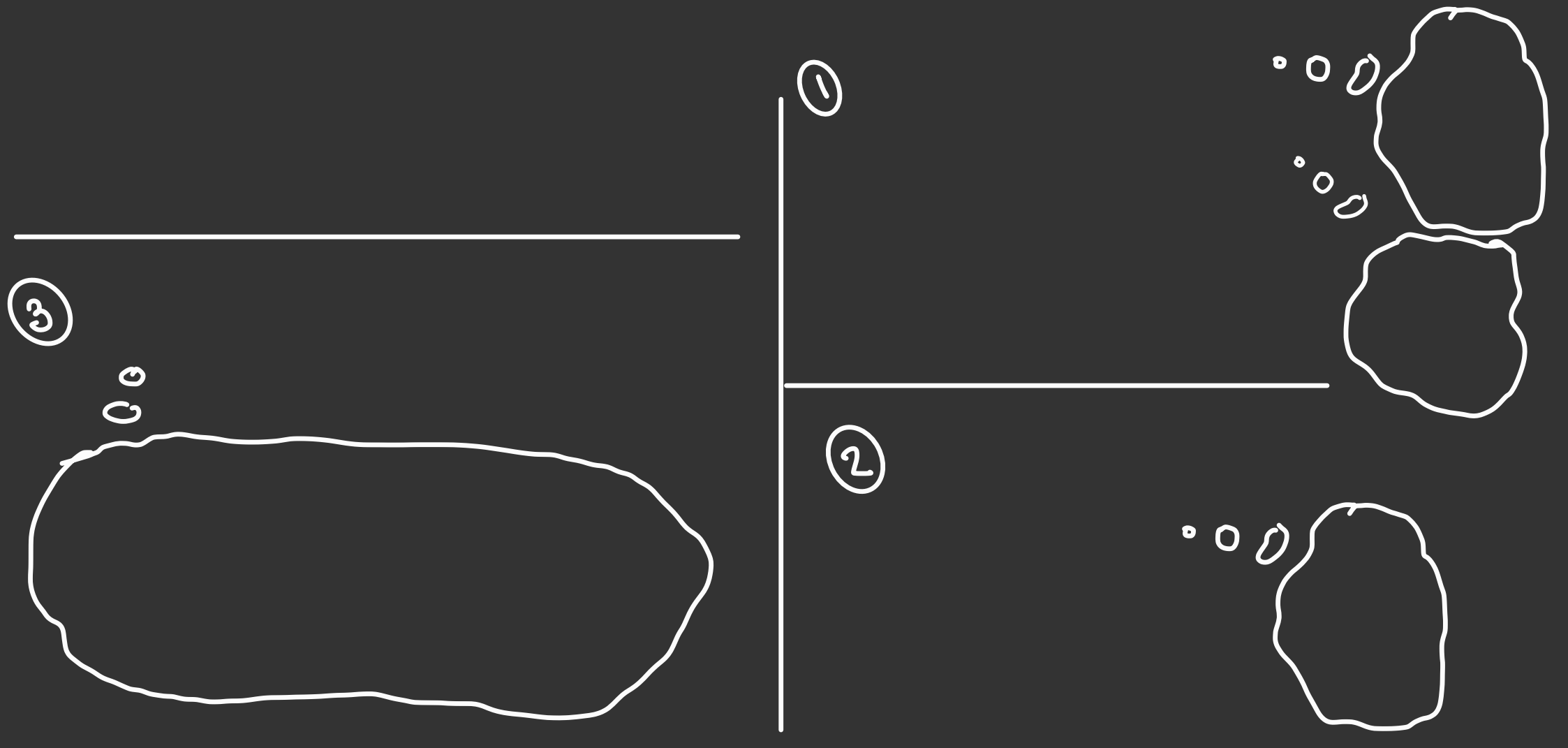
Complicial sets [Verity]

Def A complicial set is a simplicial set X



Complicial sets [Verity]

Def A complicial set is a simplicial set X with marking



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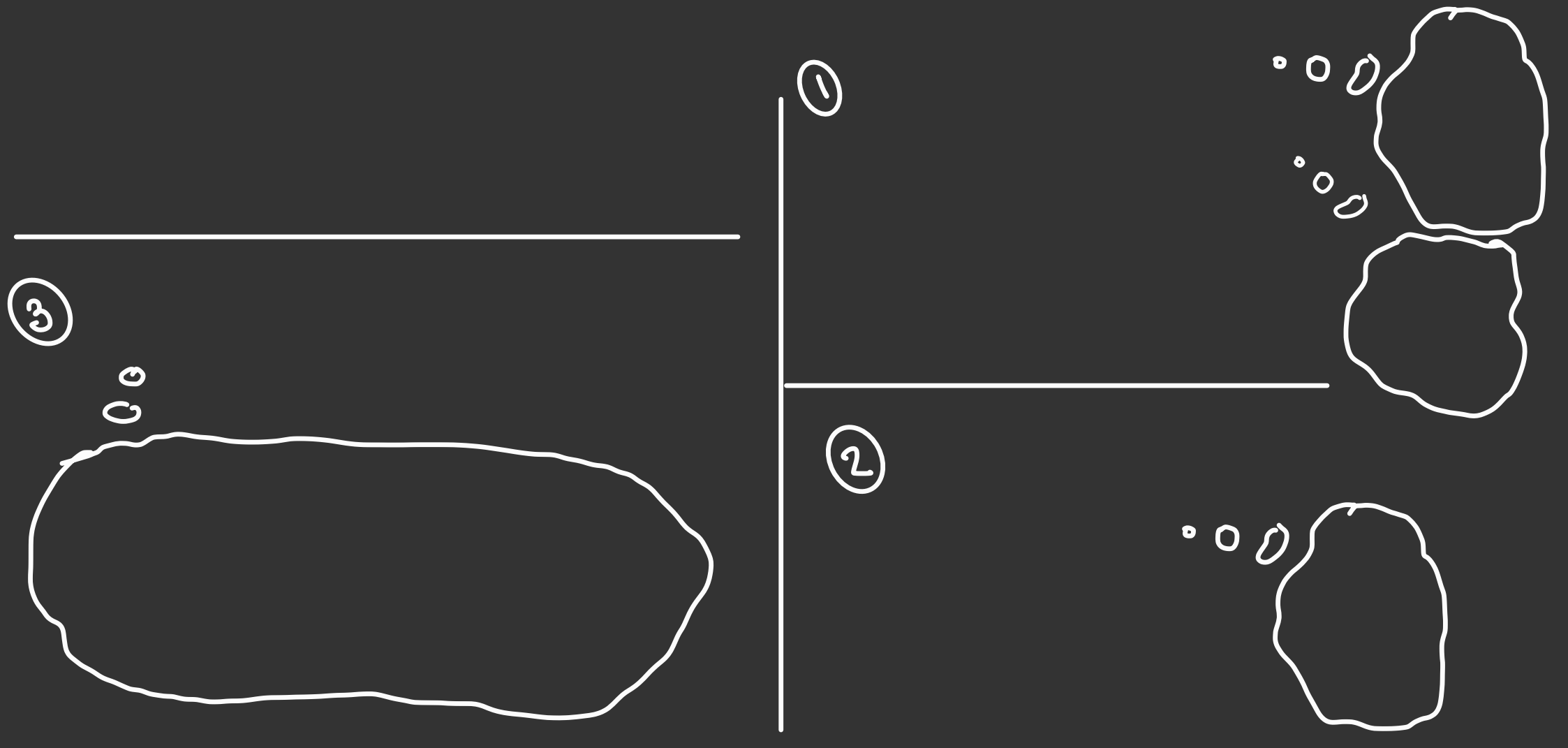
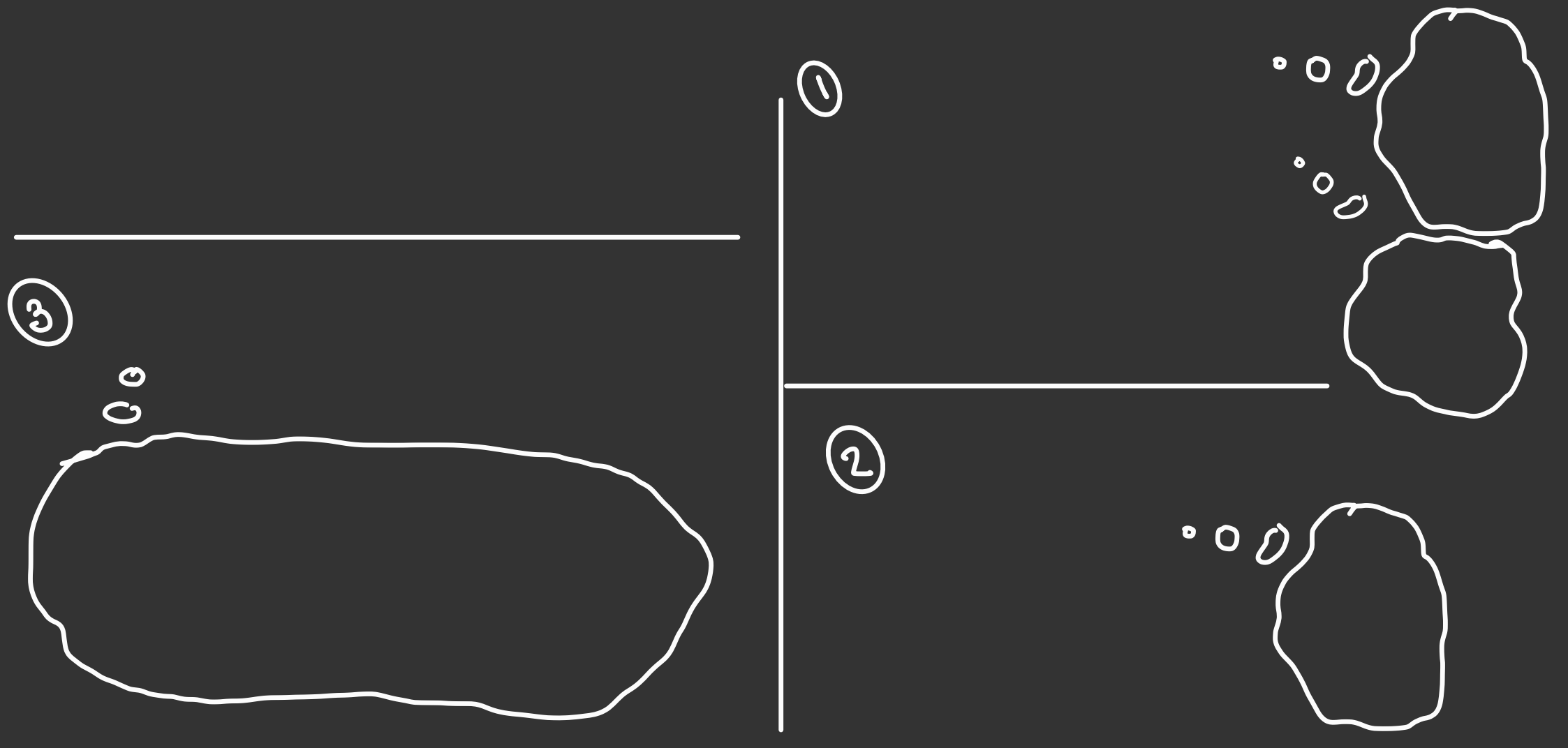
Def A complicial set is a simplicial set X with marking X_κ



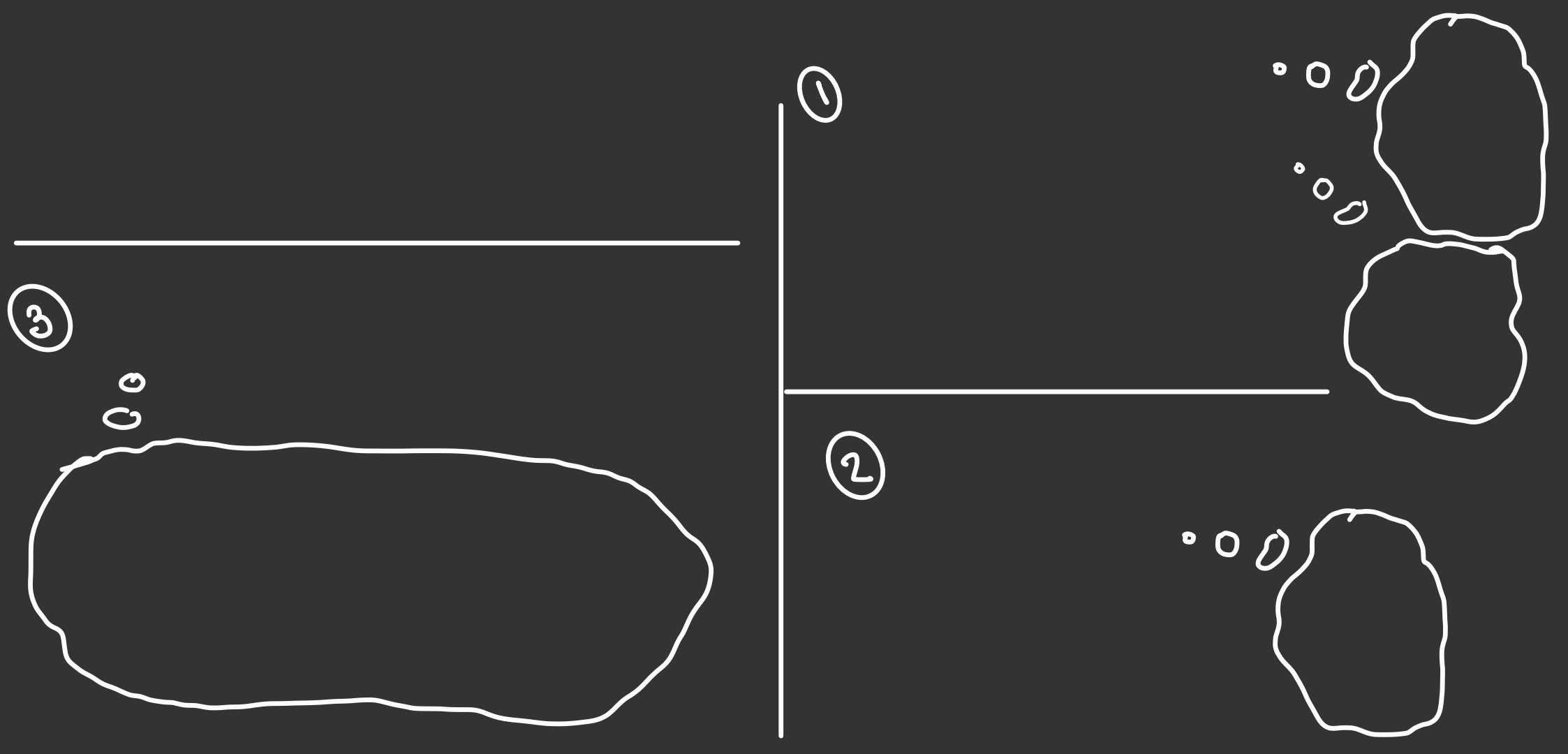
Complicial sets [Verity]

Def A complicial set is a simplicial set X with marking $X^{deg}_k \subseteq X_k$



Complicial sets [Verity]

Def A complicial set is a simplicial set X with marking $X^{deg}_k \subseteq \tilde{X}_k \subseteq X_k \ \forall k > 0$



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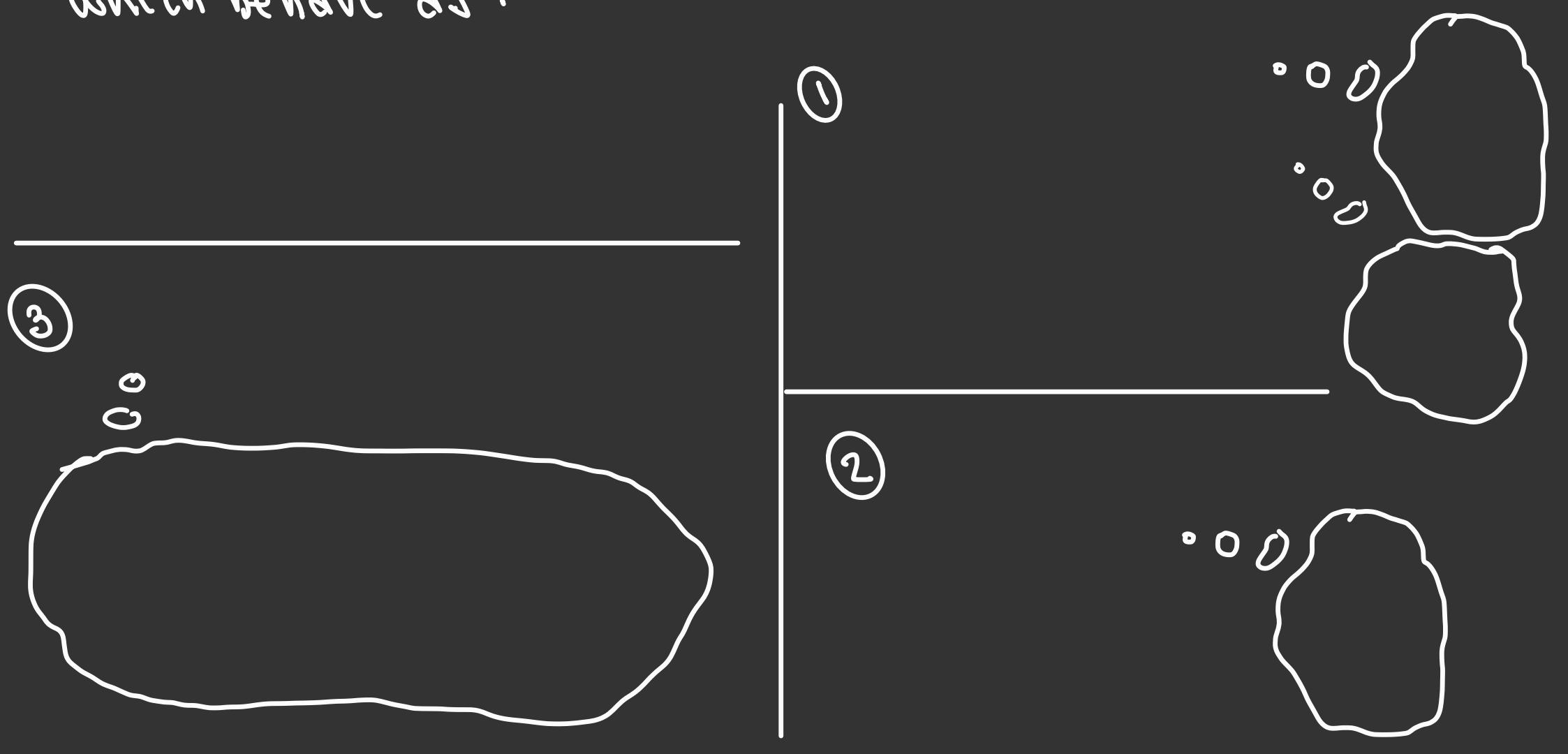
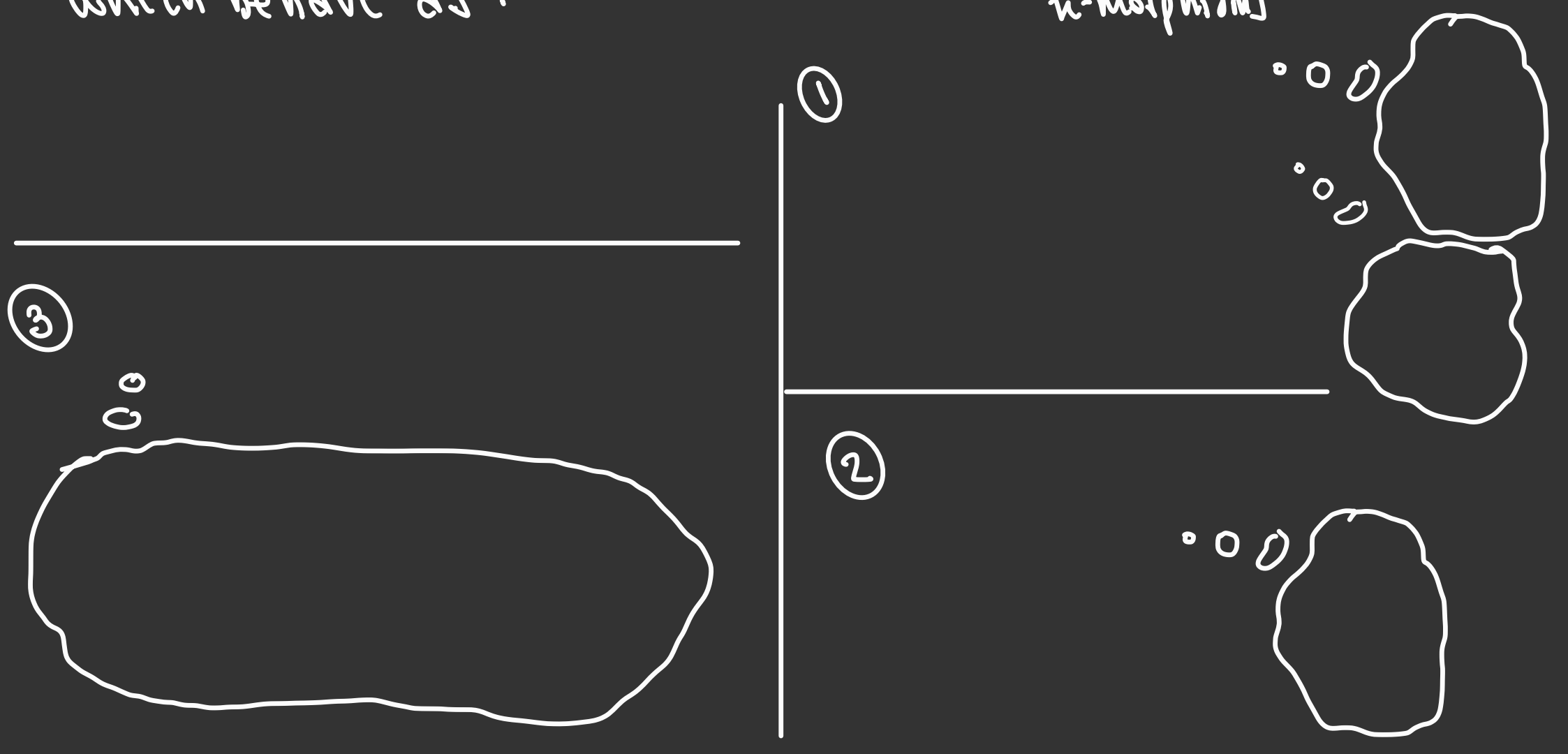


Complicial sets [Verity]

Def A complicial set is a simplicial set X with marking $X^{\flat}_k \subseteq \tilde{X}_k \subseteq X_k \ \forall k>0$ which behave as:

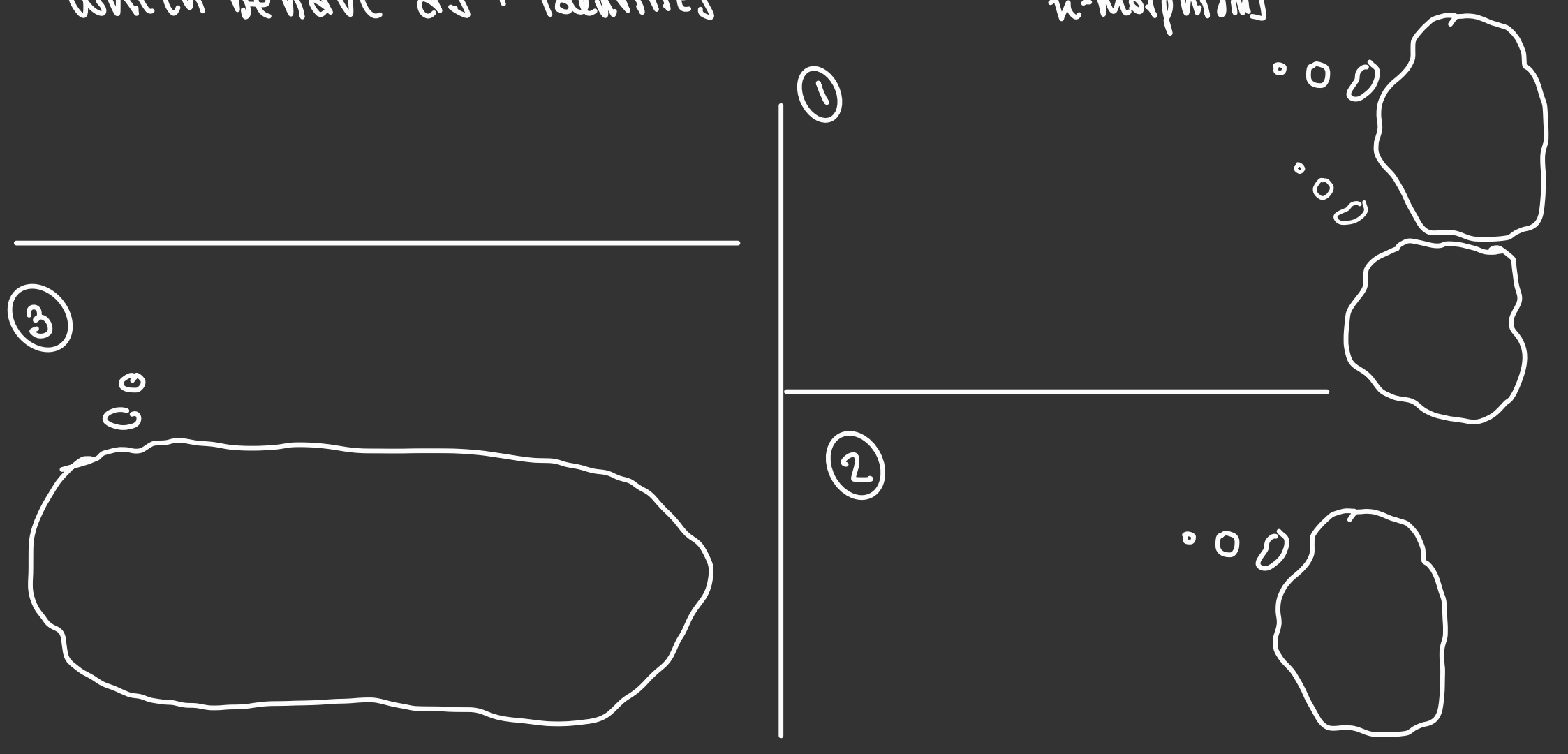
$\tilde{X}_k$: k-morphisms



Complicial sets [Verity]

Def A complicial set is a simplicial set X with marking $X^{\flat}_k \subseteq \tilde{X}_k \subseteq X_k \ \forall k>0$

which behave as: $X^{\flat}_k$ ⇒ identities, $\tilde{X}_k$ ≅ h-morphisms



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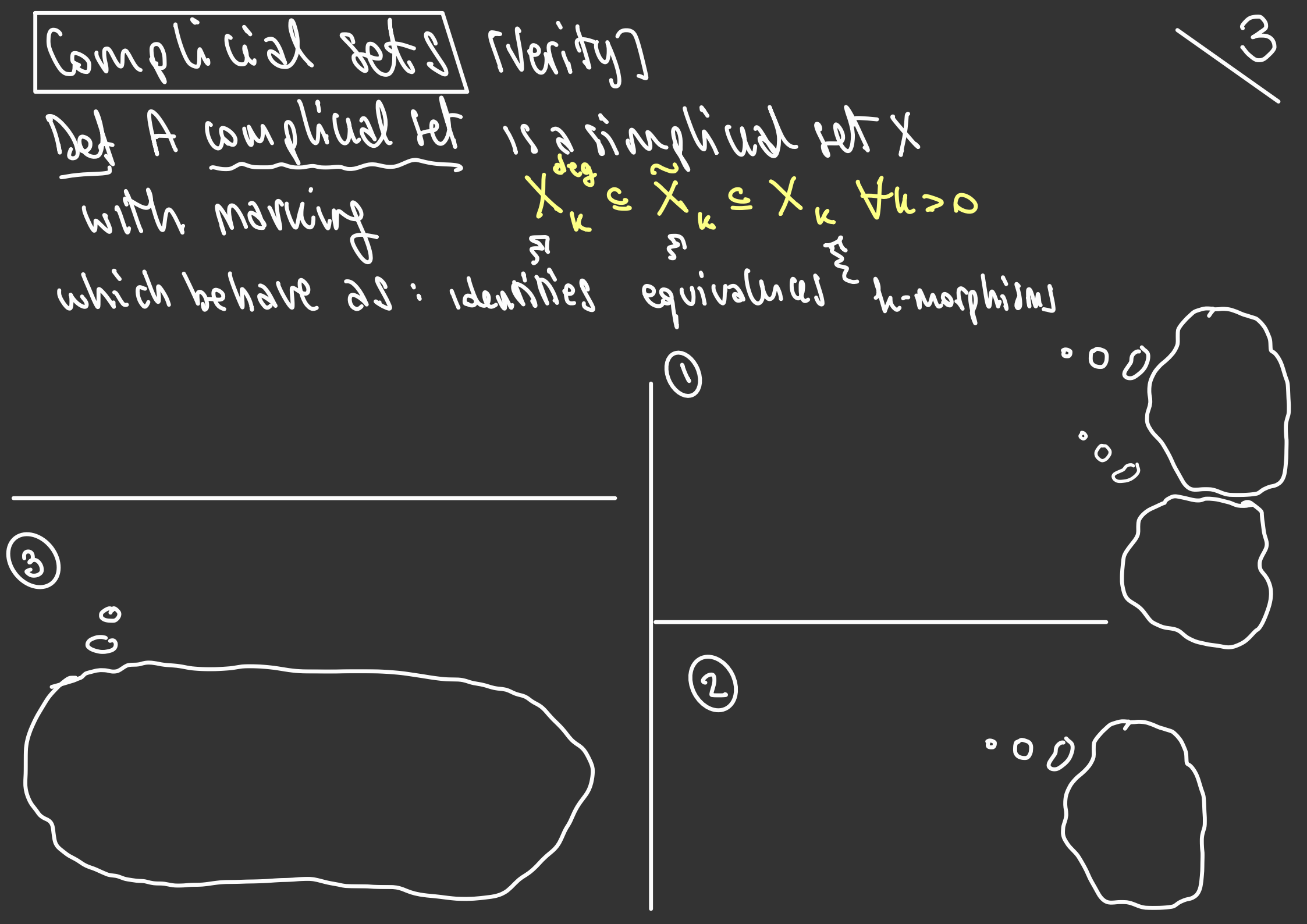

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Meaning: X has right lifting property against:



Complicial sets [Verity]

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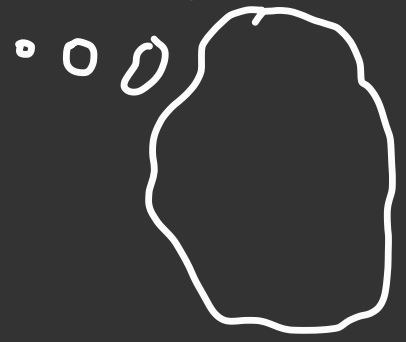
① $\Lambda^k[m] \to \Delta^k[m]$ $(0 \le k \le m)$



② $\Delta^k[m]' \to \Delta^k[m]''$ $(0 \le k \le m)$



③ $\Delta[3|\ell]^{eq} \to \Delta[3|\ell]^{\#}$ $(\ell \ge -1)$



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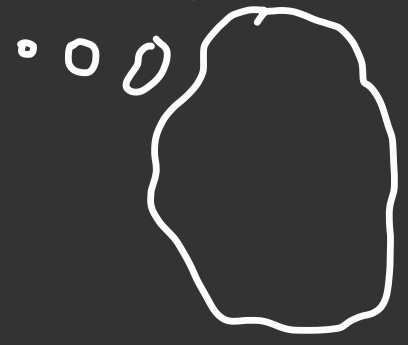
<< can compose simplices



② $\Delta^k[m]' \to \Delta^k[m]''$ $(0<k<m)$



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Meaning: X has right lifting property against:

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<< can compose simplices & marked is invertible >>



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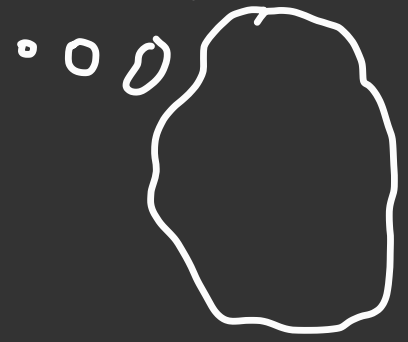

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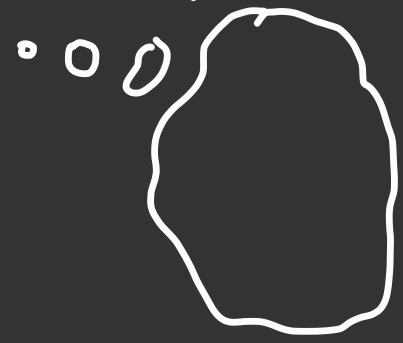

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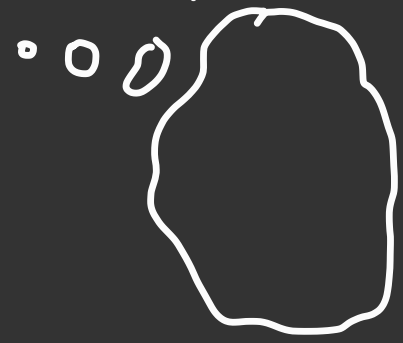

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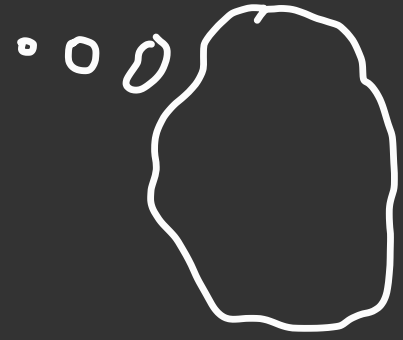

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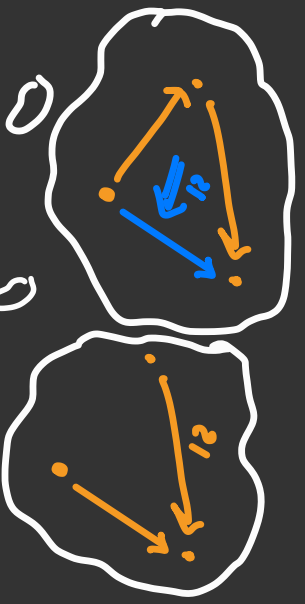

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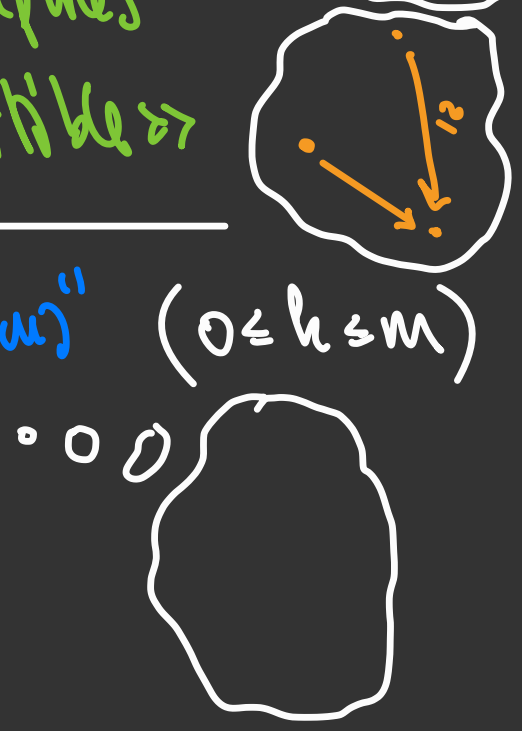

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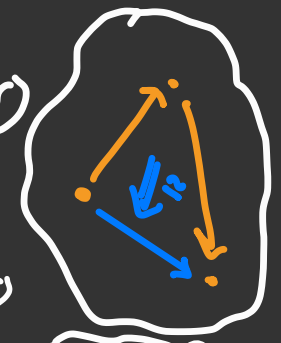

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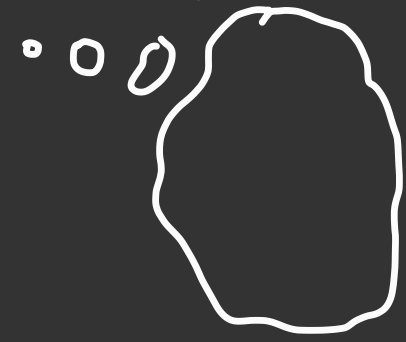

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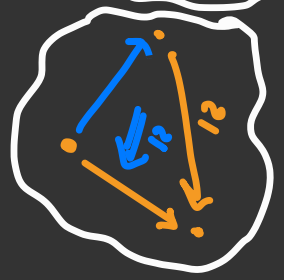

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$X_k^{\flat}$: identities; $\tilde{X}_k$: equivalences; X_k: k-morphisms

which behave as: Meaning: X has right lifting property against:

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« can compose simplices & marked is invertible »



---

② $\Delta^k[m]' \to \Delta^k[m]''$ $(0 \le k \le m)$

« composite of marked is marked »



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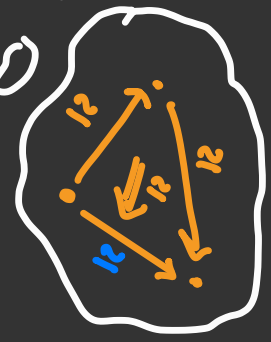

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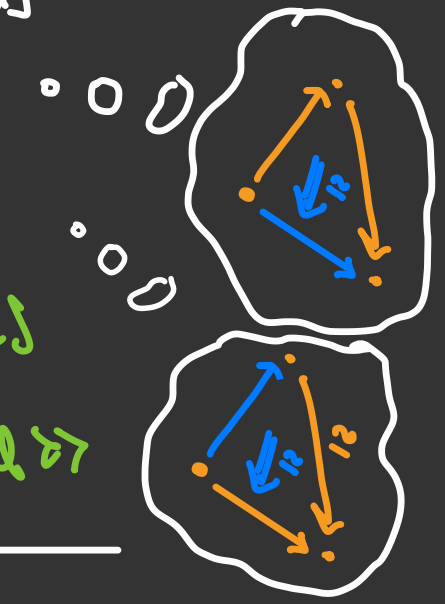

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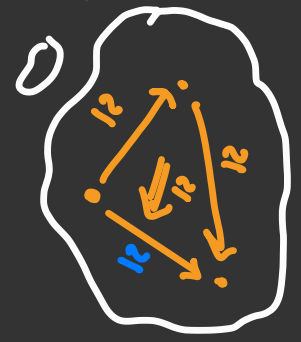

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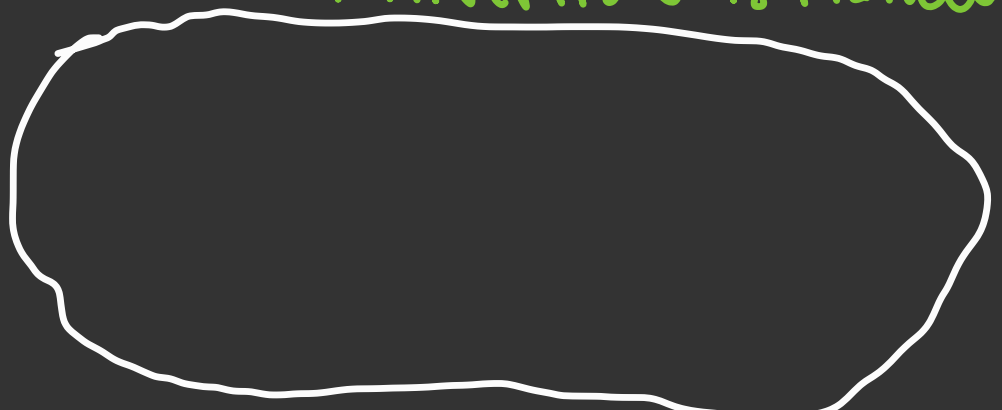

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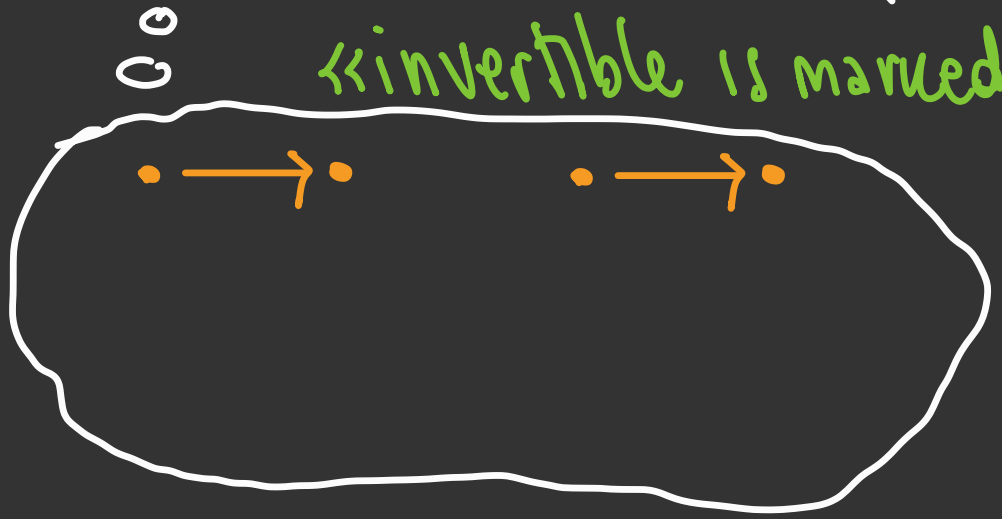

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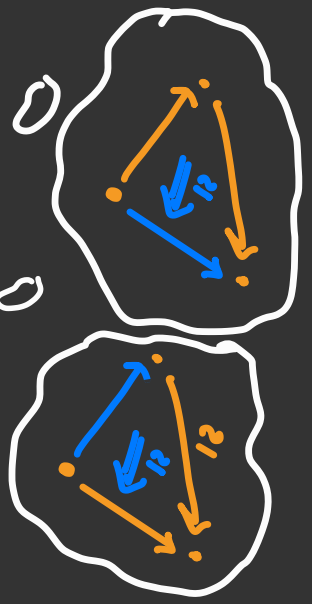

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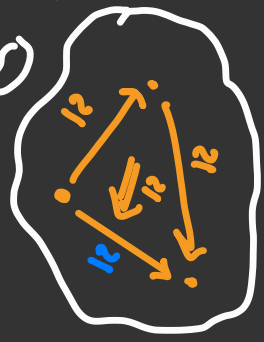

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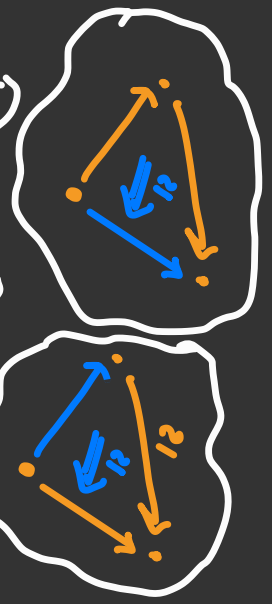

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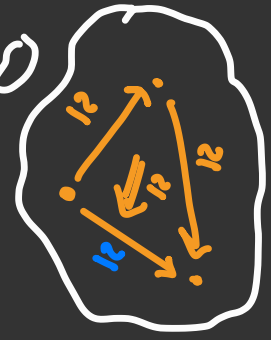
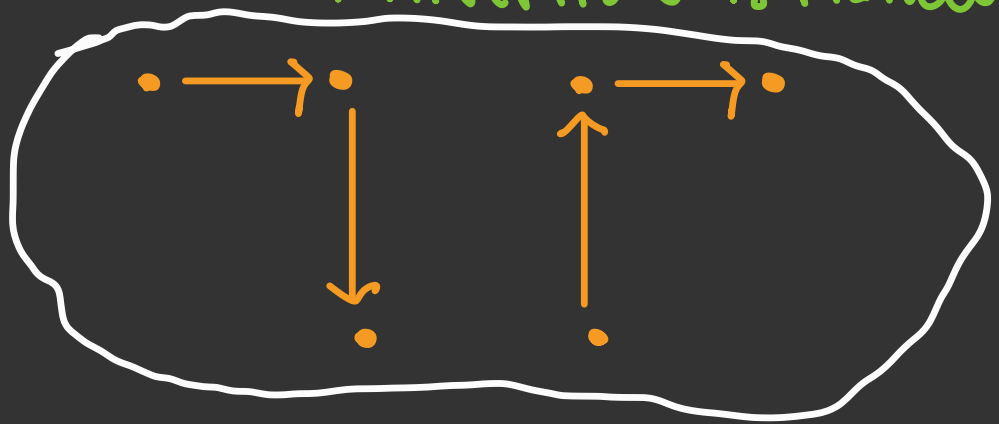
<< composite of marked is marked >>



③ $\Delta[3|\ell] \xrightarrow{eq} \Delta[3|\ell]^{\#}$ $(\ell \geq -1)$

<< invertible is marked >>



Complicial sets [Verity]

Def A complicial set is a simplicial set X with marking

$$X_k^{deg} \subseteq \tilde{X}_k \subseteq X_k \quad \forall k>0$$

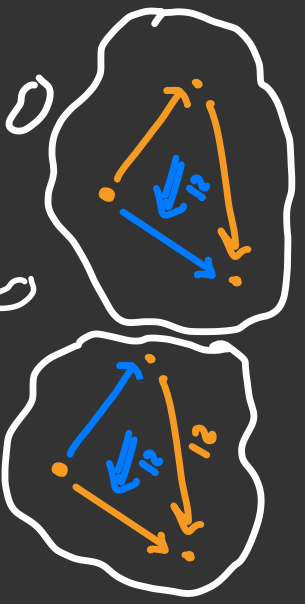
which behave as: identities ⇒ equivalences ⇒ k-morphisms

Meaning: X has right lifting property against:

---

① $\Lambda^k[m] \to \Delta^k[m]$ $(0<k<m)$

<< can compose simplices & marked is invertible >>



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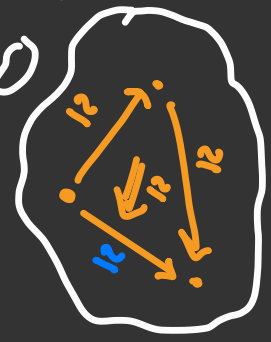
② $\Delta^k[m]' \to \Delta^k[m]''$ $(0<k<m)$

<< composite of marked is marked >>



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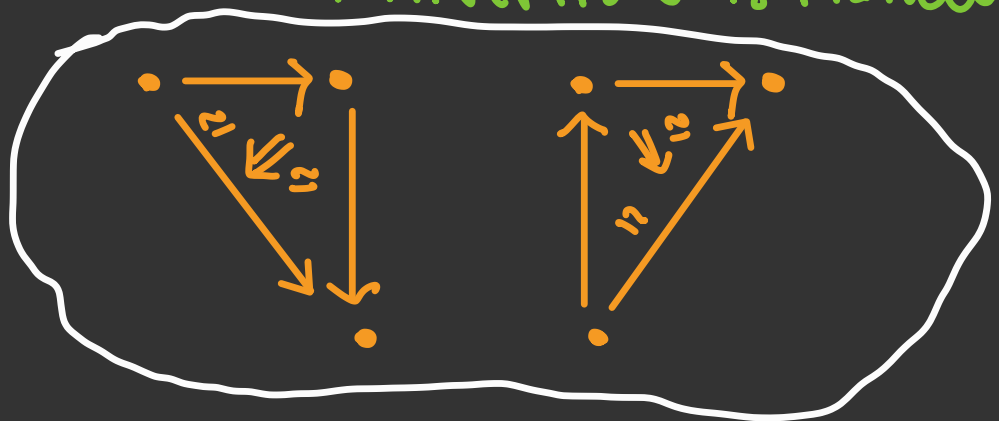

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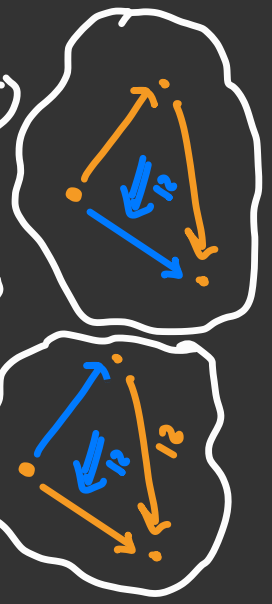

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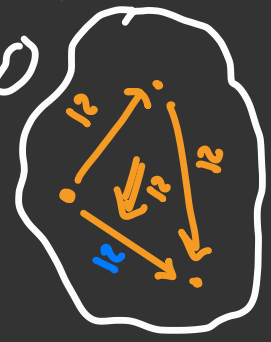

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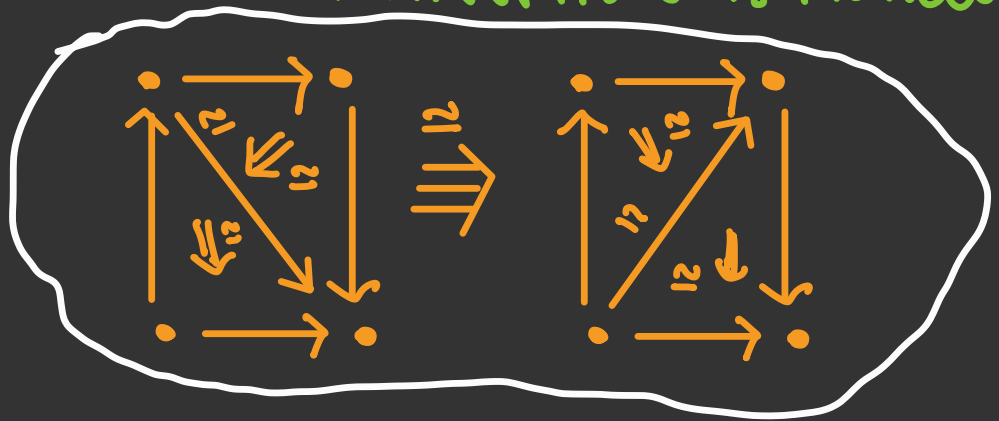

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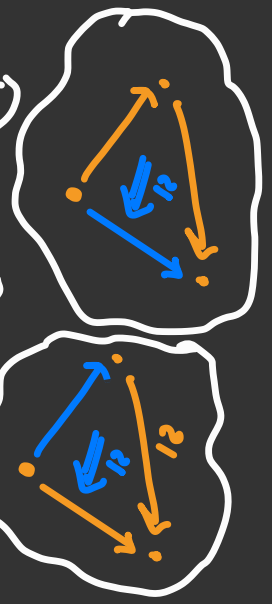

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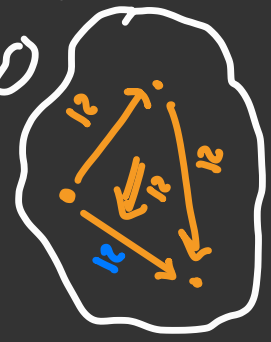

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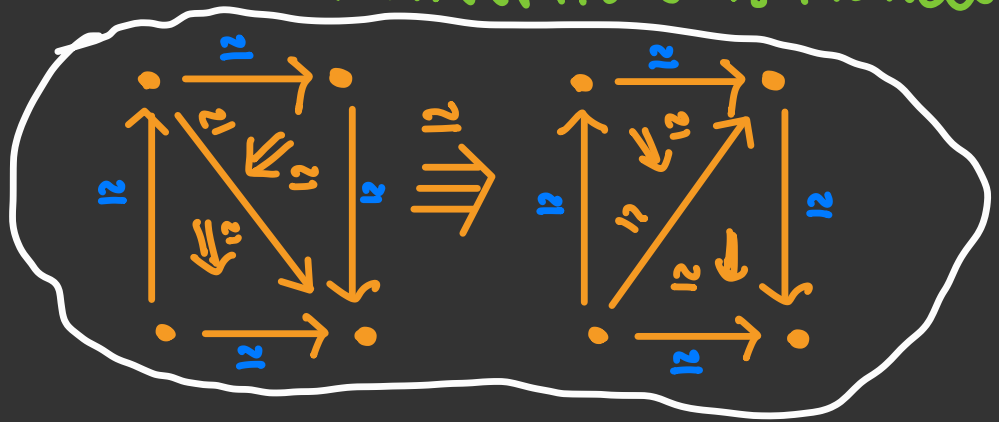

Complicial sets vs (∞,∞)-categories

Thm

Def:

Thm

Def:

Thm

Complicial sets vs (∞,∞)-categories

Thm $\mathfrak{Cat}_{(\infty,1)}$

Def:

Thm $\mathfrak{Cat}_{(\infty,\infty)}^{R}$

Def:

Thm $\mathfrak{Cat}_{(\infty,\infty)}^{L}$

Complicial sets vs (∞,∞)-categories

Thm [..., Lobatton] $\mathcal{C}at_{(\infty,n)} \simeq n\mathcal{C}omp$

$n\mathcal{C}omp$ = "$\{x \in \mathcal{C}omp \mid x \text{ fully marked in dim} > n\}$"

Def:

Thm $\mathcal{C}at^{R}_{(\infty,\infty)}$

Def:

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Complicial sets vs (∞,∞)-categories

Thm [..., Lobbaton] $\mathcal{C}at_{(\infty,n)} \simeq n\mathcal{C}omp$

$n\mathcal{C}omp$ "$=$" $\{x \in \mathcal{C}omp \mid x \text{ fully marked in dim} > n\}$

Def: $\mathcal{C}at_{(\infty,0)} \overset{\text{Gpd}}{\hookrightarrow} \mathcal{C}at_{(\infty,1)}$

Thm $\mathcal{C}at_{(\infty,\infty)}^{R}$

Def:

Thm $\mathcal{C}at_{(\infty,\infty)}^{L}$

Complicial sets vs (∞,∞)-categories

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$\{x \in \mathcal{C}omp \mid x \text{ fully marked in dim} > n\}$

Def: $\mathcal{C}at_{(\infty,0)} \xleftarrow{\text{core}_0} \mathcal{C}at_{(\infty,1)} \xleftarrow{\text{core}_1} \cdots \leftarrow \mathcal{C}at_{(\infty,n)} \xleftarrow{\text{core}_n} \cdots$

Thm $\mathcal{C}at^{R}_{(\infty,\infty)}$

Def:

Thm $\mathcal{C}at^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

Thm [..., Loubaton] $\mathcal{C}at_{(\infty,n)} \simeq n\mathcal{C}omp$

$n\mathcal{C}omp$ "$= \{x \in \mathcal{C}omp \mid x \text{ fully marked in dim} > n\}$"

Def: $\lim\left[\mathcal{C}at_{(\infty,0)} \xleftarrow{\text{core}_1} \mathcal{C}at_{(\infty,1)} \xleftarrow{} \cdots \xleftarrow{} \mathcal{C}at_{(\infty,n)} \xleftarrow{\text{core}_n} \cdots\right]$

Thm $\mathcal{C}at_{(\infty,\infty)}^{R}$

Def:

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Complicial sets vs (∞,∞)-categories

Thm [..., Loubaton] $\mathrm{Cat}_{(\infty,n)} \simeq n\mathrm{Comp}$

$n\mathrm{Comp} := \{x \in \mathrm{Comp} \mid x \text{ fully marked in dim} > n\}$

Def: $\mathrm{Cat}^{R}_{(\infty,\infty)} := \lim\left[\mathrm{Cat}_{(\infty,0)} \xleftarrow{\mathrm{core}_0} \mathrm{Cat}_{(\infty,1)} \xleftarrow{\mathrm{core}_1} \cdots \leftarrow \mathrm{Cat}_{(\infty,n)} \xleftarrow{\mathrm{core}_n} \cdots\right]$

Thm $\mathrm{Cat}^{R}_{(\infty,\infty)}$

Def:

Thm $\mathrm{Cat}^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

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$\mathcal{D} \hookrightarrow \mathcal{C}at^{R}_{(\infty,\infty)}$

Thm

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Complicial sets vs (∞,∞)-categories

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$\mathcal{D} \hookrightarrow [\mathrm{core}_0 \mathcal{D}$

Thm $\mathrm{Cat}^{R}_{(\infty,\infty)}$

Def:

Thm $\mathrm{Cat}^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

Thm [..., Lobatton] $\mathrm{Cat}_{(\infty,n)} \simeq n\text{-}\mathrm{Comp}$

$\{x \in \mathrm{Comp} \mid x \text{ fully marked in dim} > n\}$

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$\mathcal{D} \hookrightarrow [\mathrm{core}_0 \mathcal{D} \hookrightarrow \mathrm{core}_1 \mathcal{D}$

Thm $\mathrm{Cat}^{R}_{(\infty,\infty)}$

Def:

Thm $\mathrm{Cat}^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

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Thm $\mathrm{Cat}^{R}_{(\infty,\infty)}$

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Complicial sets vs (∞,∞)-categories

Thm [..., Lobster] $\mathcal{C}at_{(\infty,n)} \simeq n\mathcal{C}omp$

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$\mathcal{D} \hookrightarrow [\text{core}_0\mathcal{D} \hookrightarrow \text{core}_1\mathcal{D} \hookrightarrow \dots \hookrightarrow \text{core}_n\mathcal{D}]$

Thm [Lobster, ORV] $\mathcal{C}at^{R}_{(\infty,\infty)} \simeq \mathcal{C}omp$

Def:

Thm $\mathcal{C}at^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

Thm [..., Lobster] $\mathrm{Cat}_{(\infty,n)} \simeq n\mathrm{Comp} := \{x \in \mathrm{Comp} \mid x \text{ fully marked in dim} > n\}$

Def: $\mathrm{Cat}^R_{(\infty,\infty)} := \lim\left[\mathrm{Cat}_{(\infty,0)} \xleftarrow{\mathrm{core}_0} \mathrm{Cat}_{(\infty,1)} \xleftarrow{\mathrm{core}_1} \cdots \leftarrow \mathrm{Cat}_{(\infty,n)} \xleftarrow{\mathrm{core}_n} \cdots\right]$

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Thm [Lobster, ORW] $\mathrm{Cat}^R_{(\infty,\infty)} \simeq \mathrm{Comp}$

Def:

$$\mathrm{Cat}_{(\infty,0)} \xleftarrow{\mathrm{tr}} \mathrm{Cat}_{(\infty,1)}$$

Thm

$$\mathrm{Cat}^L_{(\infty,\infty)}$$

Complicial sets vs (∞,∞)-categories

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Thm [Lurie, Ozornova-Rovelli] $\mathrm{Cat}^{R}_{(\infty,\infty)} \simeq \mathrm{Comp}$

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$$\mathrm{Cat}_{(\infty,0)} \xleftarrow{\mathrm{tr}_0} \mathrm{Cat}_{(\infty,1)} \xleftarrow{\mathrm{tr}_1} \cdots \leftarrow \mathrm{Cat}_{(\infty,n)} \xleftarrow{\mathrm{tr}_n} \cdots$$

Thm

$$\mathrm{Cat}^{L}_{(\infty,\infty)}$$

Complicial sets vs (∞,∞)-categories

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Thm $\mathrm{Cat}^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

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Thm $\mathrm{Cat}^{L}_{(\infty,\infty)}$

Complicial sets vs (∞,∞)-categories

Thm [..., Lobster] $\mathrm{Cat}_{(\infty,n)} \simeq n\mathrm{Comp}$

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Complicial sets VS (∞,∞)-categories

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Thm [Gepner-Henne, ORV] $\mathrm{Cat}^{L}_{(\infty,\infty)} \simeq \mathrm{Loc}\,\mathrm{Comp}$

Complicial sets VS (∞,∞)-categories

Thm [..., Lobatton] $\mathrm{Cat}_{(\infty,n)} \simeq n\mathrm{Comp}$

$$\{x \in \mathrm{Comp} \mid x \text{ fully marked in dim} > n\}$$

Def: $\mathrm{Cat}^{R}_{(\infty,\infty)} := \lim\left[\mathrm{Cat}_{(\infty,0)} \xleftarrow{\mathrm{core}_0} \mathrm{Cat}_{(\infty,1)} \leftarrow \dots \leftarrow \mathrm{Cat}_{(\infty,n)} \xleftarrow{\mathrm{core}_n} \dots\right]$

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$$= \{x \in \mathrm{Comp} \mid x \simeq \mathrm{colim}\,\mathrm{tr}_n x\}$$

mark all in dim $> n$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Cat}_{(\infty,0)}$

Ex ①: $\mathrm{Cat}_{(\infty,1)}$, $\mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)}$, $\mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Cat}_{(\infty,0)}$

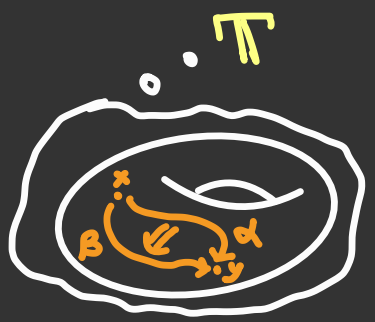


Ex ①: $\mathrm{Cat}_{(\infty,1)}$, $\mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)}$, $\mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Cat}_{(\infty,0)}$



Ex ①: $\mathrm{Cat}_{(\infty,1)}$, $\mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)}$, $\mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^*\mathcal{T} \in \mathcal{C}at_{(\infty,0)}$

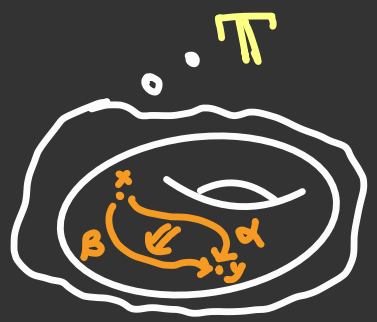


Ex ①: $\mathcal{C}at_{(\infty,1)}$, $\mathcal{C}at_{(\infty,0)}$

Ex ②: $\mathcal{C}at_{(\infty,2)}$, $\mathcal{C}at_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{*}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



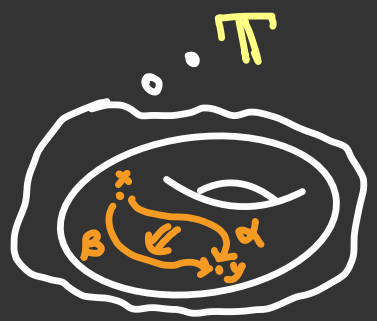
Mark everything on $\mathrm{Sing}\,\mathbb{T}$

Ex ①: $\mathrm{Cat}_{(\infty,1)}, \mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)}, \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}_0 T \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}^{\#}_0 T$

$\mathrm{Sing}^{\#}_0 T \ni$ 

$\mathrm{Sing}^{\#}_1 T \ni$ 

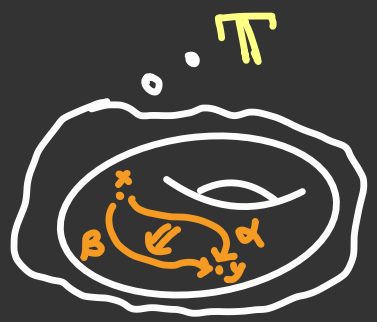
$\mathrm{Sing}^{\#}_2 T \ni$ 

Ex ①: $\mathrm{Cat}_{(\infty,1)}, \mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)}, \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}_0 T \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}^{\#}_0 T$

$\mathrm{Sing}^{\#}_0 T \ni$ 

$\mathrm{Sing}^{\#}_1 T \ni$ 

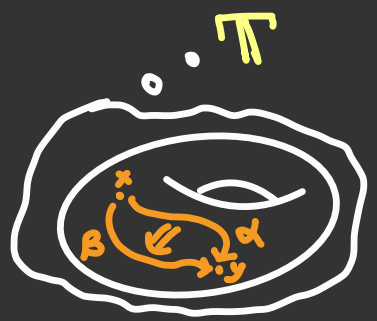
$\mathrm{Sing}^{\#}_2 T \ni$ 

Ex ①: $\mathrm{Cat}_{(\infty,1)} \supset \mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)} \supset \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathcal{T} \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}\,\mathcal{T}$

$\mathrm{Sing}_0^{\#}\mathcal{T}$ ∋



$\mathrm{Sing}_1^{\#}\mathcal{T}$ ∋



$\mathrm{Sing}_2^{\#}\mathcal{T}$ ∋

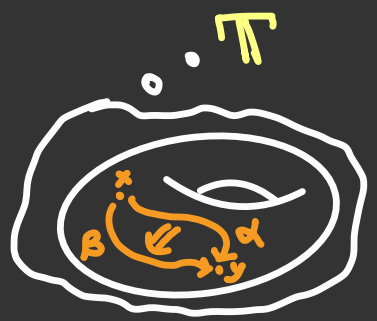


Ex ①: $\mathrm{Cat}_{(\infty,1)}$, $\mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)}$, $\mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}_0 T \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}^{\#}_0 T$

$\mathrm{Sing}^{\#}_0 T \ni$ 

$\mathrm{Sing}^{\#}_1 T \ni$ 

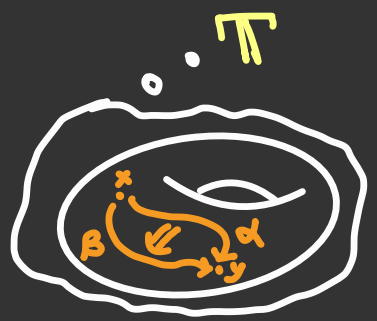
$\mathrm{Sing}^{\#}_2 T \ni$ 

Ex ①: $\mathrm{Cat}_{(\infty,1)} \supset \mathrm{Cat}_{(\infty,0)}$

Ex ②: $\mathrm{Cat}_{(\infty,2)} \supset \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex⓪: $\mathrm{Sing}^{\#}\mathcal{T} \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}\,\mathcal{T}$

$\mathrm{Sing}^{\#}_0\mathcal{T} \ni$ 

$\tilde{\mathrm{Sing}}^{\#}_1\mathcal{T} \ni$ 

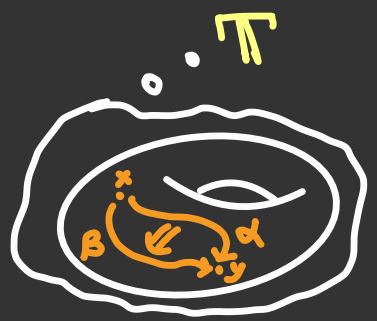
$\tilde{\mathrm{Sing}}^{\#}_2\mathcal{T} \ni$ 

Ex①: $\mathrm{Cat}_{(\infty,1)}, \mathrm{Cat}_{(\infty,0)}$

Ex②: $\mathrm{Cat}_{(\infty,2)}, \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathcal{T} \in \mathrm{Cat}_{(\infty,0)}$



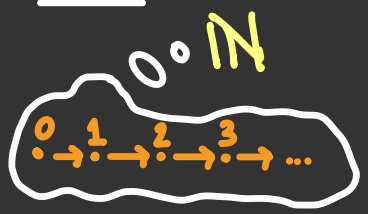
Mark everything on $\mathrm{Sing}\,\mathcal{T}$

$\mathrm{Sing}^{\#}_0\mathcal{T} \ni$ 

$\simeq \mathrm{Sing}^{\#}_1\mathcal{T} \ni$ 

$\simeq \mathrm{Sing}^{\#}_2\mathcal{T} \ni$ 

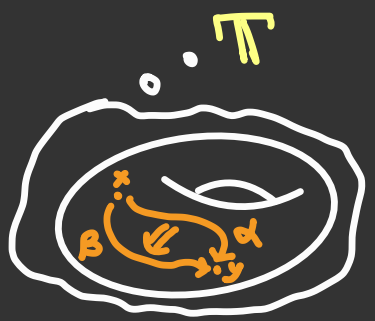
Ex ①: $\mathrm{Cat}_{(\infty,1)}, \mathrm{Cat}_{(\infty,0)}$



Ex ②: $\mathrm{Cat}_{(\infty,2)}, \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathcal{T} \in \mathrm{Cat}_{(\infty,0)}$



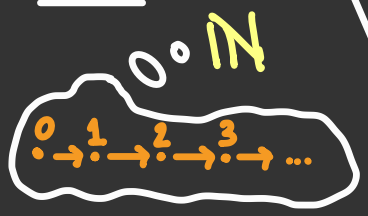
Mark everything on $\mathrm{Sing}\,\mathcal{T}$

$\mathrm{Sing}_0^{\#}\mathcal{T} \ni$ 

$\simeq$ $\mathrm{Sing}_1^{\#}\mathcal{T} \ni$ 

$\simeq$ $\mathrm{Sing}_2^{\#}\mathcal{T} \ni$ 

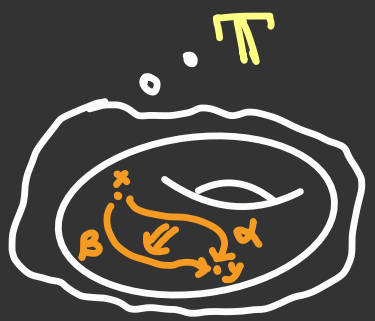
Ex ①: $B^{\mathbb{N}} \in \mathrm{Cat}_{(\infty,1)}, \mathrm{Cat}_{(\infty,0)}$



Ex ②: $\mathrm{Cat}_{(\infty,2)}, \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



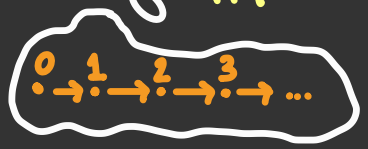
Mark everything on $\mathrm{Sing}\,\mathbb{T}$

$\mathrm{Sing}^{\#}_{0}\mathbb{T} \ni$ 

$\tilde{\mathrm{Sing}}^{\#}_{1}\mathbb{T} \ni$ 

$\tilde{\mathrm{Sing}}^{\#}_{2}\mathbb{T} \ni$ 

Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$

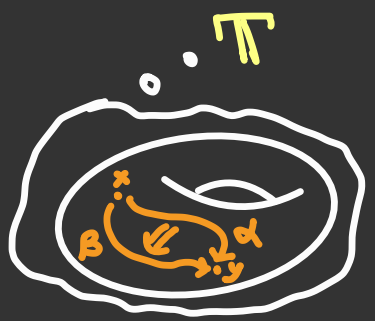


Mark every thing in dim >1 on $B\mathbb{N}$

Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$

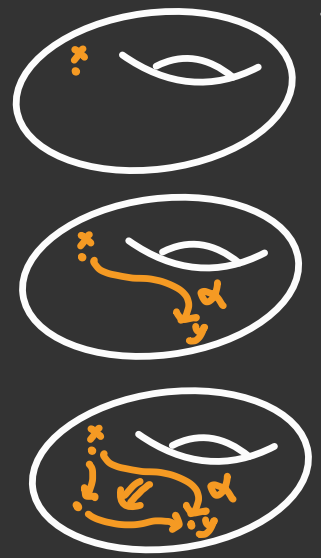


Mark everything on $\mathrm{Sing}\,\mathbb{T}$

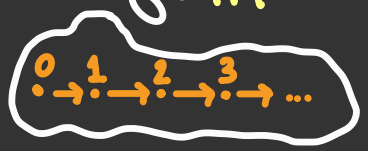
$\mathrm{Sing}^{\#}_0\mathbb{T} \ni$

$\simeq \mathrm{Sing}^{\#}_1\mathbb{T} \ni$

$\simeq \mathrm{Sing}^{\#}_2\mathbb{T} \ni$



Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



Mark every thing in dim >1 on $B\mathbb{N}$

$B^{\natural}_0\mathbb{N} \ni$

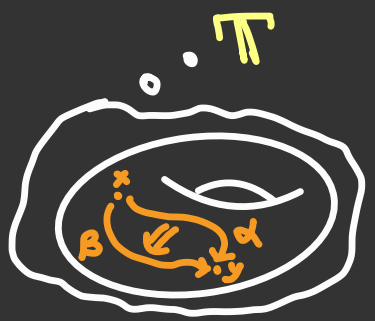
$B^{\natural}_1\mathbb{N} \ni$

$B^{\natural}_2\mathbb{N} \ni$

Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



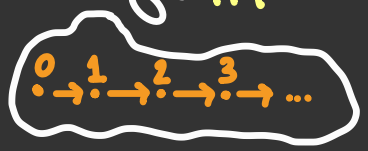
Mark everything on $\mathrm{Sing}\,\mathbb{T}$

$\mathrm{Sing}^{\#}_0\mathbb{T} \ni$ 

$\simeq$ $\mathrm{Sing}^{\#}_1\mathbb{T} \ni$ 

$\simeq$ $\mathrm{Sing}^{\#}_2\mathbb{T} \ni$ 

Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



Mark every thing in dim >1 on $B\mathbb{N}$

$B^{\natural}_0\mathbb{N} \ni 0$

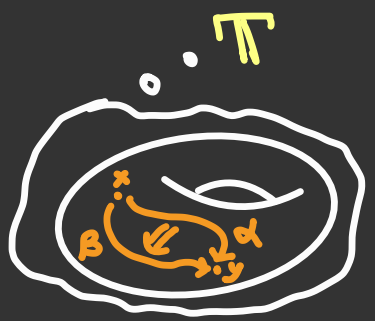
$B^{\natural}_1\mathbb{N} \ni$

$B^{\natural}_2\mathbb{N} \ni$

Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



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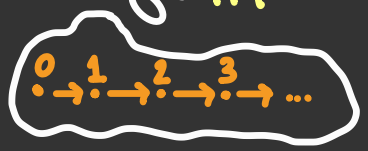
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



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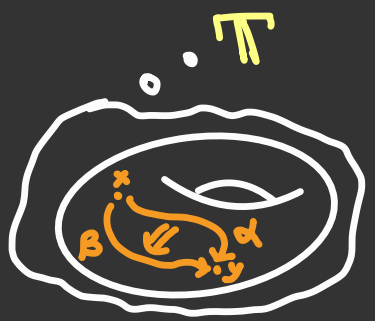
$B^{\natural}_1\mathbb{N} \ni 0, 1, 2, 3, \ldots$

$B^{\natural}_2\mathbb{N} \ni$

Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



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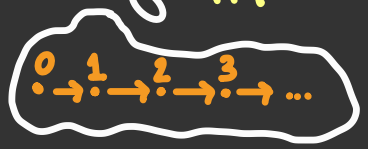

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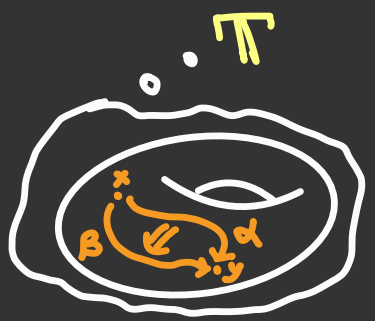
Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}\,\mathbb{T}$

$\mathrm{Sing}^{\#}_0\mathbb{T} \ni$ 

$\sim$

$\mathrm{Sing}^{\#}_1\mathbb{T} \ni$ 

$\sim$

$\mathrm{Sing}^{\#}_2\mathbb{T} \ni$ 

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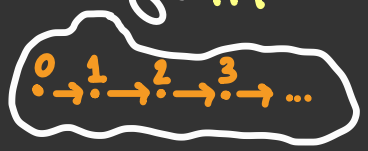

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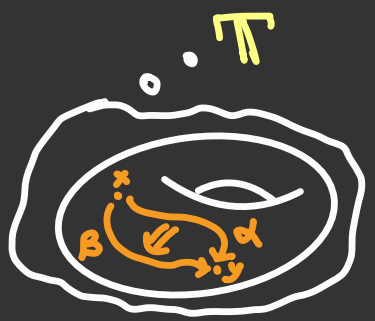
Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



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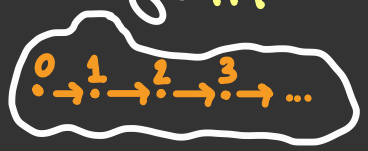
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



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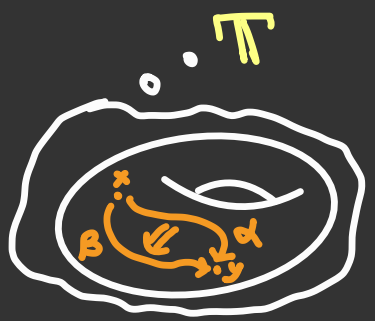
Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}\,\mathbb{T}$

$\mathrm{Sing}^{\#}_0\mathbb{T} \ni$ 

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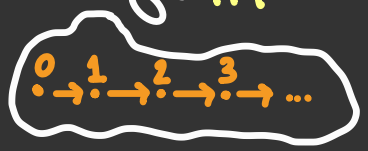
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



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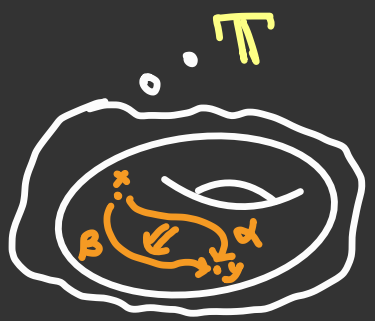
Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathcal{T} \in \mathrm{Cat}_{(\infty,0)}$



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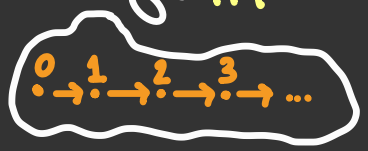
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



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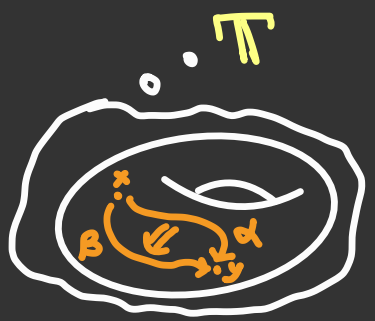
Ex ②: $\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

$\mathrm{Ab}^{\otimes}$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}\,\mathbb{T}$

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$\simeq$ $\mathrm{Sing}^{\#}_1\mathbb{T} \ni$ 

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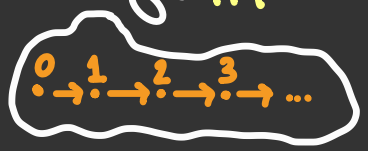
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



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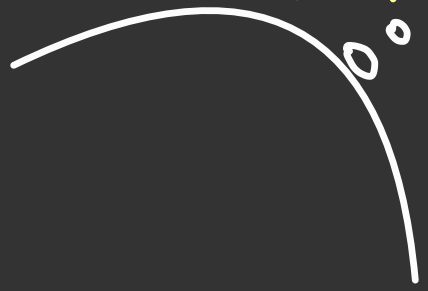
Ex ②:

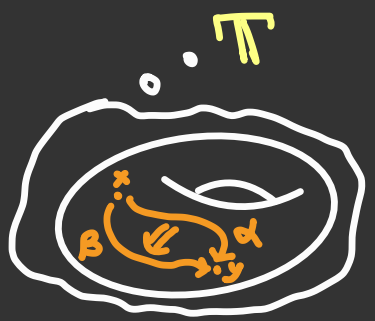
$\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

$\mathfrak{M}(\mathrm{Ab}^{\otimes})$



Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^* T \in \mathrm{Cat}_{(\infty,0)}$



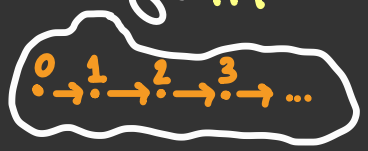
Mark everything on $\mathrm{Sing}^* T$

$\mathrm{Sing}^*_0 T \ni$ 

$\simeq \mathrm{Sing}^*_1 T \ni$ 

$\simeq \mathrm{Sing}^*_2 T \ni$ 

Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



Mark every thing in dim >1 on $B\mathbb{N}$

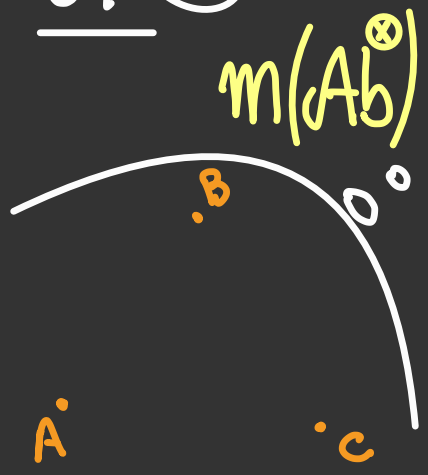
$B^{\natural}_0\mathbb{N} \ni 0$

$B^{\natural}_1\mathbb{N} \ni 0, 1, 2, 3, \ldots$

$B^{\natural}_2\mathbb{N} \ni (2,3)$

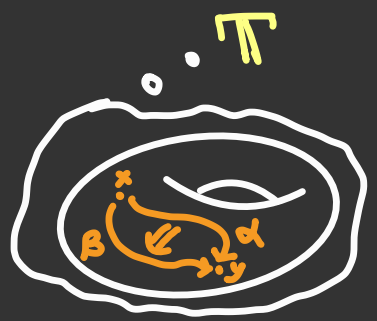
$\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Ex ②:



Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^* T \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}^* T$

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$\simeq$ $\mathrm{Sing}^*_1 T \ni$ 

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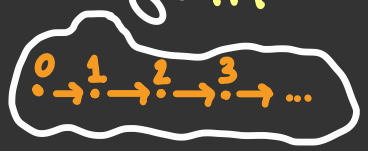
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



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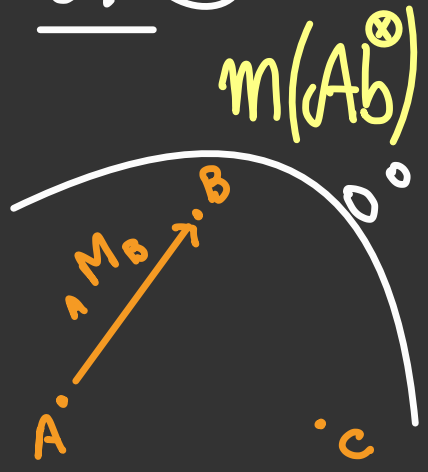
$B^{\natural}_2\mathbb{N} \ni (2,3)$

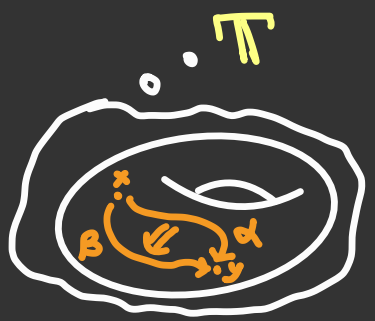
$\mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

Ex ②:



Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^* T \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}^* T$

$\mathrm{Sing}^*_0 T \ni$



$\simeq \mathrm{Sing}^*_1 T \ni$



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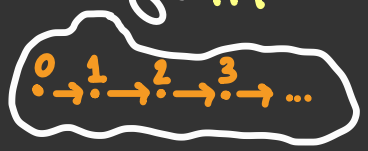
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



Mark every thing in dim >1 on $B\mathbb{N}$

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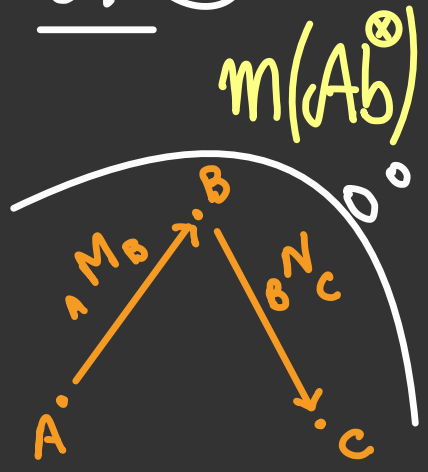

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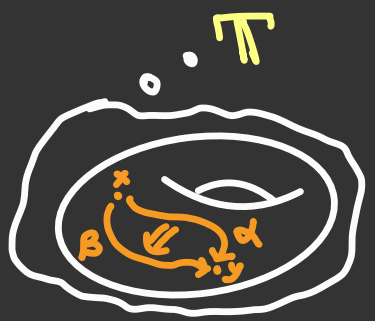

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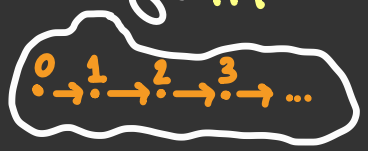

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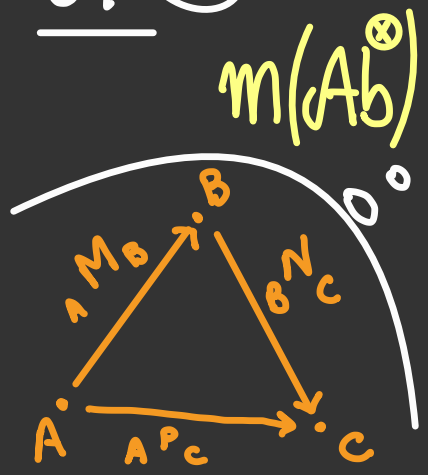

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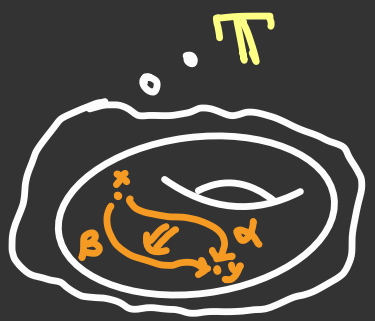

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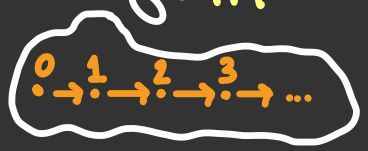

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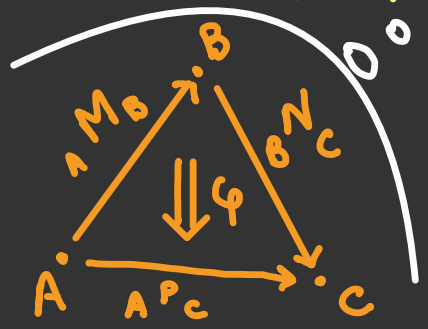
Ex ②:

$\mathfrak{m}(\mathrm{Ab}^{\otimes})$



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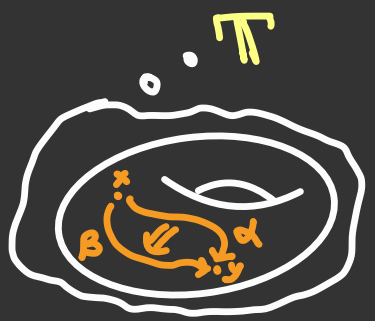

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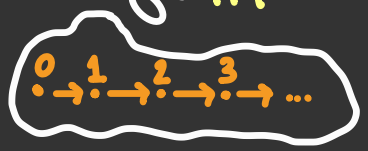

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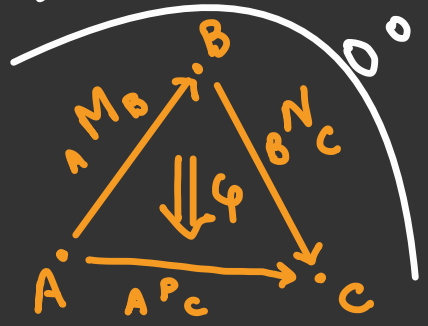
[Gurski, DLRS] $\mathfrak{m}(\mathrm{Ab}^{\otimes})$



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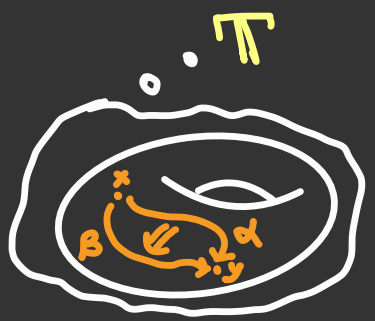

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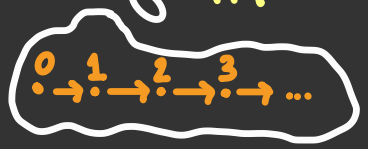

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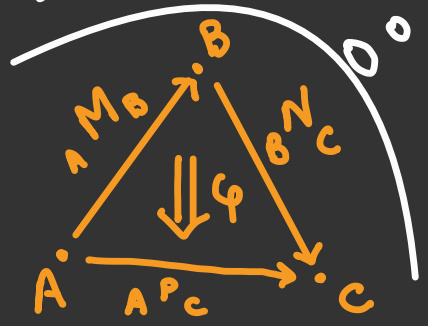
[Gurski, DLRS]

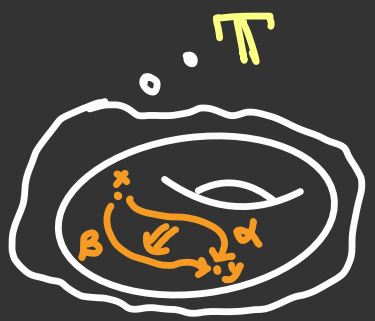
$\mathcal{M}(\mathrm{Ab}^{\otimes})$



Mark on $N(\mathcal{M}(\mathrm{Ab}^{\otimes}))$

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



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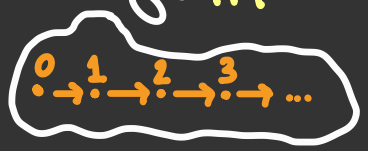

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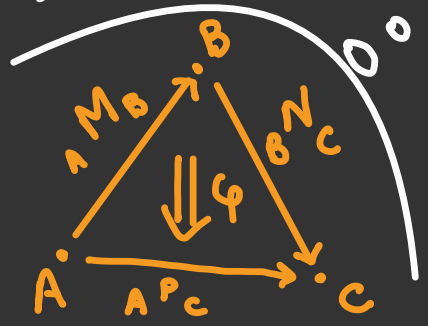
[Gurski, DLRS]

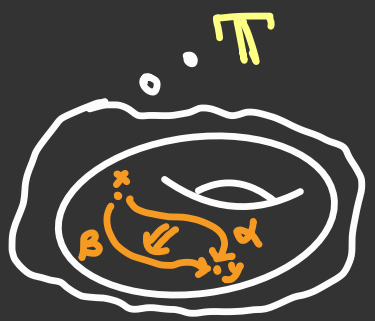
$\mathfrak{m}(\mathrm{Ab}^{\otimes})$ •



Mark on $N(\mathfrak{m}(\mathrm{Ab}^{\otimes}))$ all in dim >2

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



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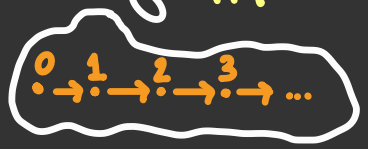

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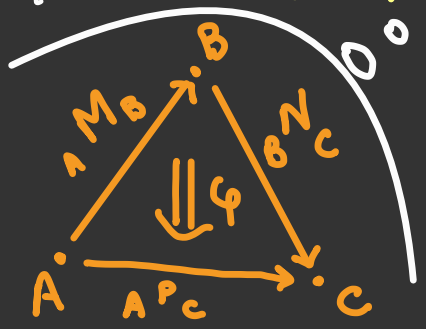
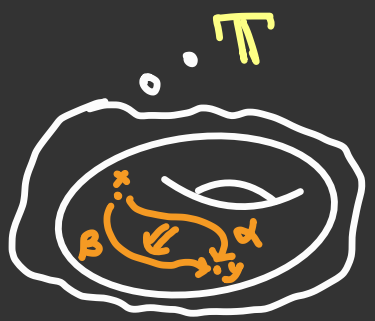


Mark on $N(\mathcal{M}(\mathrm{Ab}^{\otimes}))$ all in dim ≥ 2 bimodule isos in dim 2

Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



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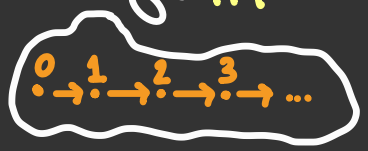

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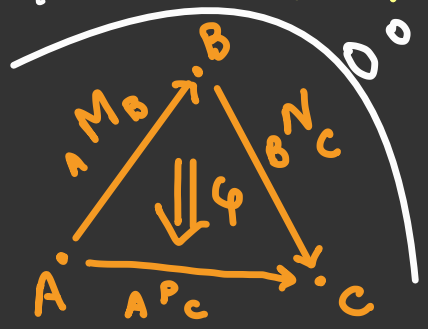

Mark on $N(\mathfrak{M}(\mathrm{Ab}^{\otimes}))$
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Examples of (∞,n)-categories

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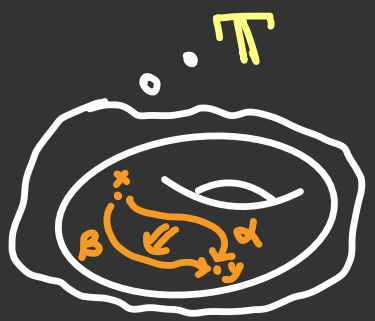

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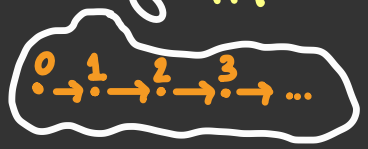

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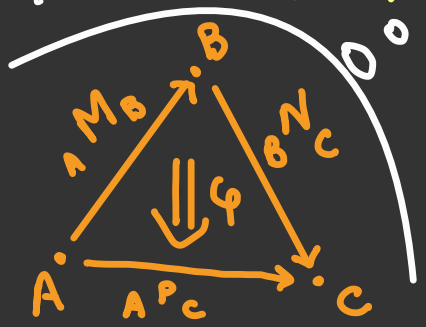

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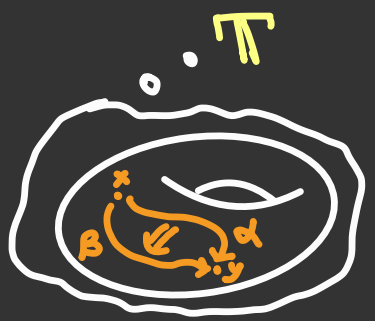

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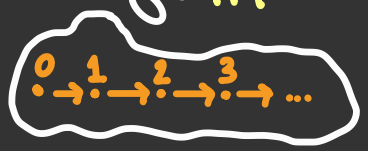

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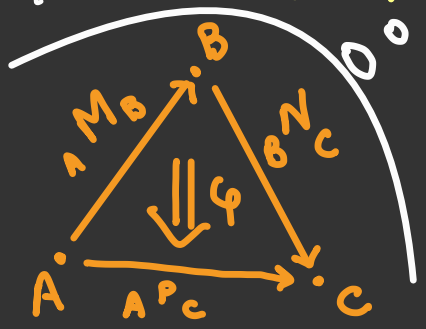

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Examples of (∞,n)-categories

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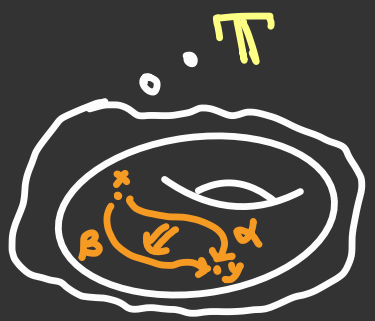

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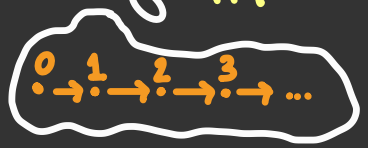

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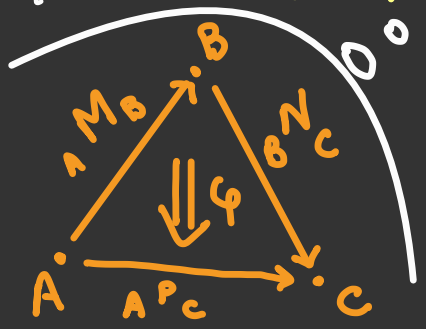

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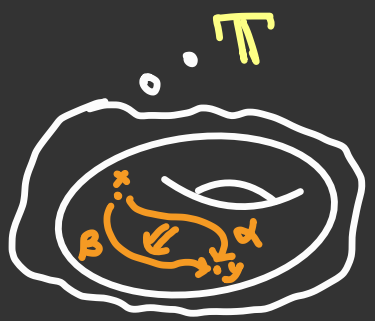
Examples of (∞,n)-categories

Ex ⓪: $\mathrm{Sing}^{\#}\mathbb{T} \in \mathrm{Cat}_{(\infty,0)}$



Mark everything on $\mathrm{Sing}^{\#}\mathbb{T}$

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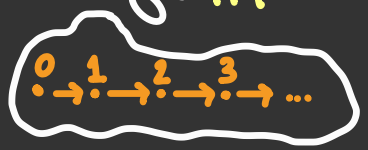
Ex ①: $B^{\natural}\mathbb{N} \in \mathrm{Cat}_{(\infty,1)} \setminus \mathrm{Cat}_{(\infty,0)}$



Mark every thing in dim >1 on $B\mathbb{N}$

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$B^{\natural}_1\mathbb{N} \ni 0, 1, 2, 3, \ldots$

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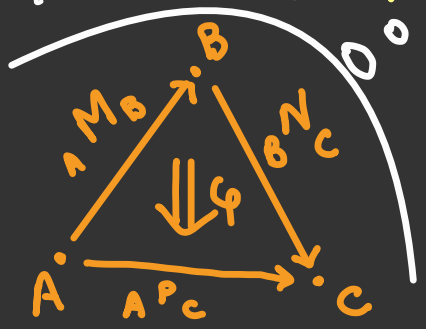
Ex ②: $N^{\natural}\mathfrak{m}(\mathrm{Ab}^{\otimes}) \in \mathrm{Cat}_{(\infty,2)} \setminus \mathrm{Cat}_{(\infty,1)}$

[Gurski, DLRS]

$\mathfrak{m}(\mathrm{Ab}^{\otimes})$



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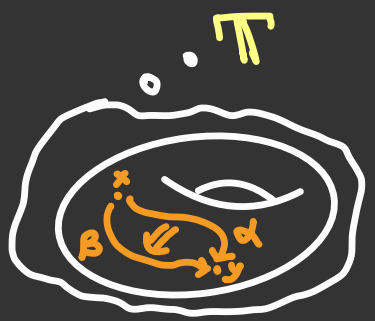

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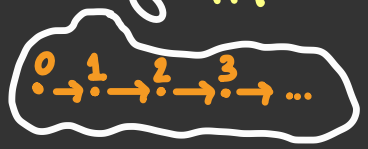

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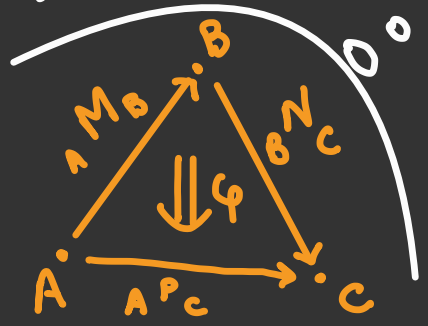

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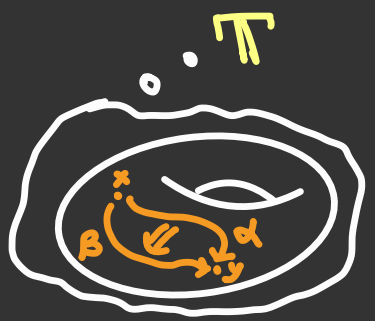

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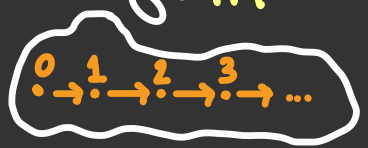

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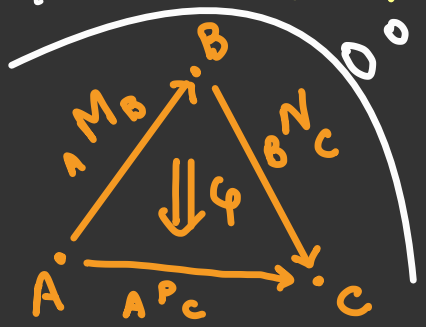

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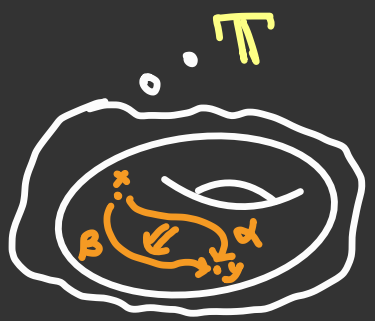

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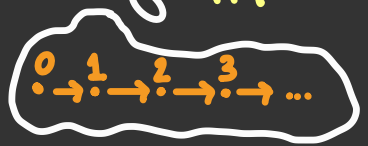

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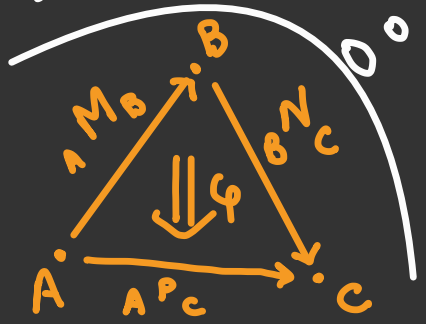

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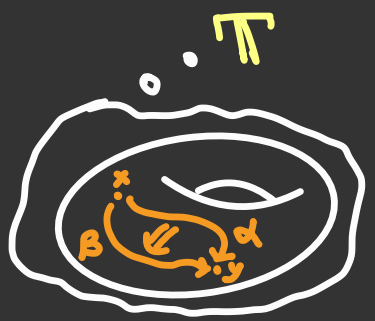

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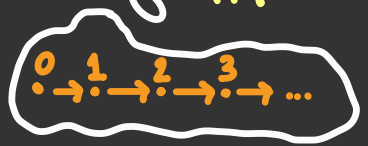

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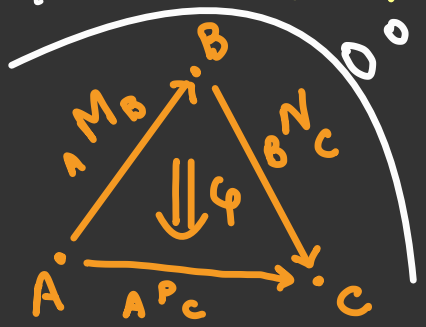

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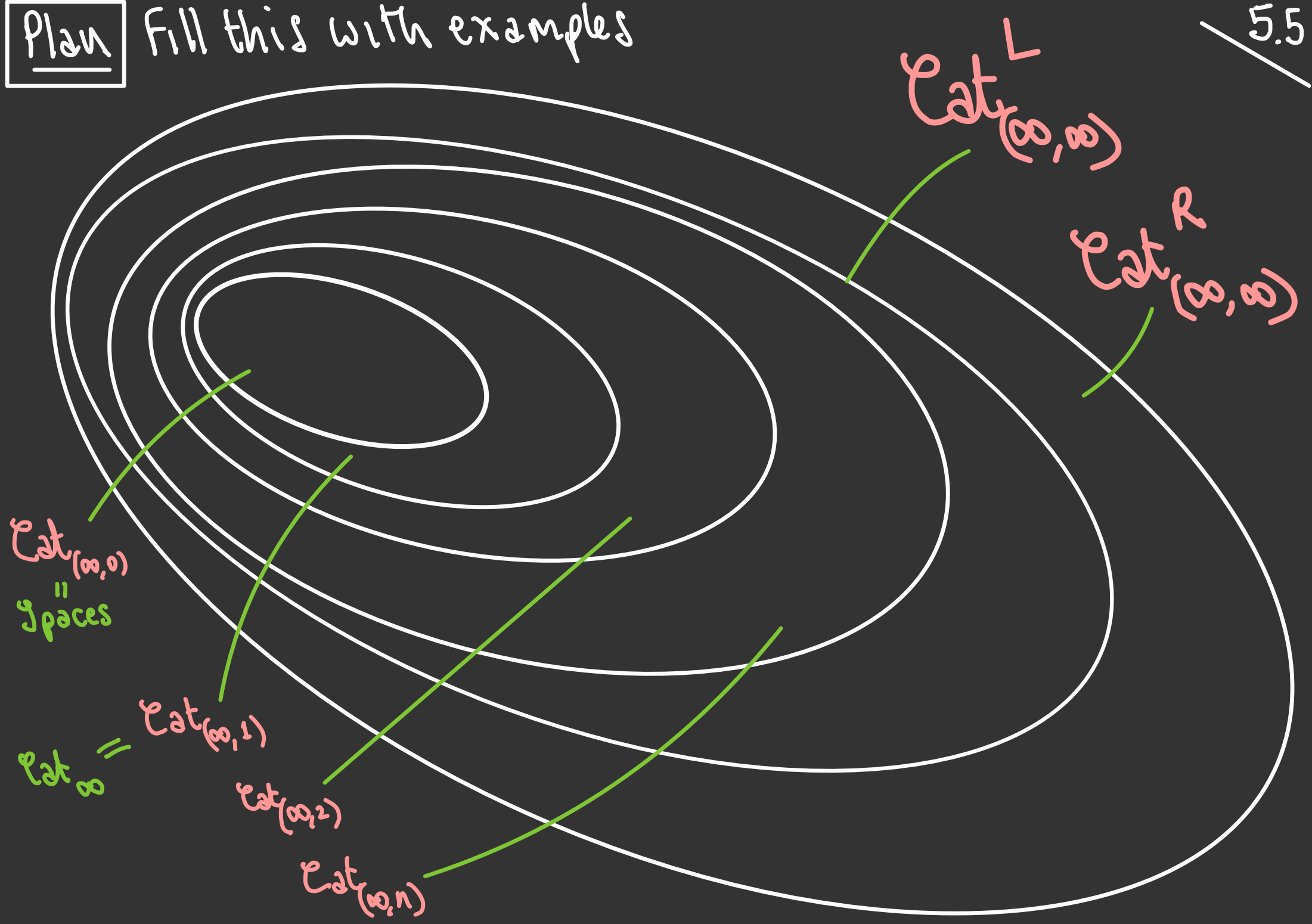
Plan Fill this with examples



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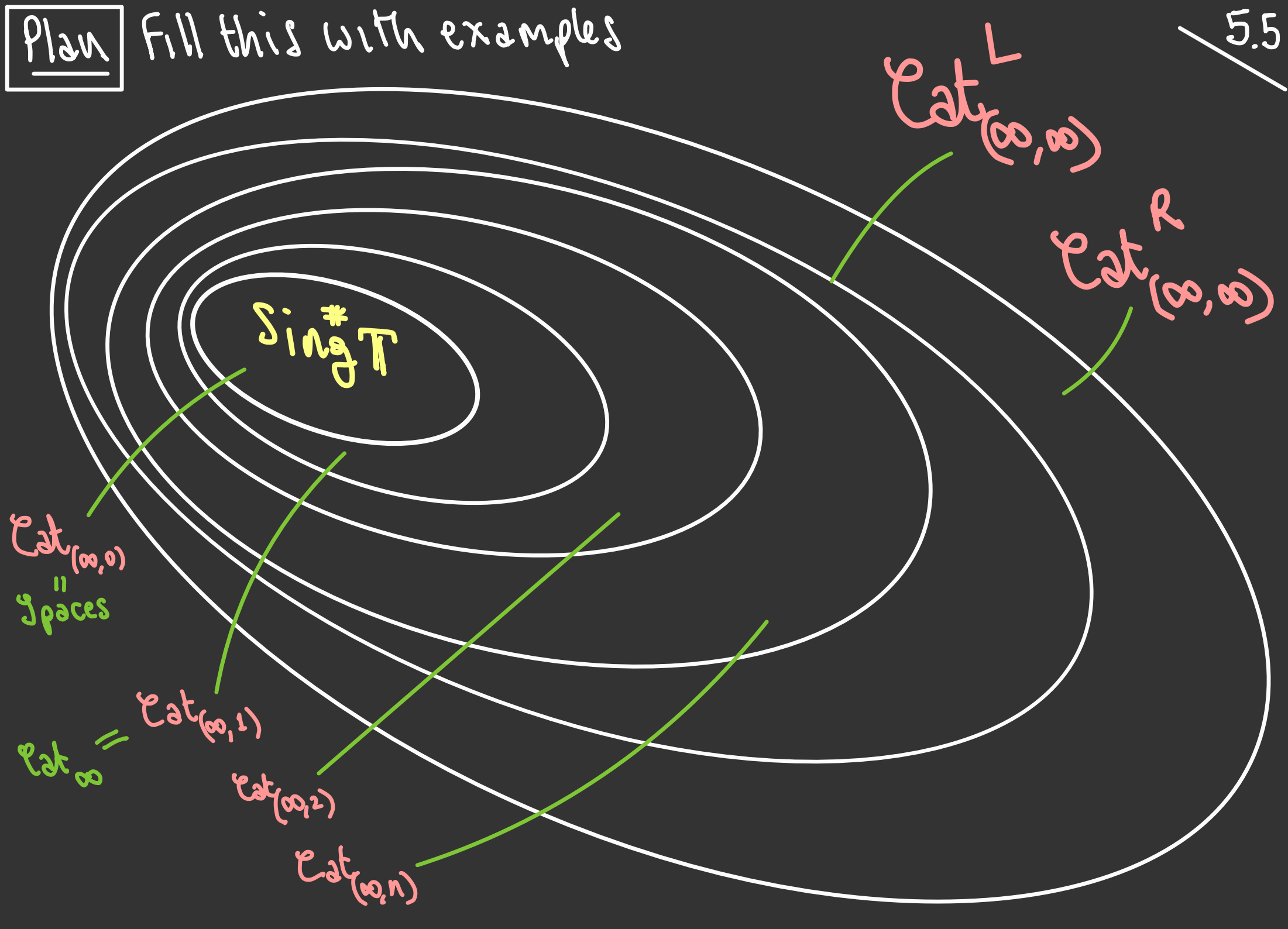

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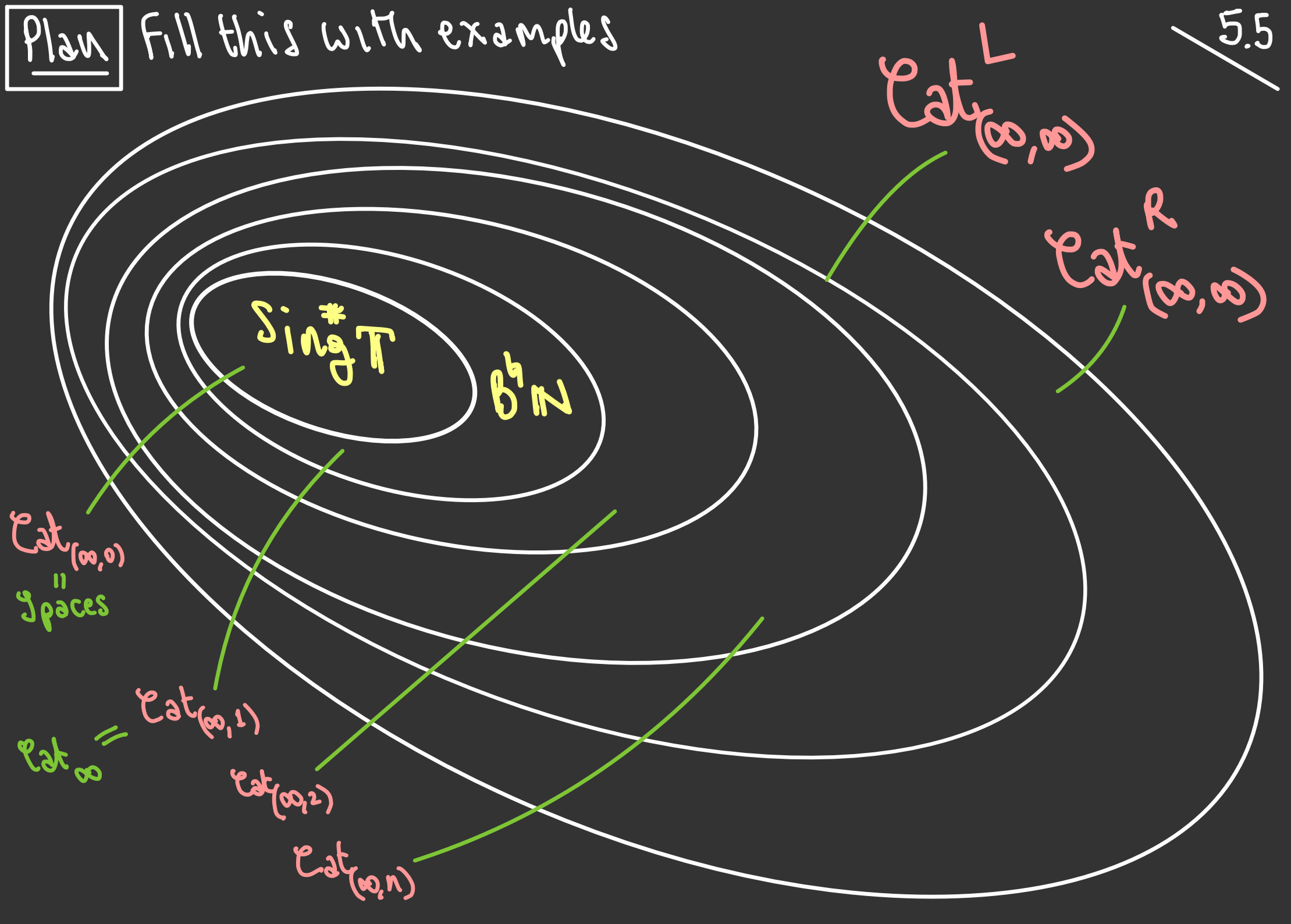

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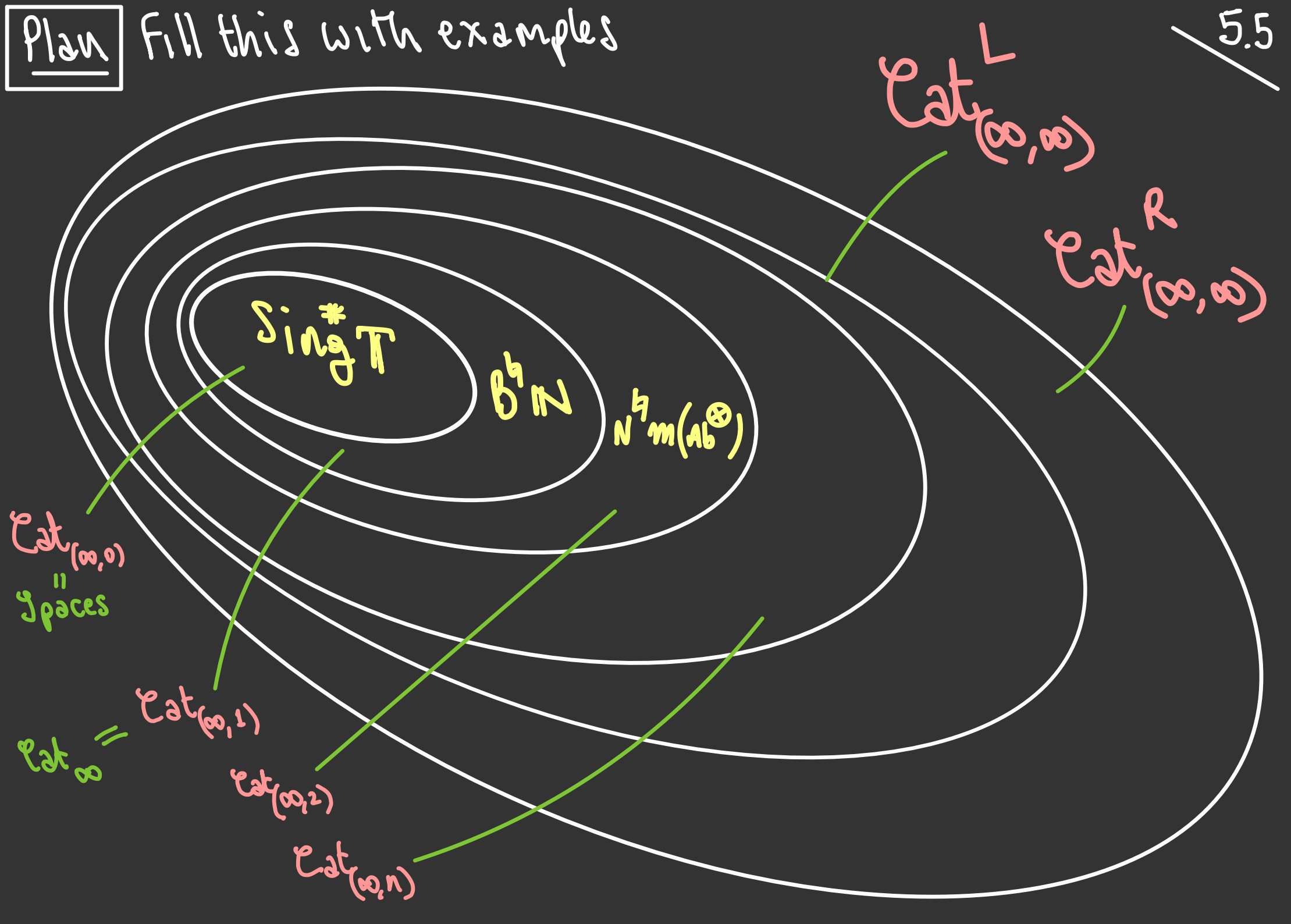

ω-categories

Note:

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- objects $\langle$ • 2

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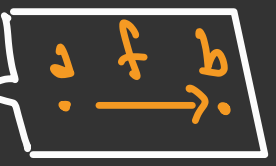
- morphisms 
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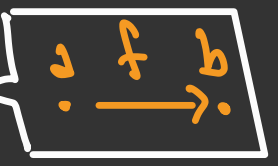
- morphisms 

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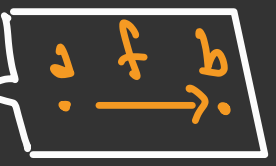
- morphisms 



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equality

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$+ (\tilde{\eta}_f), (\tilde{\varepsilon}_f)$

Note: Many notions of sameness in an ω-category

$\exists\, f : a \to b$
$g : b \to a$ + $id_a = gf$, $fg = id_b$

$\exists\, f : a \to b$
$g : b \to a$ + $id_a \cong gf$, $fg \cong id_b$

$a = b \Rightarrow a \cong b \Rightarrow a \simeq b \Rightarrow a \overset{2}{\simeq} b \Rightarrow \dots \Rightarrow a \overset{\omega}{\simeq} b$

equality — isomorphism — equivalence — 2-equivalence — ω-equivalence

ω-categories

An ω-category consists of

- objects $\langle a$
- morphisms $\langle a \xrightarrow{f} b$
- 2-morphisms
- 3-morphisms
- n morphisms $\forall n$

with strict composition and identities



$\exists f : a \to b$
$+ \tilde{f} : b \to a$
$+ \eta_f : id_a \Rightarrow \tilde{f} f$
$+ \varepsilon_f : f \tilde{f} \Rightarrow id_b$
$+ (\eta_f), (\varepsilon_f)$
$+ \eta_{(\eta_f)}, \eta_{(\varepsilon_f)}$

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$+\ (\tilde{\eta_f}), (\tilde{\varepsilon_f})$
$+\ \eta_{(\eta_f)}, \eta_{(\varepsilon_f)}$
$+\ \varepsilon_{(\eta_f)}, \eta_{(\varepsilon_f)}$
$+\ \ldots$

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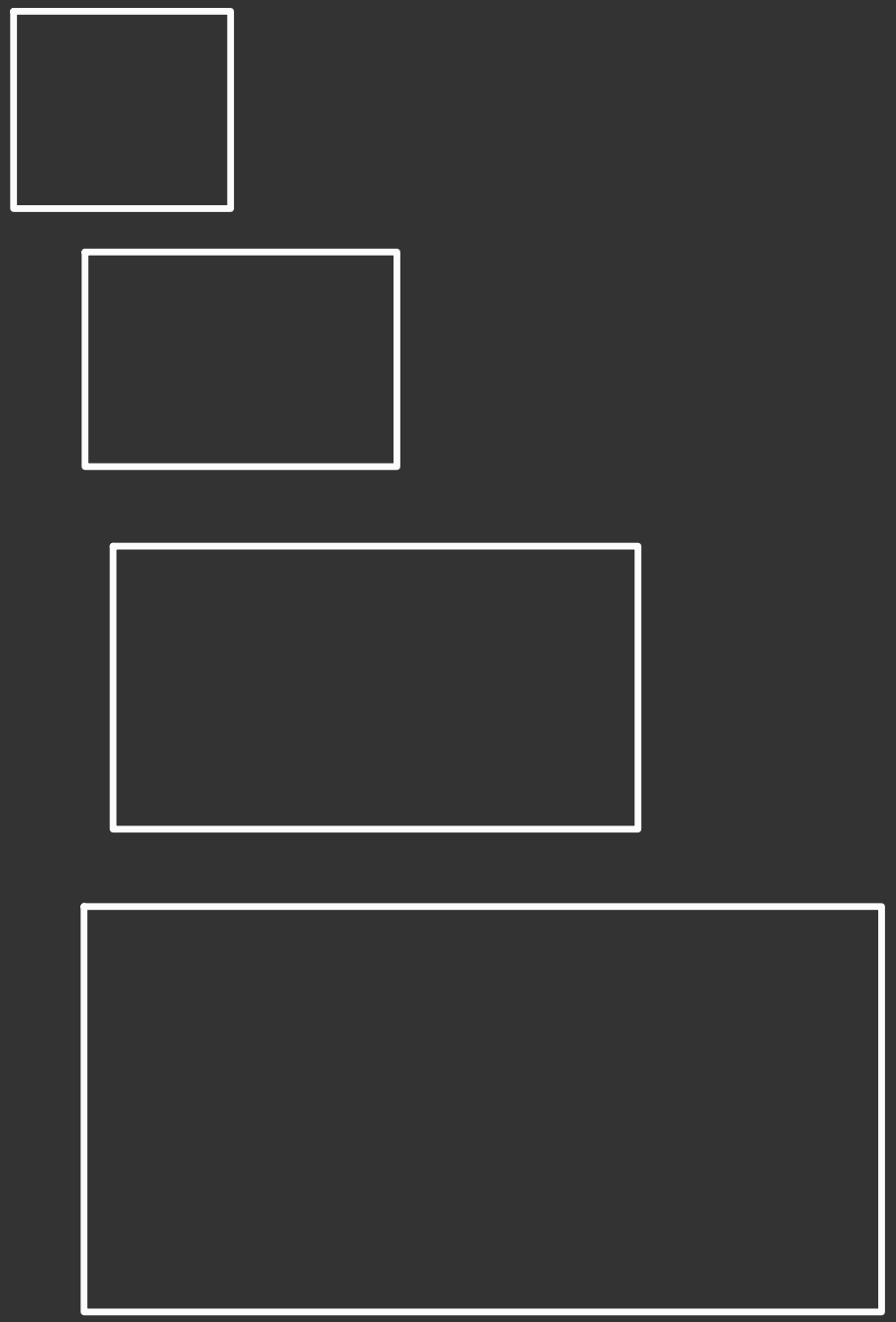
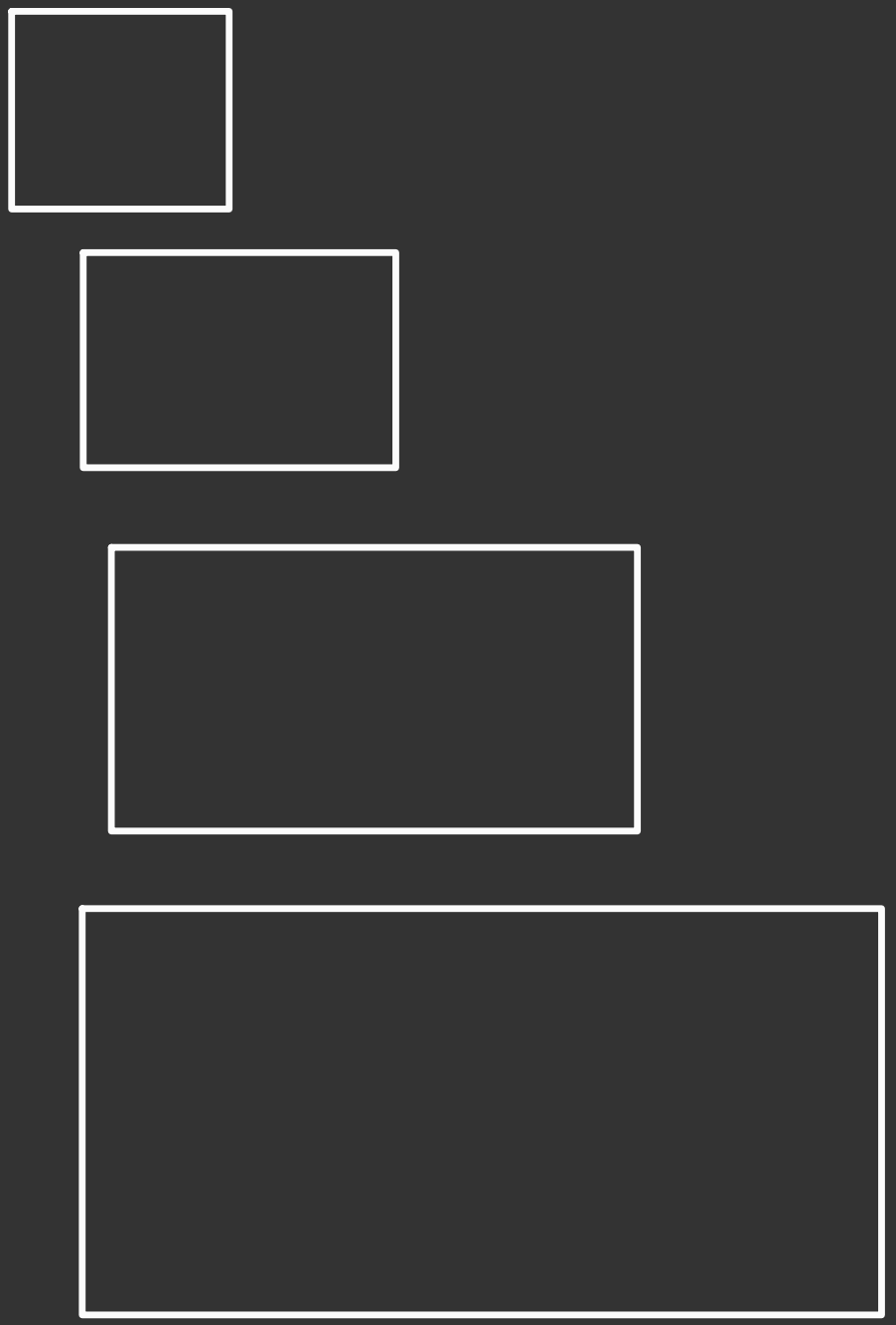
equality — isomorphism — equivalence — 2-equivalence — ω-equivalence

Nerves of ω-categories



Nerves of ω-categories

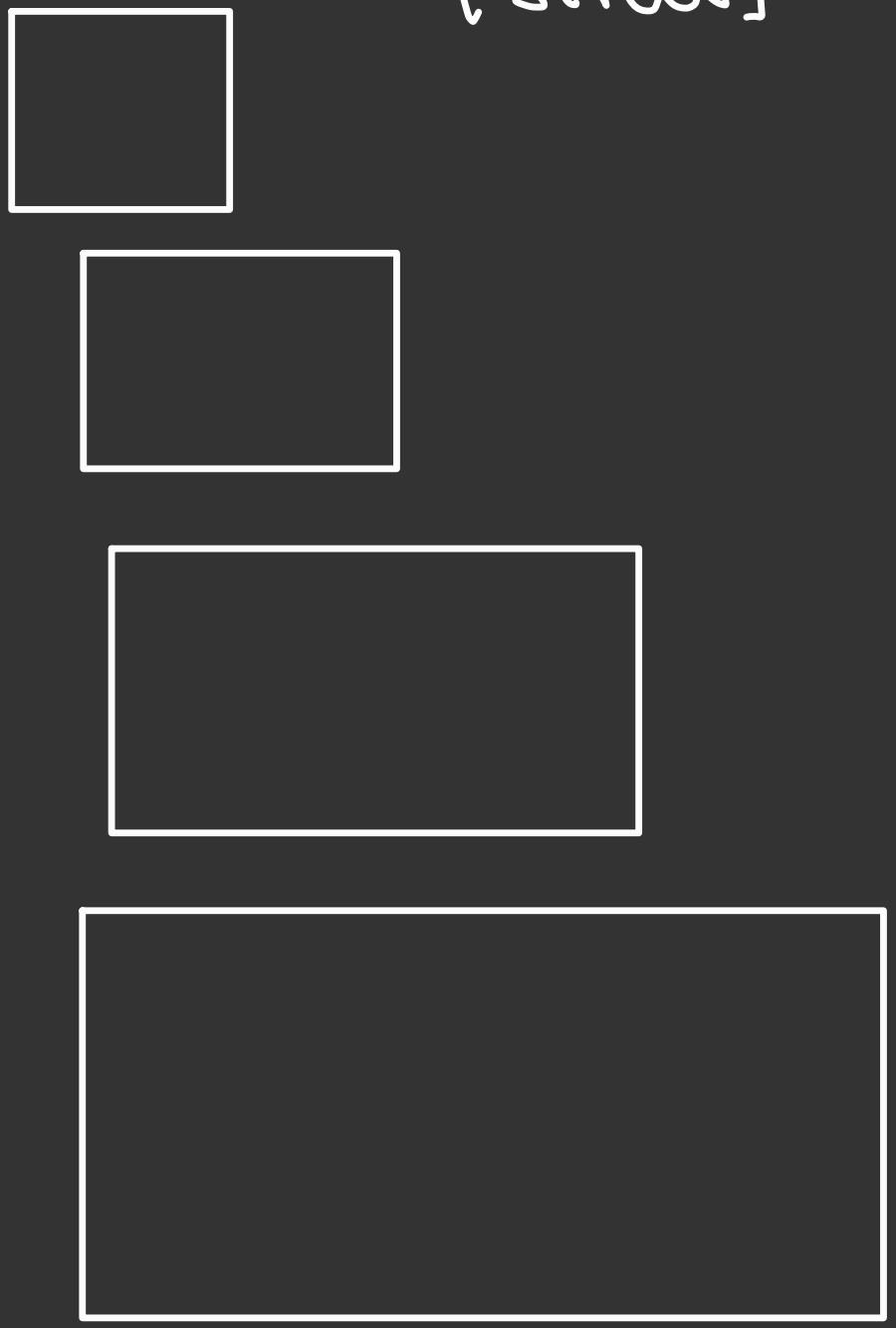
$\mathcal{D} \in \omega\mathbf{Cat}$



Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$

[Street]

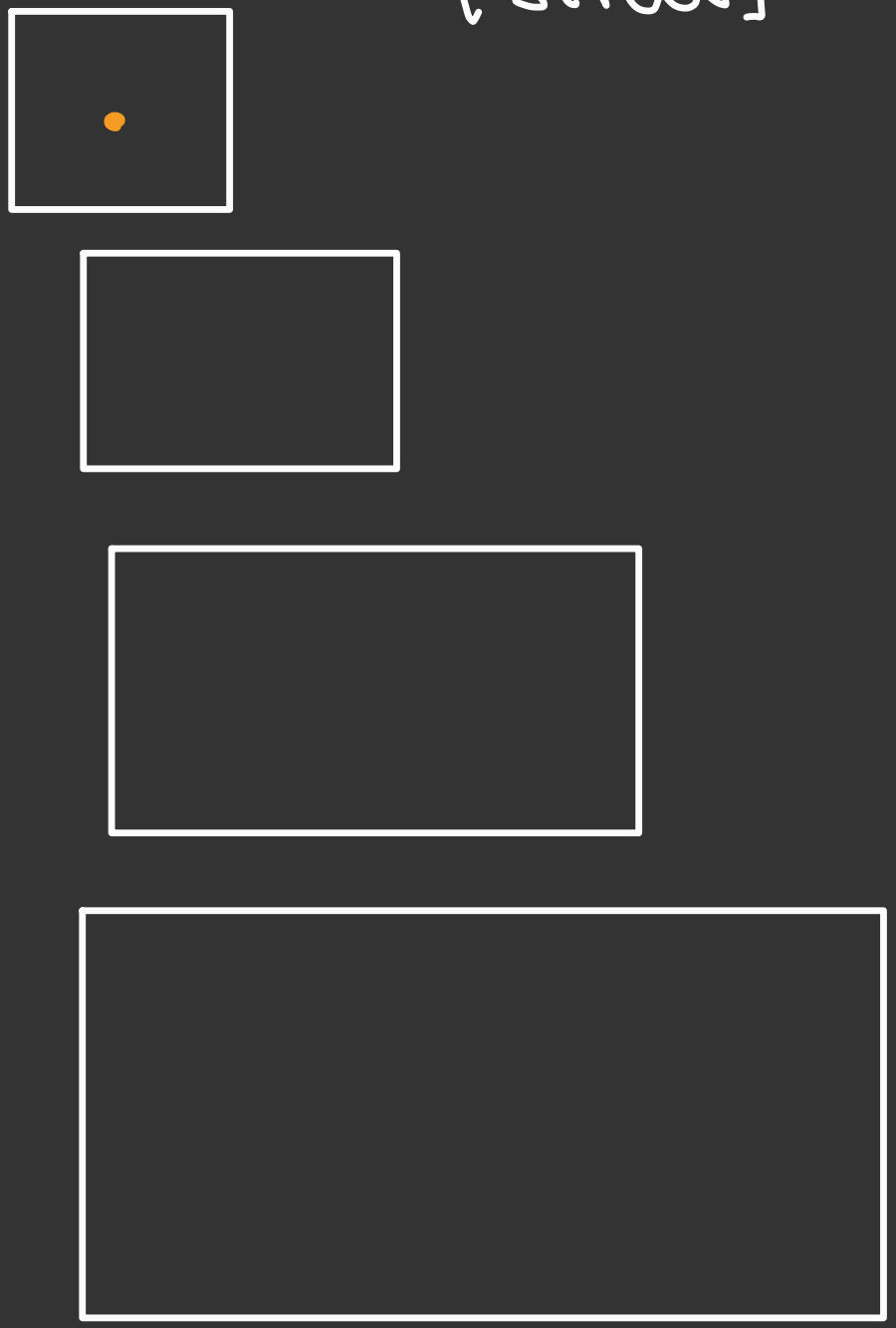


Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$

[Street]

$\mathcal{D}[0]$



Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$

[Street]

$\{ \mathcal{D}[0] \to \mathcal{D} \} \ni$ 







Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$$









Nerves of ω-categories

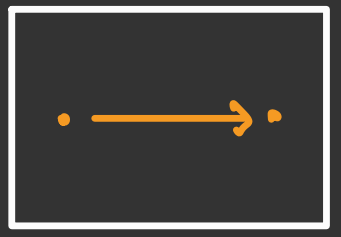
$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$

↖ [Street]

$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$



$\mathcal{D}[1]$







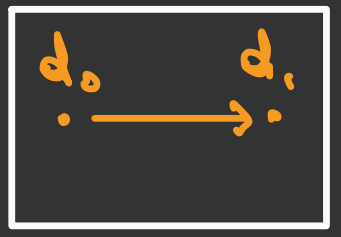
Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$



$\{ \mathcal{D}[1] \to \mathcal{D} \} \ni$







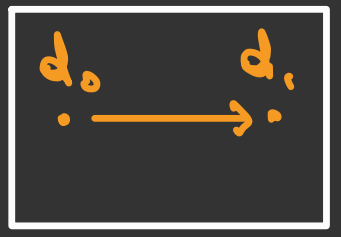
Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$$



$$N_1\mathcal{D} = \{ \mathcal{D}[1] \to \mathcal{D} \} \ni$$







Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$



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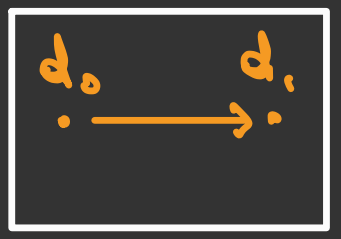
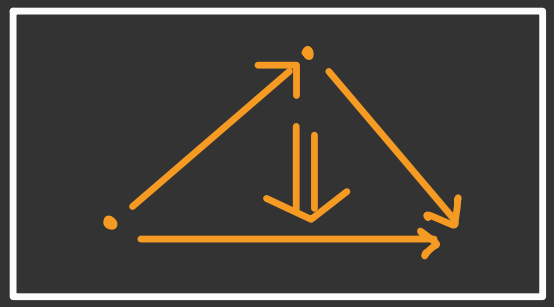


$\mathcal{D}[2]$





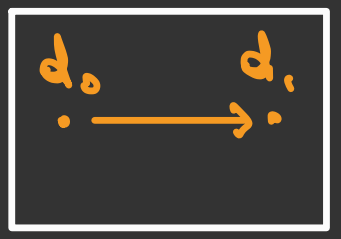
Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

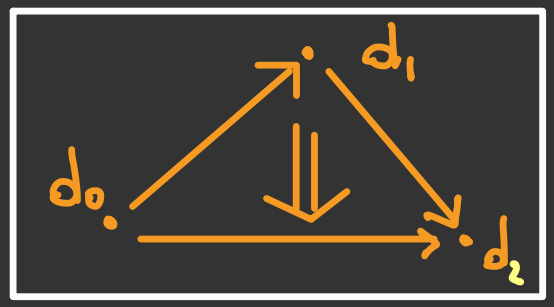
$$N_0\mathcal{D} = \{\Theta[0] \to \mathcal{D}\} \ni$$



$$N_1\mathcal{D} = \{\Theta[1] \to \mathcal{D}\} \ni$$



$$\{\Theta[2] \to \mathcal{D}\} \ni$$



Nerves of ω-categories

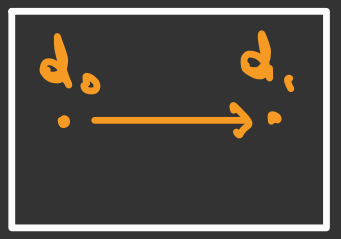
$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$

[Street]

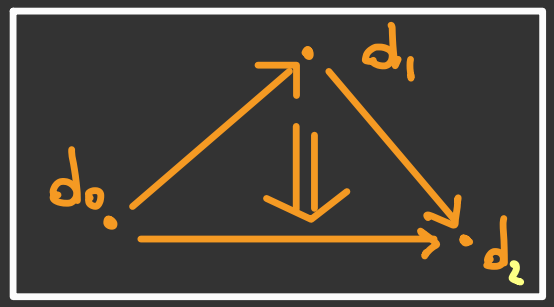
$$N_0\mathcal{D} = \{ \mathcal{O}[0] \to \mathcal{D} \} \ni$$



$$N_1\mathcal{D} = \{ \mathcal{O}[1] \to \mathcal{D} \} \ni$$



$$N_2\mathcal{D} = \{ \mathcal{O}[2] \to \mathcal{D} \} \ni$$



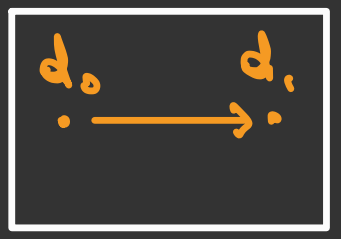
Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$$



$$N_1\mathcal{D} = \{ \mathcal{D}[1] \to \mathcal{D} \} \ni$$



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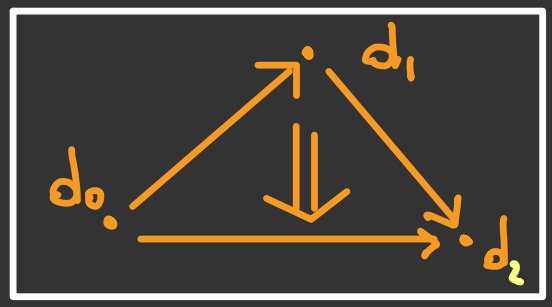
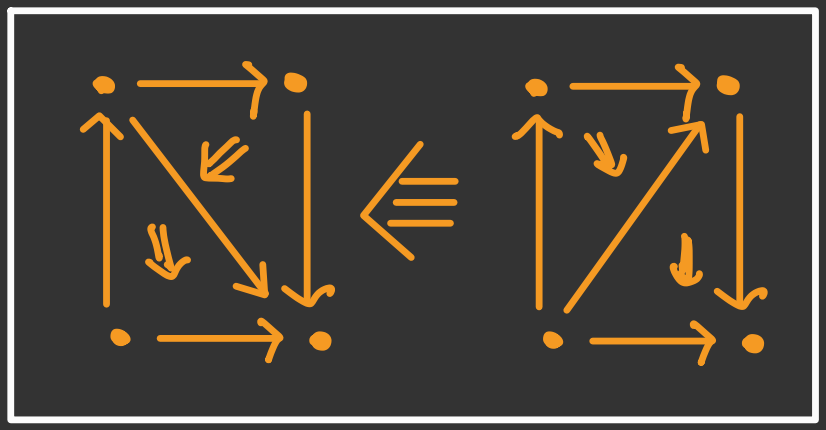


$\mathcal{D}[3]$



Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$$N_0\mathcal{D} = \{ \mathcal{O}[0] \to \mathcal{D} \} \ni$$



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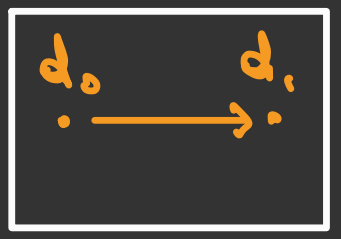
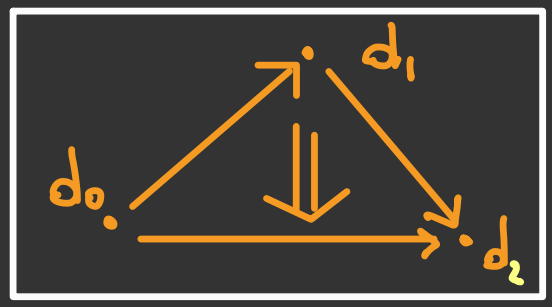
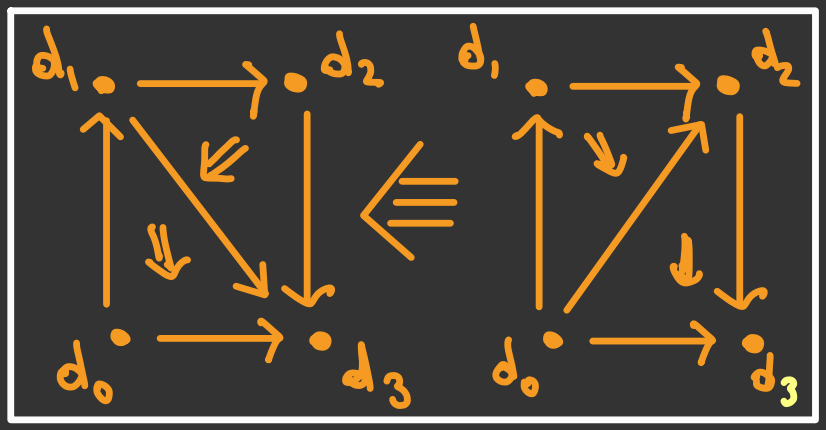


$$N_2\mathcal{D} = \{ \mathcal{O}[2] \to \mathcal{D} \} \ni$$



$$\{ \mathcal{O}[3] \to \mathcal{D} \} \ni$$



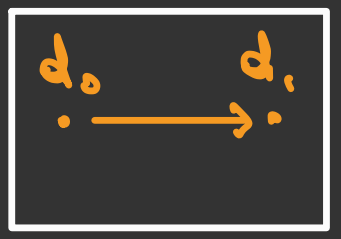
Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

$$N_0\mathcal{D} = \{ \mathcal{D}[0] \to \mathcal{D} \} \ni$$



$$N_1\mathcal{D} = \{ \mathcal{D}[1] \to \mathcal{D} \} \ni$$



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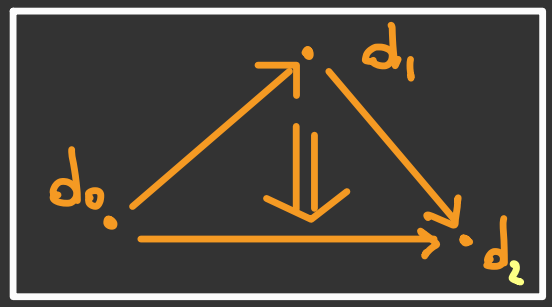
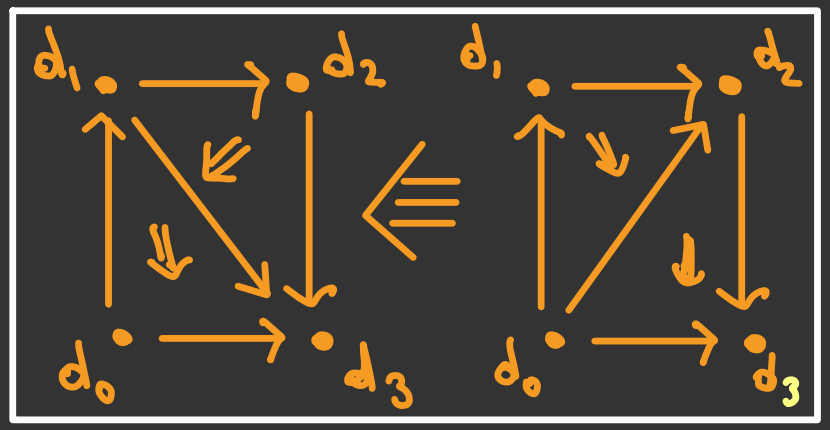


$$N_3\mathcal{D} = \{ \mathcal{D}[3] \to \mathcal{D} \} \ni$$



Nerves of ω-categories

$\mathcal{D} \in \omega\mathbf{Cat} \mapsto N\mathcal{D} \in s\mathbf{Set}$ [Street]

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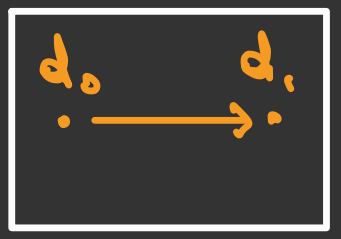

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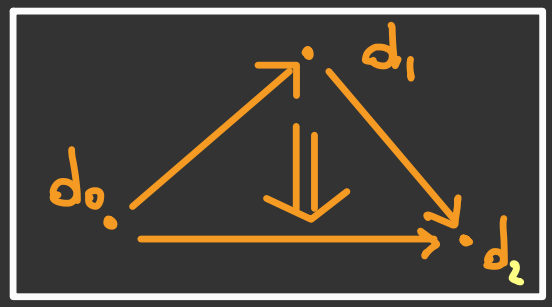
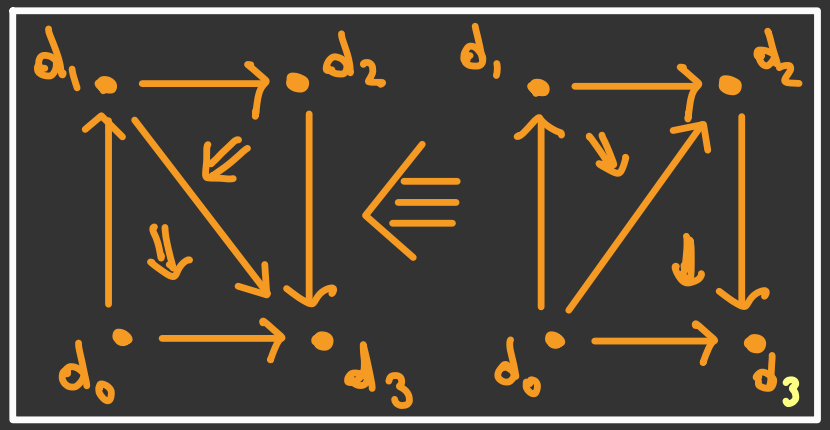
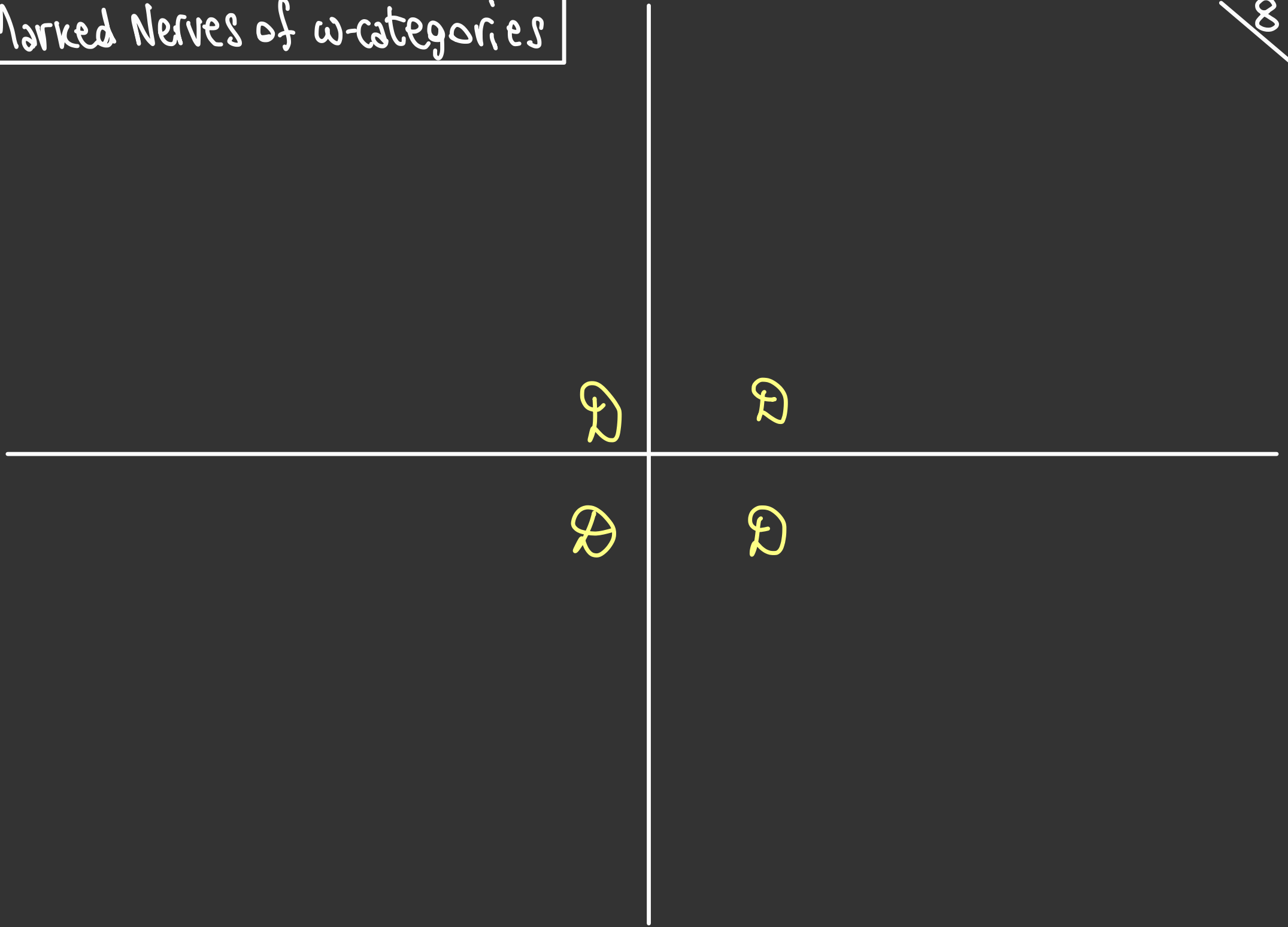


$$N_3\mathcal{D} = \{ \mathcal{D}[3] \to \mathcal{D} \} \ni$$

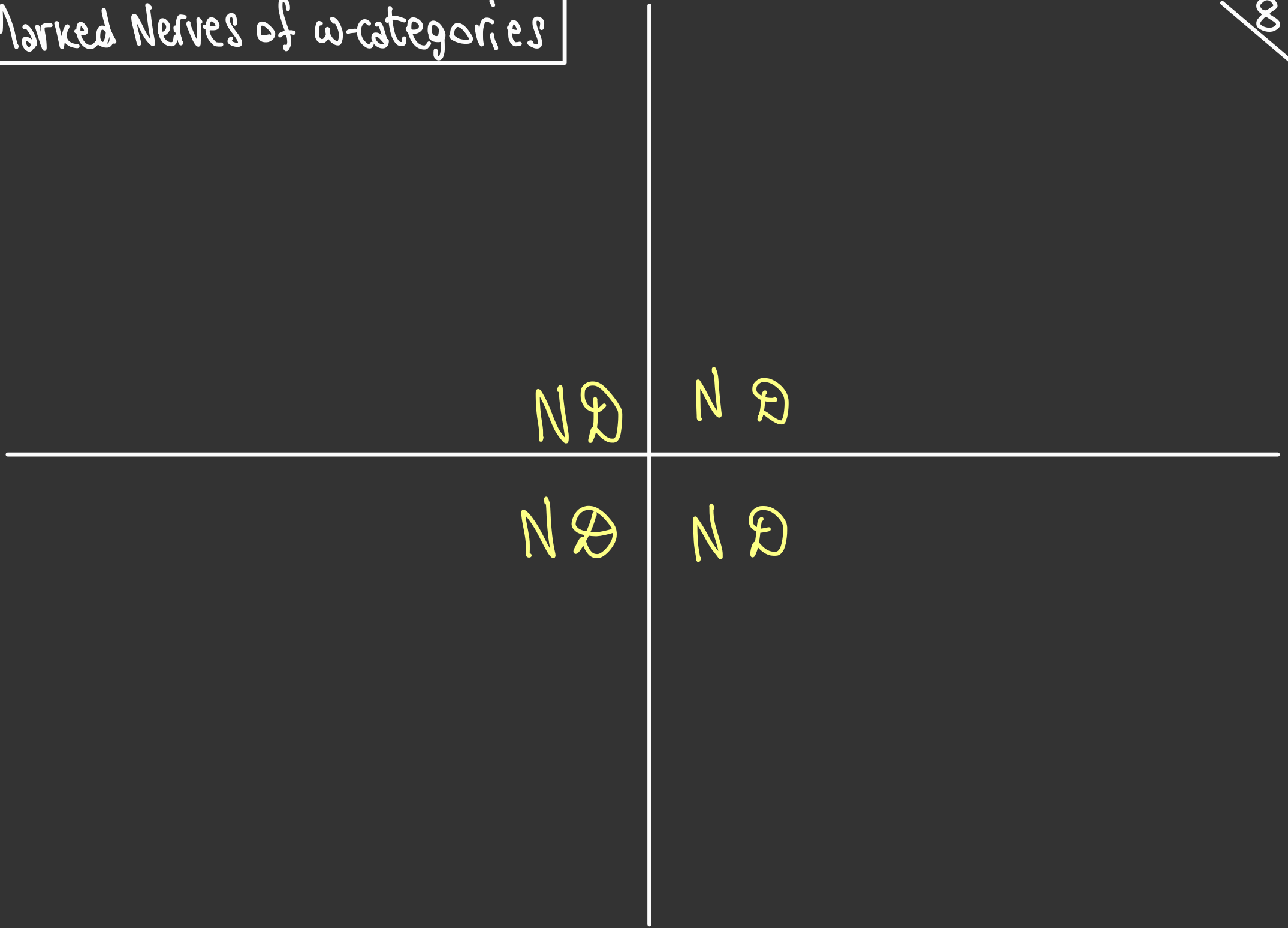
$$N_n\mathcal{D} = \{ \mathcal{D}[n] \to \mathcal{D} \}$$



Marked Nerves of ω-categories



Marked Nerves of ω-categories



Marked Nerves of ω-categories

$N^{\flat}\mathcal{D}$ $N^{RS}\mathcal{D}$

$N^{\#}\mathcal{D}$ $N^{\natural}\mathcal{D}$

Marked Nerves of ω-categories

$$N^{b}\mathcal{D} \subseteq N^{RS}\mathcal{D}$$

$$\text{in}$$

$$N^{\sharp}\mathcal{D} \supseteq N^{\flat}\mathcal{D}$$

Marked Nerves of ω-categories

Mark Nothing $\dashrightarrow$ $N^{b}\mathcal{D} \subseteq N^{RS}\mathcal{D}$

$\cap$ $\quad$ $\cap$

$N^{\#}\mathcal{D} \supseteq N^{\flat}\mathcal{D}$

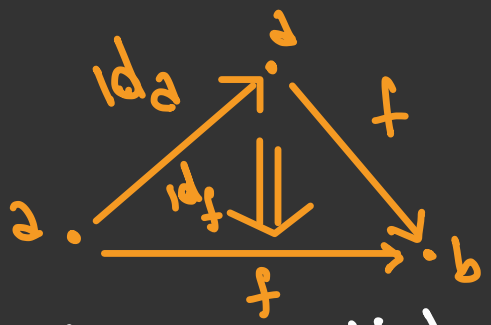
Marked Nerves of ω-categories

Mark ~~nothing~~ deg $\dashrightarrow$ $N^{b}\mathcal{D} \subseteq N^{RS}\mathcal{D}$

$\cap$ $\quad$ $\cap$

$N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$

Marked Nerves of ω-categories

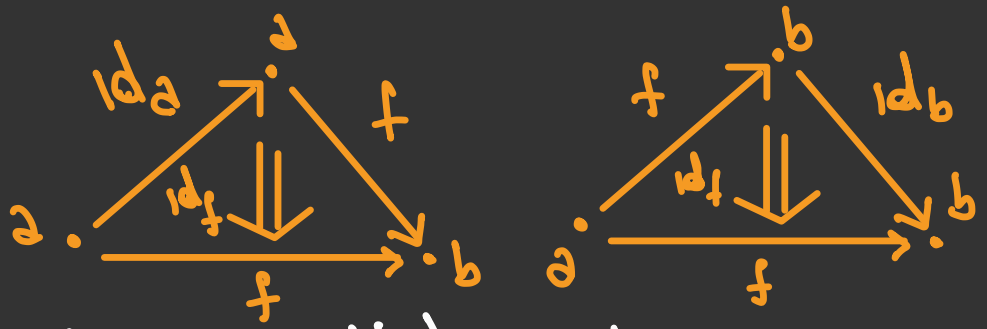


Mark ~~nothing~~ deg $\dashrightarrow$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$

in

$N^{\#}\mathcal{D} \supseteq N^{\flat}\mathcal{D}$

Marked Nerves of ω-categories

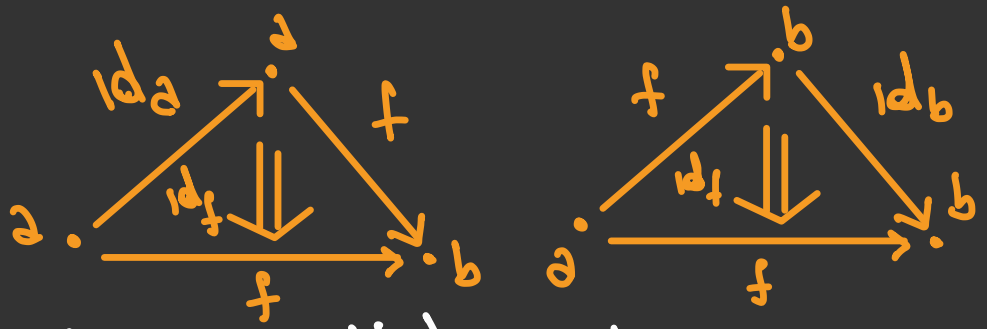


Mark ~~nothing~~ deg $\rightsquigarrow$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$

in

$N^{\#}\mathcal{D} \supseteq N^{\flat}\mathcal{D}$

Marked Nerves of ω-categories

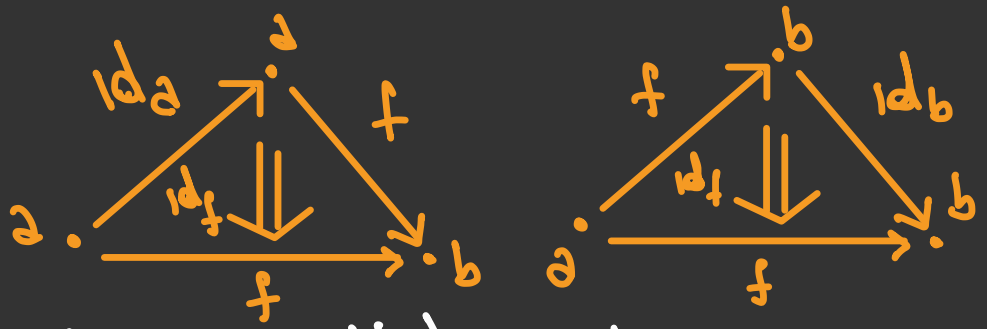


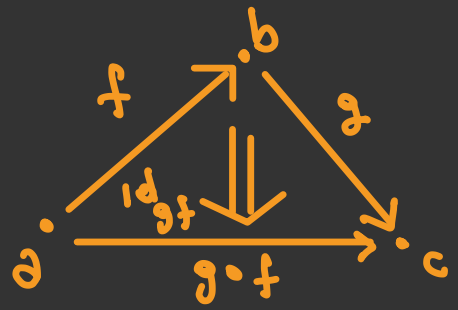
Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

in

$N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$

Marked Nerves of ω-categories





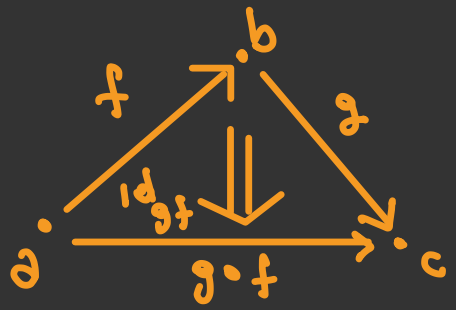
Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

$\cap$

$N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$

Marked Nerves of ω-categories



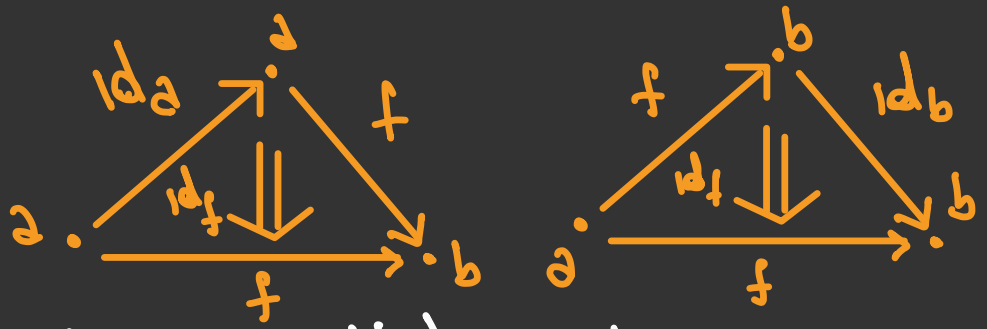


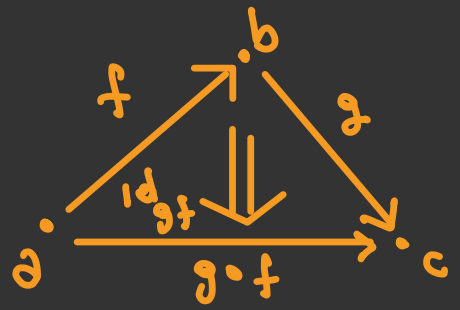
Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

$\cap\!\!|\!\!|$

$N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$ $\hookleftarrow$ Mark equivalences

Marked Nerves of ω-categories

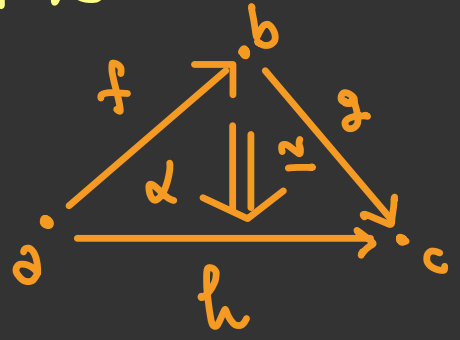




Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

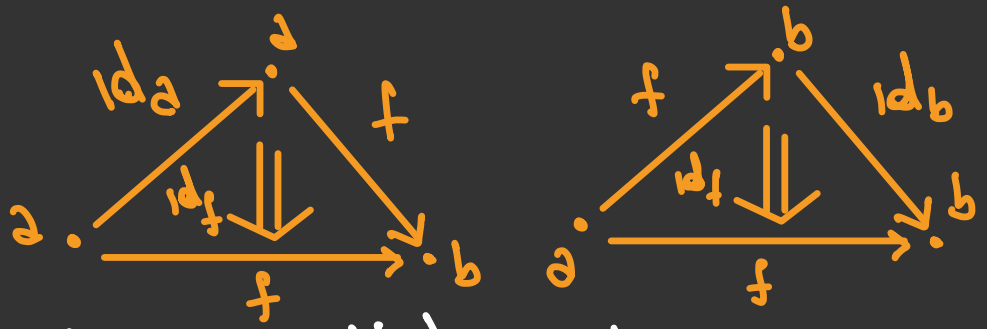
in

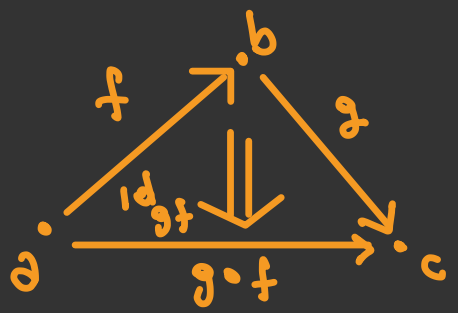
$N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$ $\hookleftarrow$ Mark equivalences



$\alpha : gf \xrightarrow{\simeq} h$

Marked Nerves of ω-categories

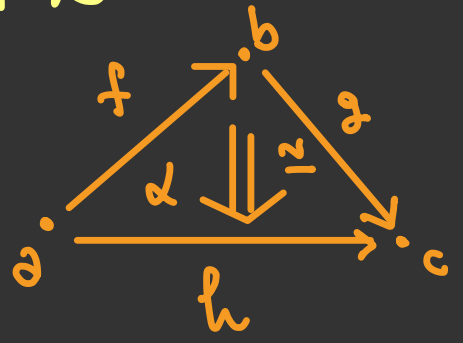




Mark ~~nothing~~ deg $\rightarrowtail N^{b}\mathcal{D} \subseteq N^{RS}\mathcal{D} \hookleftarrow$ Mark identities

in

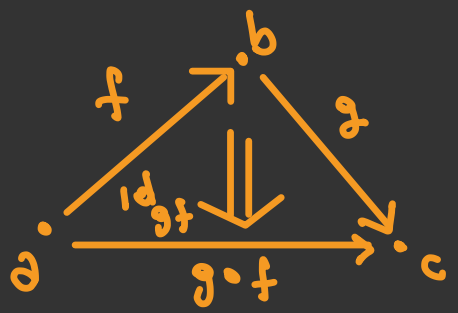
Mark all $\rightarrowtail N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D} \hookleftarrow$ Mark equivalences



$\alpha : gf \xrightarrow{\simeq} h$

Marked Nerves of ω-categories



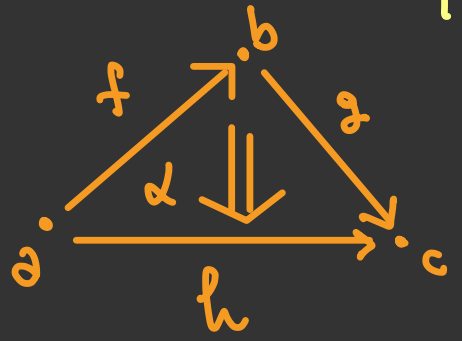


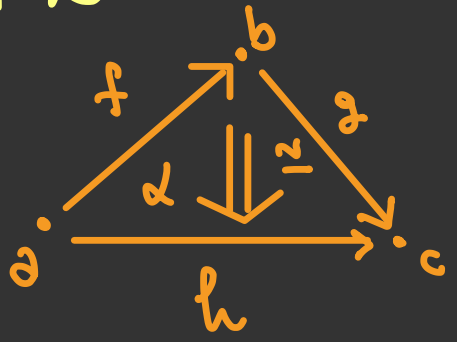
Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

in

Mark all $\rightarrowtail$ $N^{\#}\mathcal{D} \supseteq N^{\eta}\mathcal{D}$ $\hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$

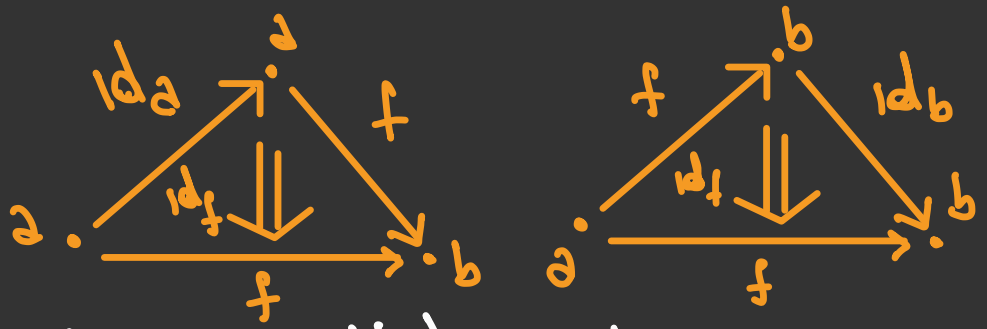


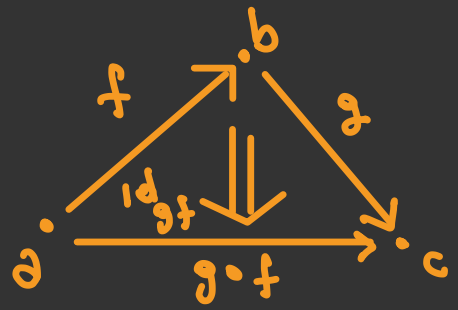


$\alpha : gf \xRightarrow{\simeq} h$

Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial





Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\leftarrowtail$ Mark identities

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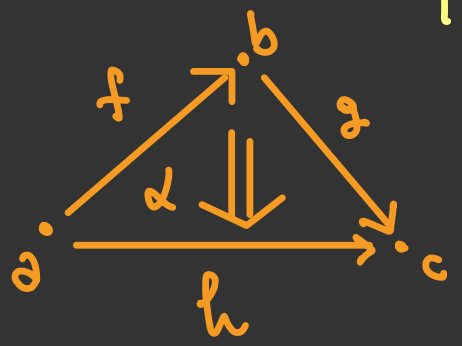
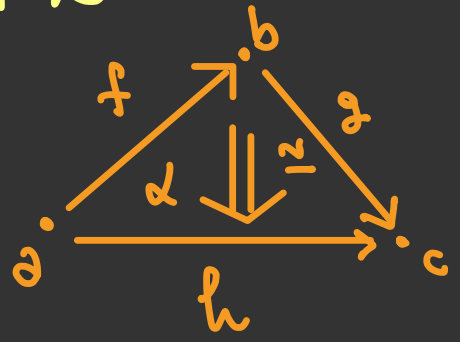
Mark all $\rightarrowtail$ $N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$ $\leftarrowtail$ Mark equivalences

$\alpha : gf \Rightarrow h$

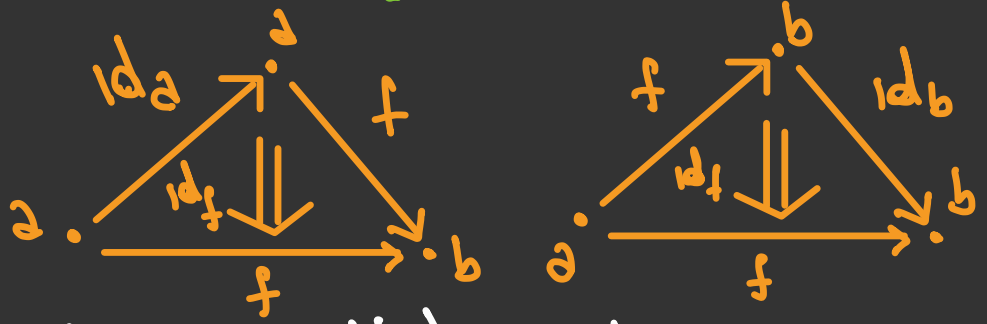


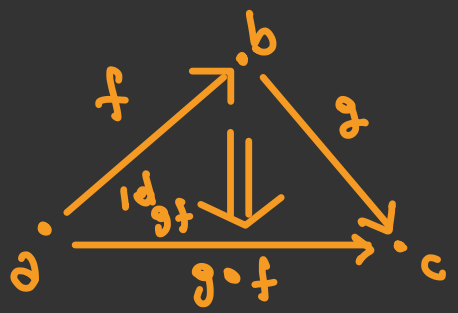


$\alpha : gf \xRightarrow{\simeq} h$

Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]





Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

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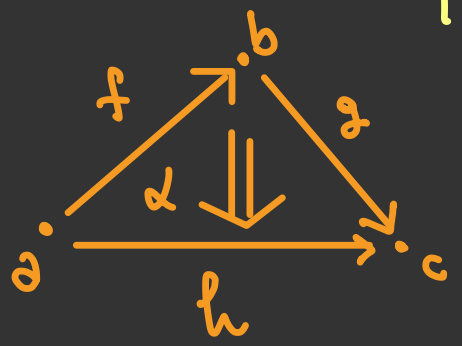
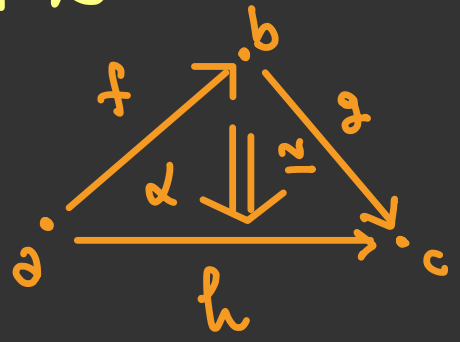
Mark all $\rightarrowtail$ $N^{\#}\mathcal{D} \supseteq N^{\eta}\mathcal{D}$ $\hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$

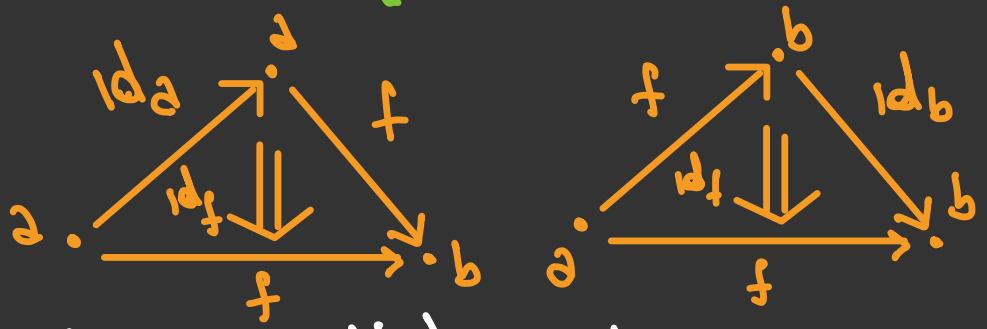




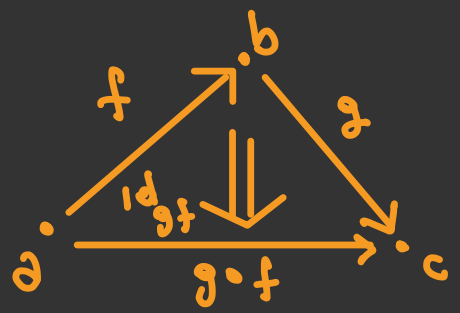
$\alpha : gf \xRightarrow{\simeq} h$

Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]



Thm [Verity] $N^{RS}\mathcal{D}$ is complicial if $\mathcal{D}$ has no equivalences



Mark ~~nothing~~ deg $\rightarrowtail N^b\mathcal{D} \subseteq N^{RS}\mathcal{D} \hookleftarrow$ Mark identities

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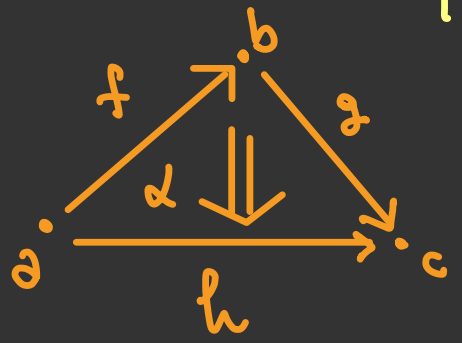
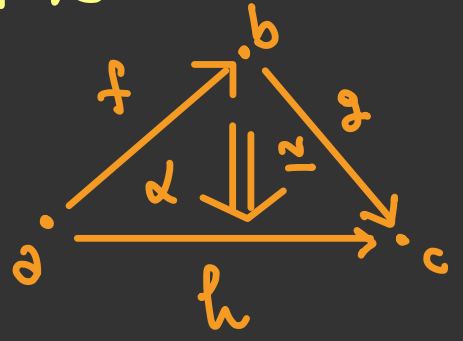
Mark all $\rightarrowtail N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D} \hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$

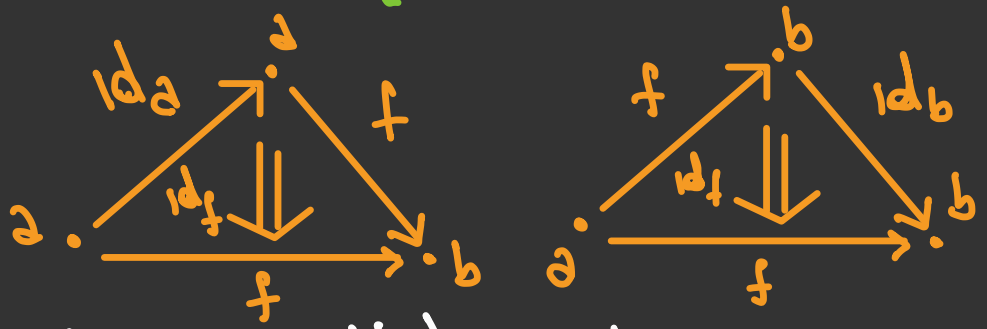




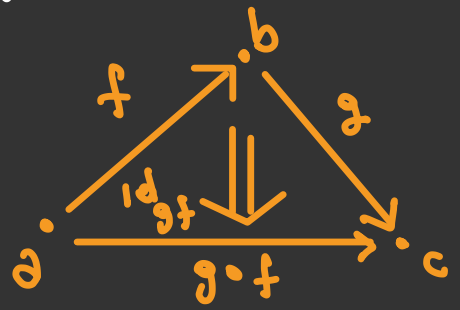
$\alpha : gf \xRightarrow{\simeq} h$

Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]



Thm [Verity] $N^{RS}\mathcal{D}$ is complicial if $\mathcal{D}$ has no non-identity equivalences



Mark ~~nothing~~ deg $\rightarrowtail$ $N^b\mathcal{D} \subseteq N^{RS}\mathcal{D}$ $\hookleftarrow$ Mark identities

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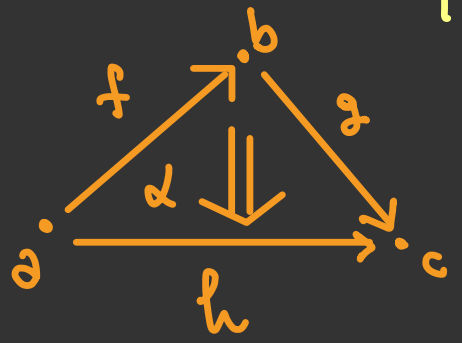
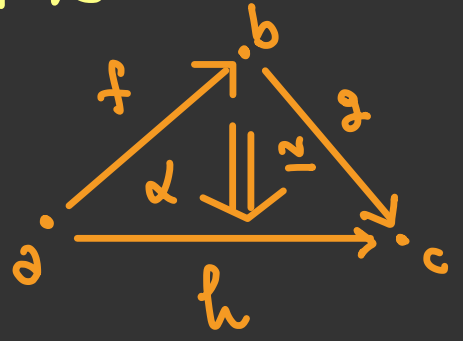
Mark all $\rightarrowtail$ $N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D}$ $\hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$

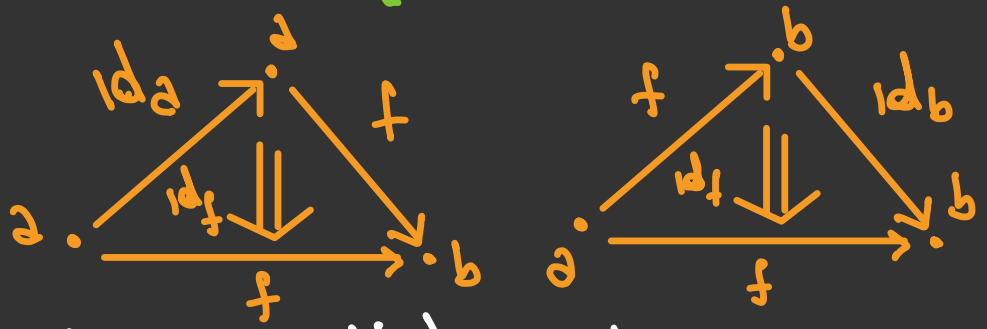




$\alpha : gf \xRightarrow{\simeq} h$

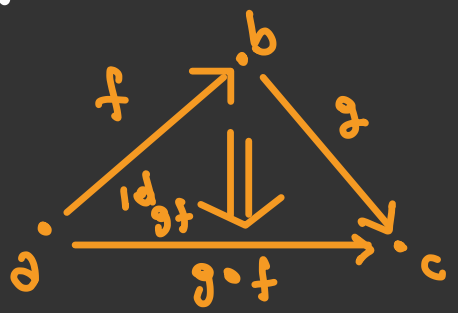
Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]



Thm [Verity] $N^{RS}\mathcal{D}$ is complicial if $\mathcal{D}$ has no non-identity equivalences

[otherwise not saturated]

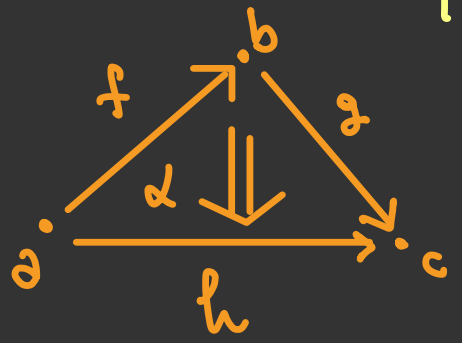


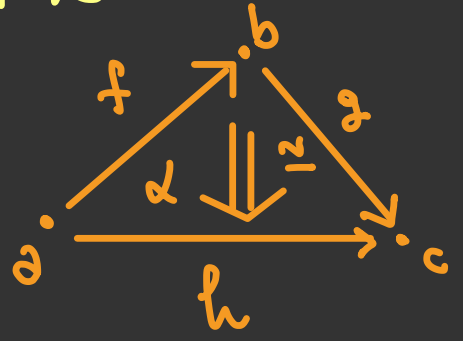
Mark ~~nothing~~ deg $\rightarrowtail N^b\mathcal{D} \subseteq N^{RS}\mathcal{D} \hookleftarrow$ Mark identities

in

Mark all $\rightarrowtail N^{\#}\mathcal{D} \supseteq N^{\simeq}\mathcal{D} \hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$

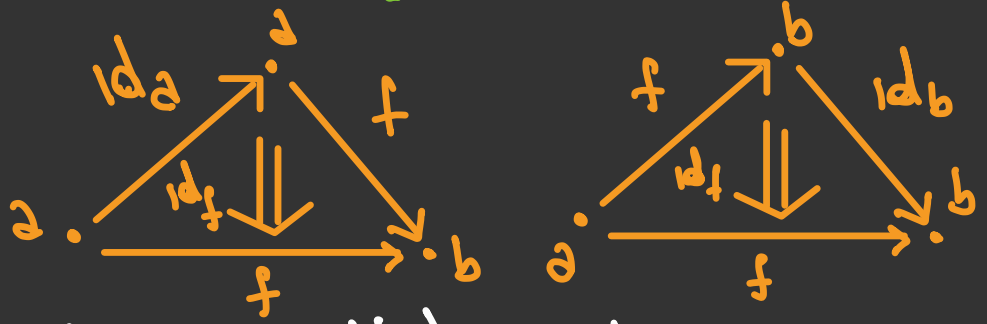




$\alpha : gf \xRightarrow{\simeq} h$

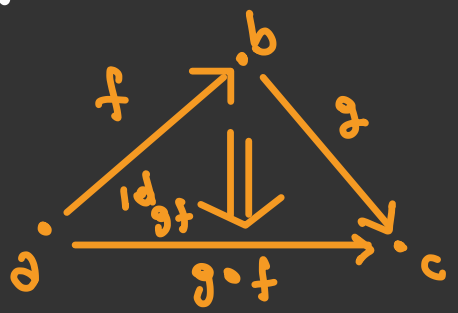
Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]



Thm [Verity] $N^{RS}\mathcal{D}$ is complicial if $\mathcal{D}$ has no non-identity equivalences

[Otherwise not saturated]

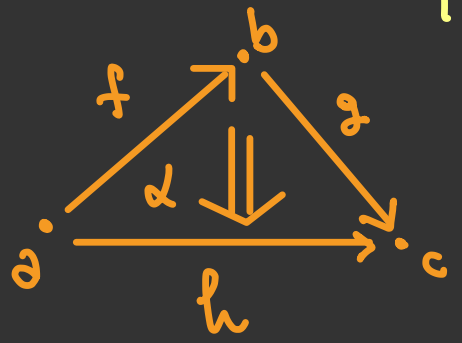


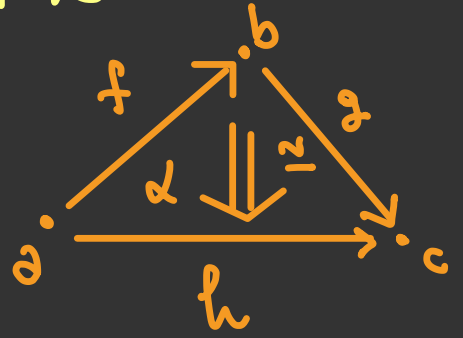
Mark ~~nothing~~ deg $\rightarrowtail N^b\mathcal{D} \subseteq N^{RS}\mathcal{D} \hookleftarrow$ Mark identities

in

Mark all $\rightarrowtail N^{\#}\mathcal{D} \supseteq N^{\flat}\mathcal{D} \hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$





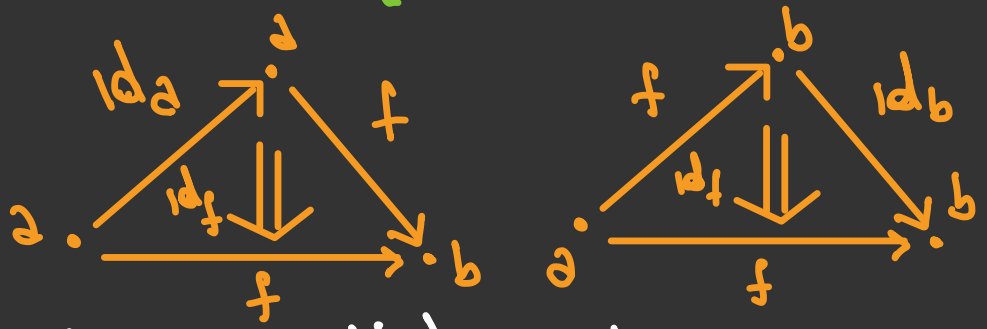
$\alpha : gf \xRightarrow{\simeq} h$

Thm [Henry-Loubaton]

$N^{\flat}\mathcal{D}$ is complicial

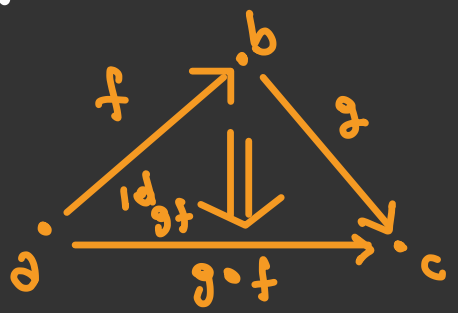
Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]



Thm [Verity] $N^{RS}\mathcal{D}$ is complicial if $\mathcal{D}$ has no non-identity equivalences

[Otherwise not saturated]



Mark ~~nothing~~ deg $\rightarrowtail N^b\mathcal{D} \subseteq N^{RS}\mathcal{D} \hookleftarrow$ Mark identities

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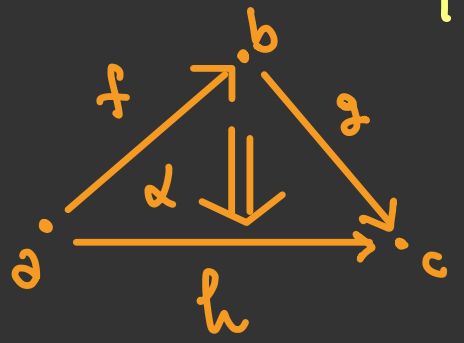
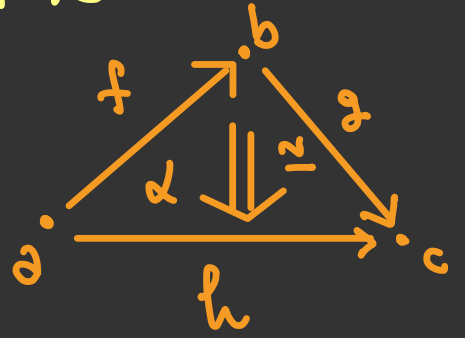
Mark all $\rightarrowtail N^{\#}\mathcal{D} \supseteq N^{\natural}\mathcal{D} \hookleftarrow$ Mark equivalences

$\alpha : gf \Rightarrow h$



$\alpha : gf \xRightarrow{\sim} h$



Thm [Joyal]

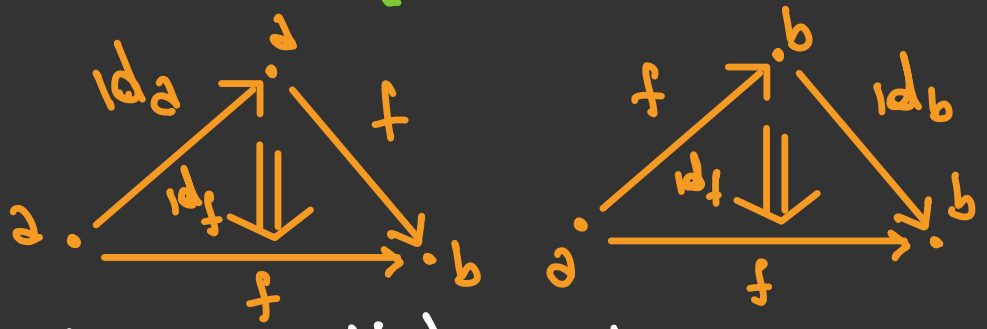
$N^{\#}\mathcal{D}$ complicial $\Leftrightarrow$ $N\mathcal{D}$ Kan cplx

Thm [Henry-Joyal]

$N^{\natural}\mathcal{D}$ is complicial

Marked Nerves of ω-categories

Note: $N^b\mathcal{D}$ is "never" complicial [lacks horn fillers]



Thm [Verity] $N^{RS}\mathcal{D}$ is complicial if $\mathcal{D}$ has no non-identity equivalences

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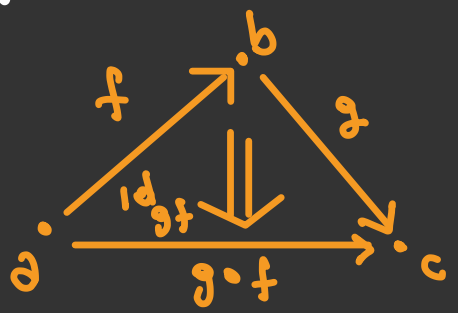

Mark ~~nothing~~ deg $\rightarrowtail N^b\mathcal{D} \subseteq N^{RS}\mathcal{D} \hookleftarrow$ Mark identities

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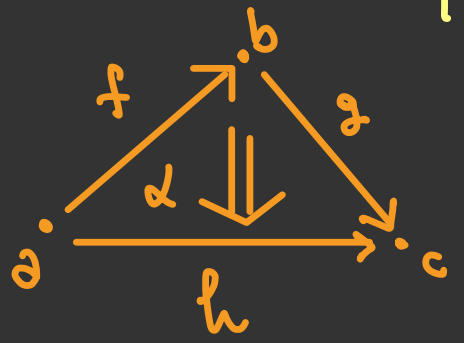
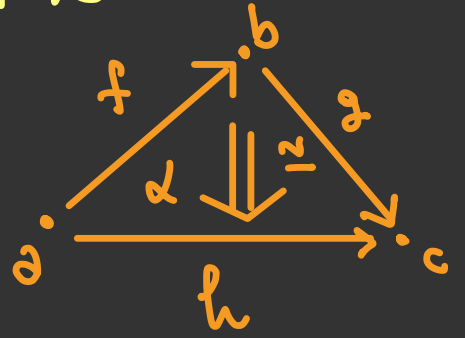


Thm [Joubaton] $\mathcal{D}$ ω-groupoid $\Downarrow$ $N^{\#}\mathcal{D}$ complicial $\Leftrightarrow$ $N\mathcal{D}$ Kan cplx



$\alpha : gf \xRightarrow{\simeq} h$

Thm [Henry-Joubaton] $N^{\natural}\mathcal{D}$ is complicial

Marked Nerves of Disks

Have :

Prop :

Have :

Prop :

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathbf{Cat}$ $\simeq$ n-disk

Have:

Prop:

Have:

Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\simeq$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have:

Prop:

Have:

Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\simeq$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have:

Prop:

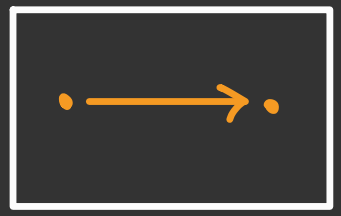
Have:

Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\simeq$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have:

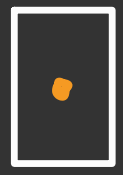
Prop:

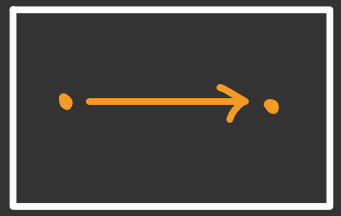
Have:

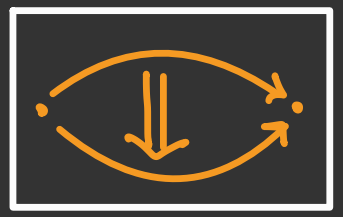
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\simeq$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have:

Prop:

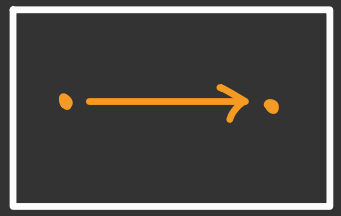
Have:

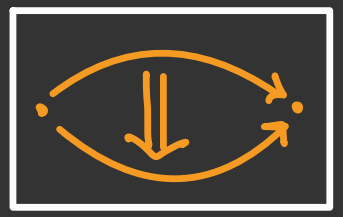
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\simeq$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have:

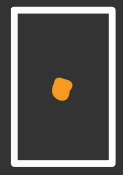
Prop:

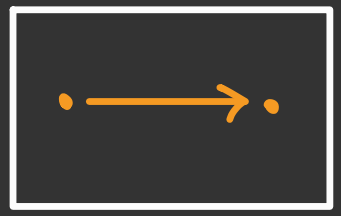
Have:

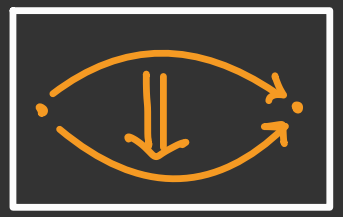
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\simeq$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b \mathcal{D}_n$ $N^{RS} \mathcal{D}_n$ $N^{\natural} \mathcal{D}_n$ $N^{\#} \mathcal{D}_n$

Prop:

Have:

Prop:

Marked Nerves of Disks

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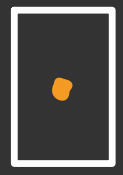
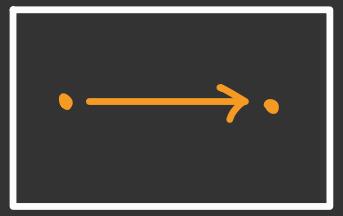
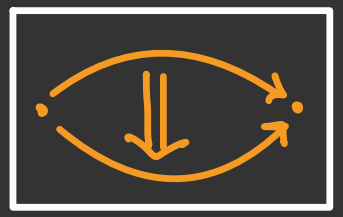
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b \mathcal{D}_n \subsetneq N^{RS} \mathcal{D}_n$ $N^{\natural} \mathcal{D}_n$ $N^{\#} \mathcal{D}_n$

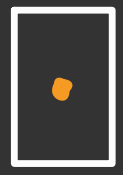
Prop:

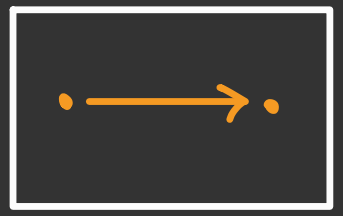
Have:

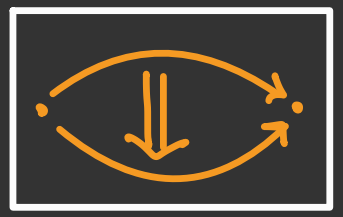
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ $\leftrightsquigarrow$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

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Marked Nerves of Disks

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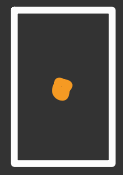
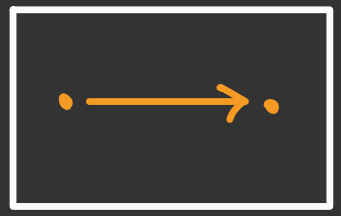
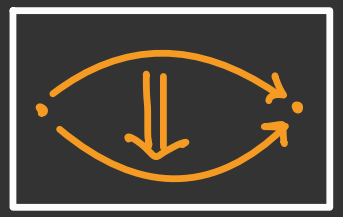
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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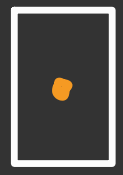
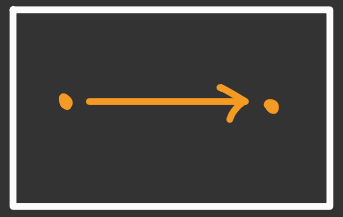
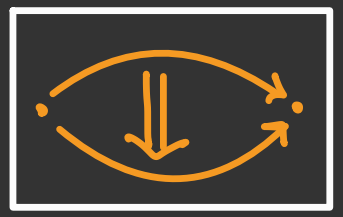
$\mathcal{D}_0 =$  $\mathcal{D}_1 =$  $\mathcal{D}_2 =$  $\mathcal{D}_3 =$ 

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complicated?

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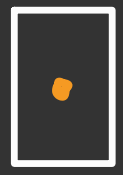
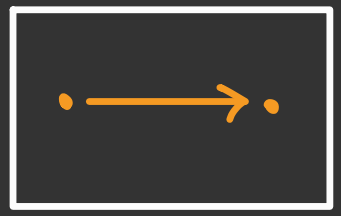
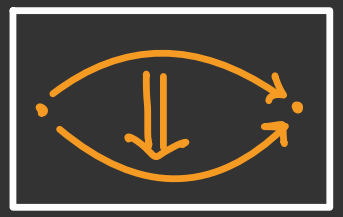
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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complicated? ×

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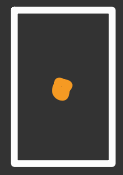
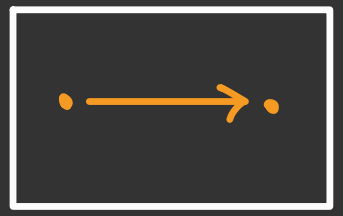
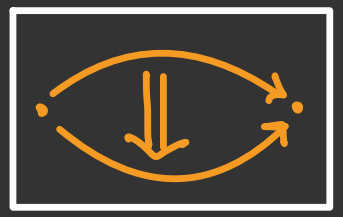
$\mathcal{D}_0 =$  $\mathcal{D}_1 =$  $\mathcal{D}_2 =$  $\mathcal{D}_3 =$ 

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complicated? ✗ ✓ ✓

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Marked Nerves of Disks

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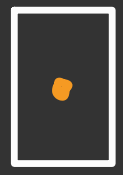
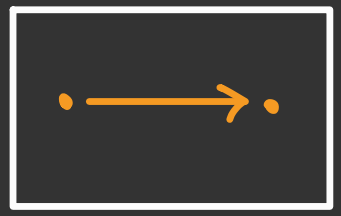
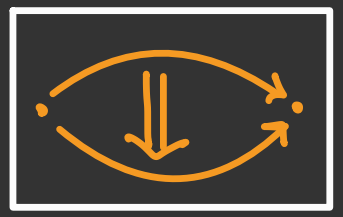
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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complicial? ✗ ✓ ✓ ✗

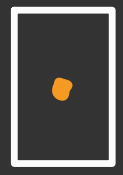
Prop:

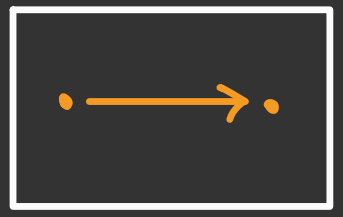
Have:

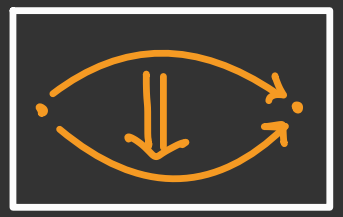
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathcal{C}at$ ← n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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complicial? ✗ ✓ ✓ ✗

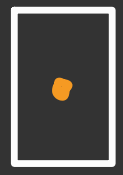
Prop: $N^{\natural}\mathcal{D}_n \in \mathcal{C}at_{(\infty,n)}$

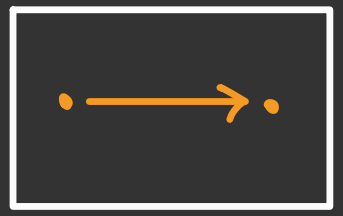
Have:

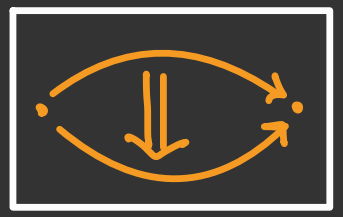
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathcal{C}at$ ← n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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complicial? ✗ ✓ ✓ ✗

Prop: $N^{\natural}\mathcal{D}_n \in \mathcal{C}at_{(\infty,n)} \setminus \mathcal{C}at_{(\infty,n-1)}$

Have:

Prop:

Marked Nerves of Disks

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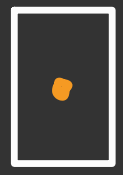
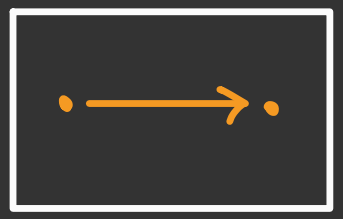
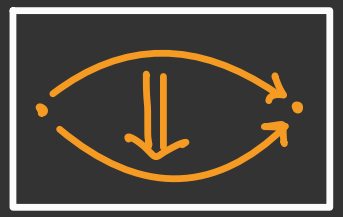
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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complicial? ✗ ✓ ✓ ✗

Prop: $N^{\natural}\mathcal{D}_n \in \mathcal{C}at_{(\infty,n)} \setminus \mathcal{C}at_{(\infty,n-1)}$

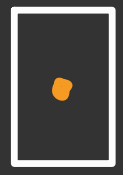
$\mathcal{D}_{cs} =$  $\in w\mathcal{C}at$

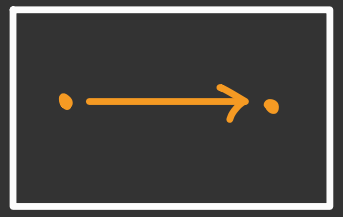
Have:

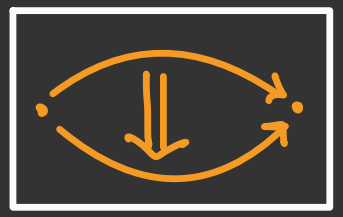
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathcal{C}at$ $\leftrightsquigarrow$ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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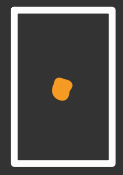
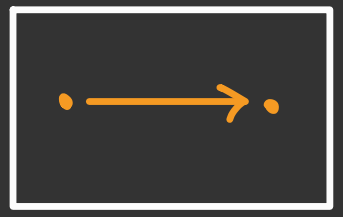
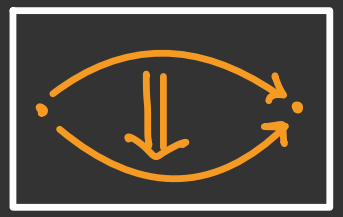
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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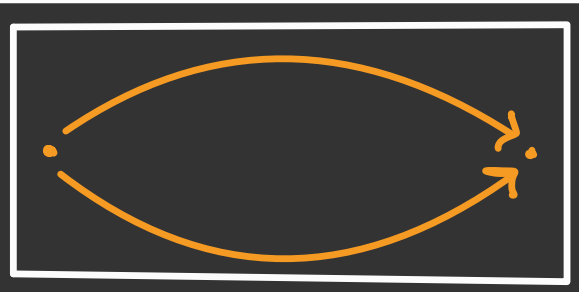
$\mathcal{D}_{cs} =$  $\in w\mathcal{C}at$

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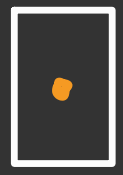
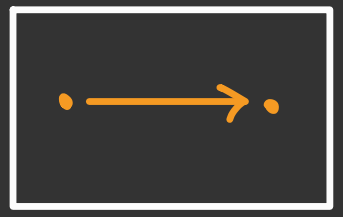
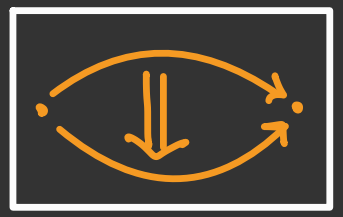
$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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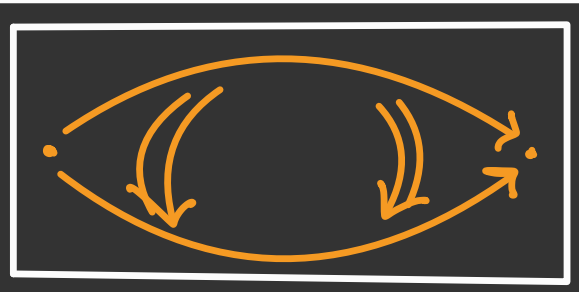
$\mathcal{D}_{cs} =$  $\in w\mathcal{C}at$

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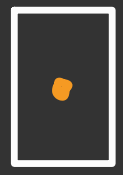
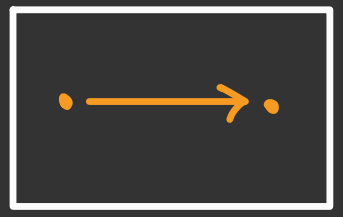
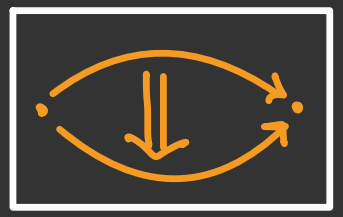
$\mathcal{D}_n \in n\mathcal{C}at$ ≃ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

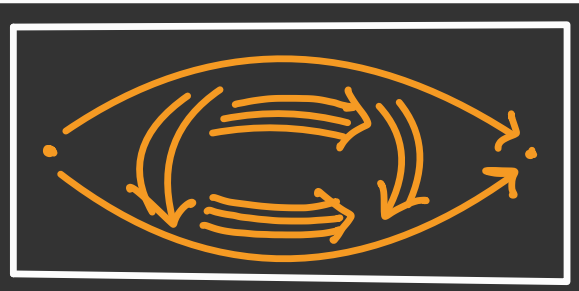
$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b\mathcal{D}_n \subsetneq N^{RS}\mathcal{D}_n = N^{\natural}\mathcal{D}_n \subsetneq N^{\#}\mathcal{D}_n$

complicial? ✗ ✓ ✓ ✗

Prop: $N^{\natural}\mathcal{D}_n \in \mathcal{C}at_{(\infty,n)} \setminus \mathcal{C}at_{(\infty,n-1)}$

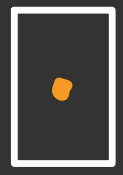
$\mathcal{D}_{ws} =$  $\in w\mathcal{C}at$

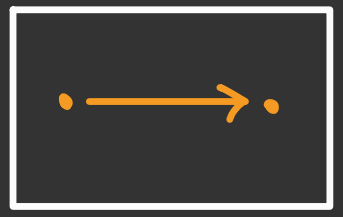
Have:

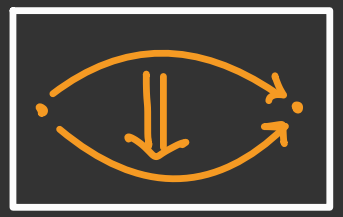
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathcal{C}at$ ← n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b\mathcal{D}_n \subsetneq N^{RS}\mathcal{D}_n = N^{\natural}\mathcal{D}_n \subsetneq N^{\#}\mathcal{D}_n$

complicial? ✗ ✓ ✓ ✗

Prop: $N^{\natural}\mathcal{D}_n \in \mathcal{C}at_{(\infty,n)} \setminus \mathcal{C}at_{(\infty,n-1)}$

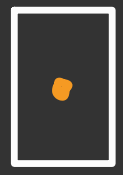
$\mathcal{D}_{\omega} =$  $\in \omega\mathcal{C}at$

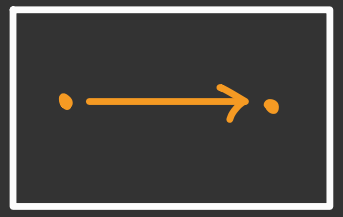
Have:

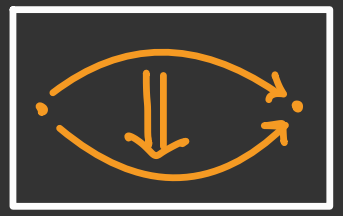
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathcal{C}at$ ≃ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

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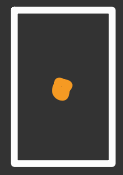
$\mathcal{D}_\omega =$  $\in \omega\mathcal{C}at$

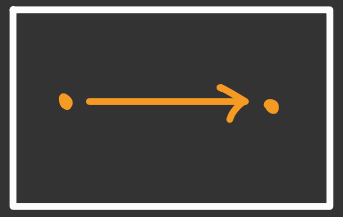
Have: $N^b\mathcal{D}_\omega \subsetneq N^{RS}\mathcal{D}_\omega = N^{\natural}\mathcal{D}_\omega \subsetneq N^{\#}\mathcal{D}_\omega$

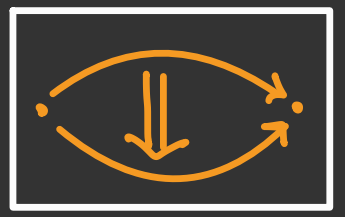
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathcal{C}at$ ≃ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b\mathcal{D}_n \subsetneq N^{RS}\mathcal{D}_n = N^{\natural}\mathcal{D}_n \subsetneq N^{\#}\mathcal{D}_n$

complicial? ✗ ✓ ✓ ✗

Prop: $N^{\natural}\mathcal{D}_n \in \mathcal{C}at_{(\infty,n)} \setminus \mathcal{C}at_{(\infty,n-1)}$

$\mathcal{D}_\omega =$  $\in \omega\mathcal{C}at$

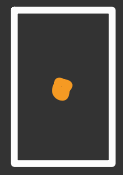
Have: $N^b\mathcal{D}_\omega \subsetneq N^{RS}\mathcal{D}_\omega = N^{\natural}\mathcal{D}_\omega \subsetneq N^{\#}\mathcal{D}_\omega$

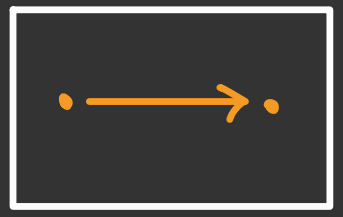
complicial? ✗ ✓ ✓ ✗

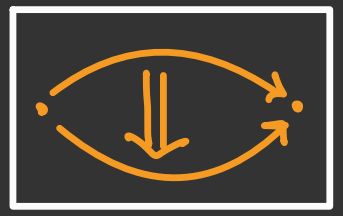
Prop:

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ ≃ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b\mathcal{D}_n \subseteq N^{RS}\mathcal{D}_n = N^{\natural}\mathcal{D}_n \subseteq N^{\#}\mathcal{D}_n$

complicial? ✗ ✓ ✓ ✗

Prop: $N^{\natural}\mathcal{D}_n \in \mathrm{Cat}_{(\infty,n)} \setminus \mathrm{Cat}_{(\infty,n-1)}$

$\mathcal{D}_\omega =$  $\in \omega\mathrm{Cat}$

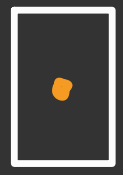
Have: $N^b\mathcal{D}_\omega \subseteq N^{RS}\mathcal{D}_\omega = N^{\natural}\mathcal{D}_\omega \subseteq N^{\#}\mathcal{D}_\omega$

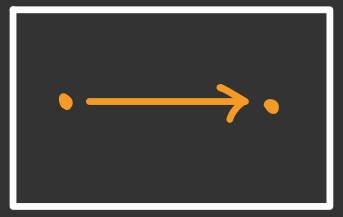
complicial? ✗ ✓ ✓ ✗

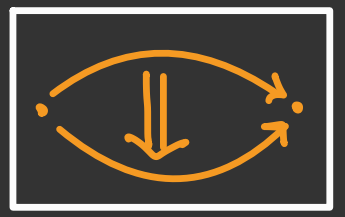
Prop: $N^{\natural}\mathcal{D}_n \in \mathrm{Cat}^{L}_{(\infty,\omega)}$

Marked Nerves of Disks

$\mathcal{D}_n \in n\mathrm{Cat}$ ≃ n-disk

$\mathcal{D}_0 =$ 

$\mathcal{D}_1 =$ 

$\mathcal{D}_2 =$ 

$\mathcal{D}_3 =$ 

Have: $N^b\mathcal{D}_n \subsetneq N^{RS}\mathcal{D}_n = N^\flat\mathcal{D}_n \subsetneq N^\#\mathcal{D}_n$

complicial? ✗ ✓ ✓ ✗

Prop: $N^\flat\mathcal{D}_n \in \mathrm{Cat}_{(\infty,n)} \setminus \mathrm{Cat}_{(\infty,n-1)}$

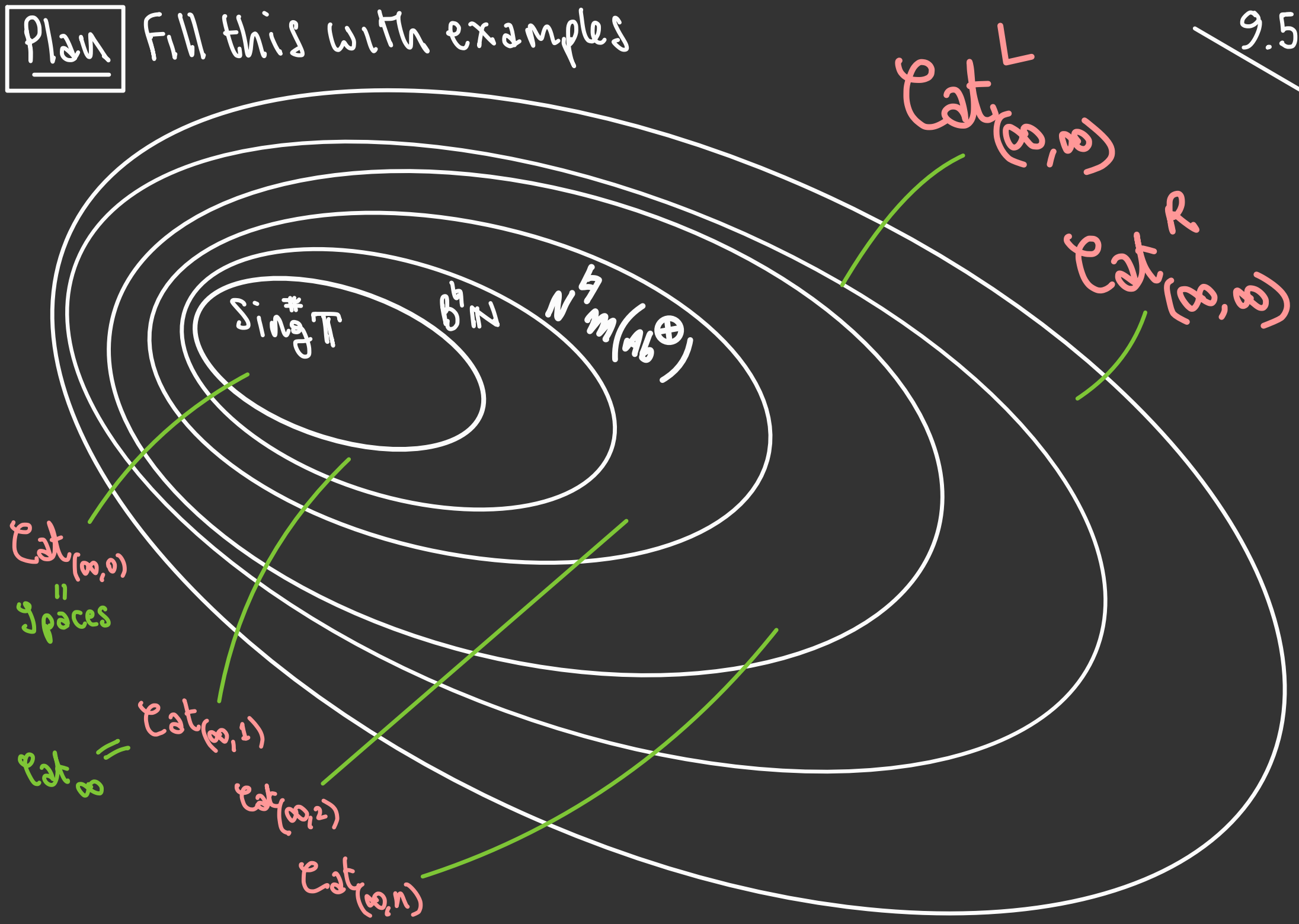
$\mathcal{D}_\omega =$  $\in \omega\mathrm{Cat}$

Have: $N^b\mathcal{D}_\omega \subsetneq N^{RS}\mathcal{D}_\omega = N^\flat\mathcal{D}_\omega \subsetneq N^\#\mathcal{D}_\omega$

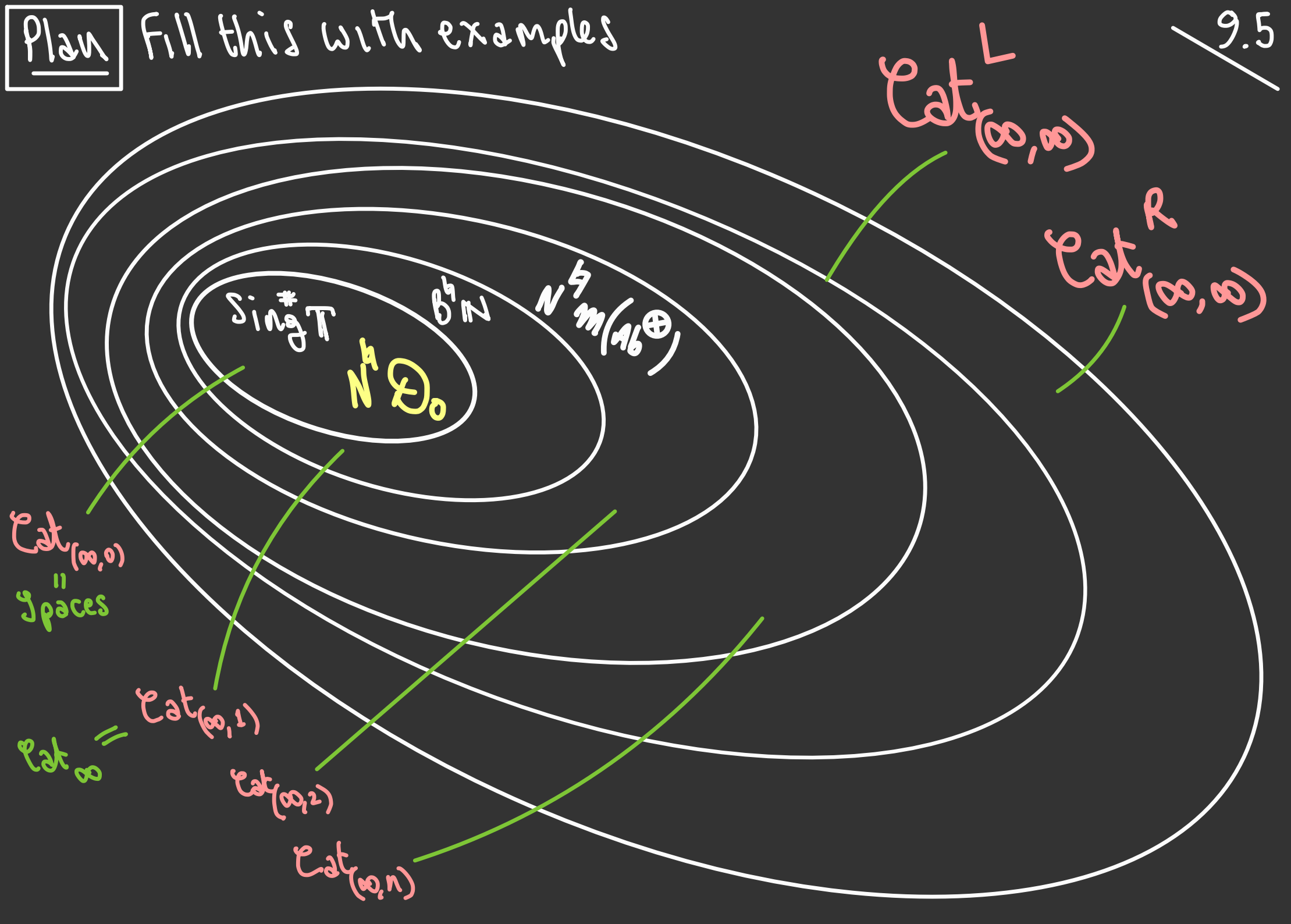
complicial? ✗ ✓ ✓ ✗

Prop: $N^\flat\mathcal{D}_n \in \mathrm{Cat}^L_{(\infty,\omega)} \setminus \mathrm{Cat}_{(\infty,n)}$

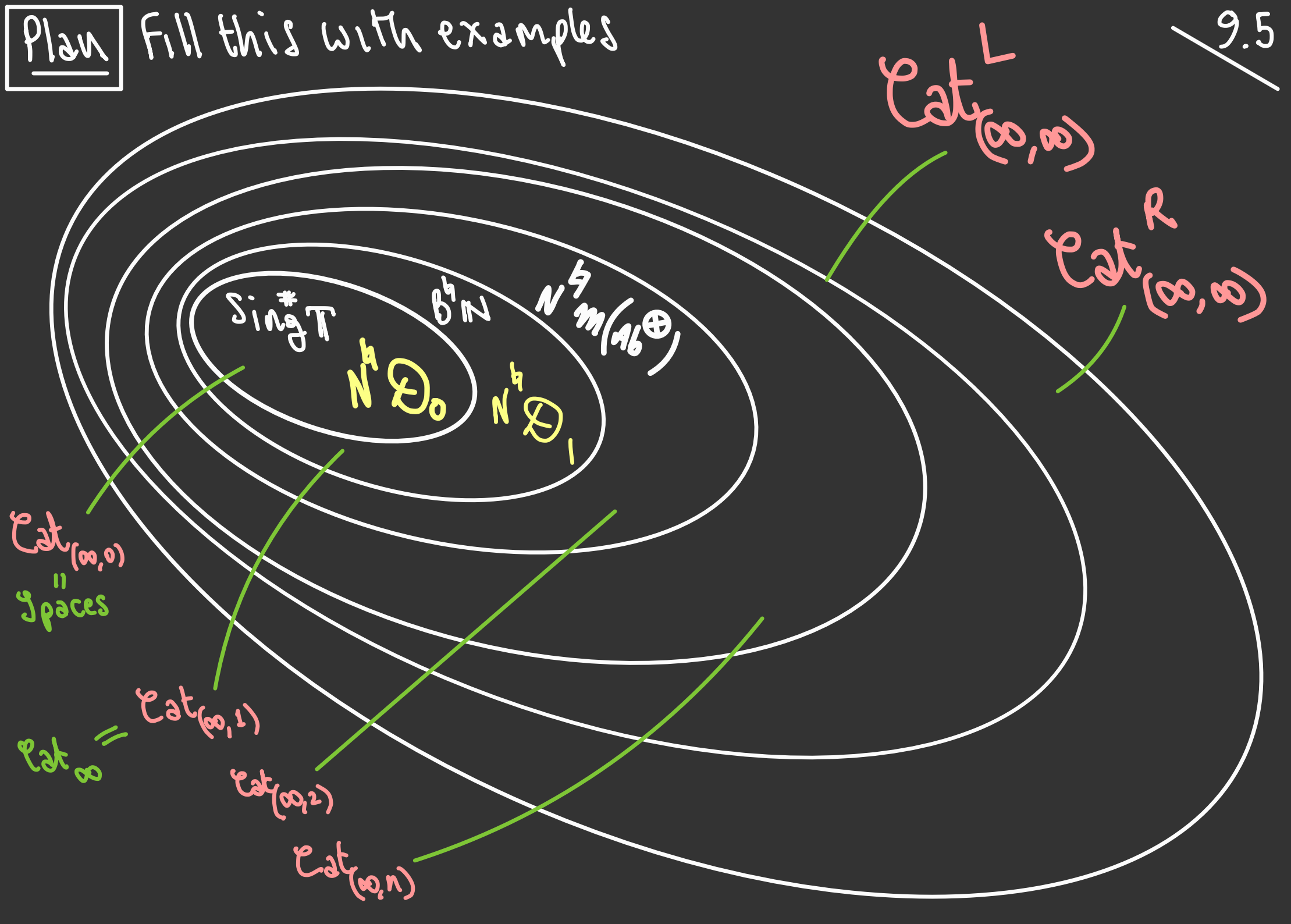
Plan Fill this with examples



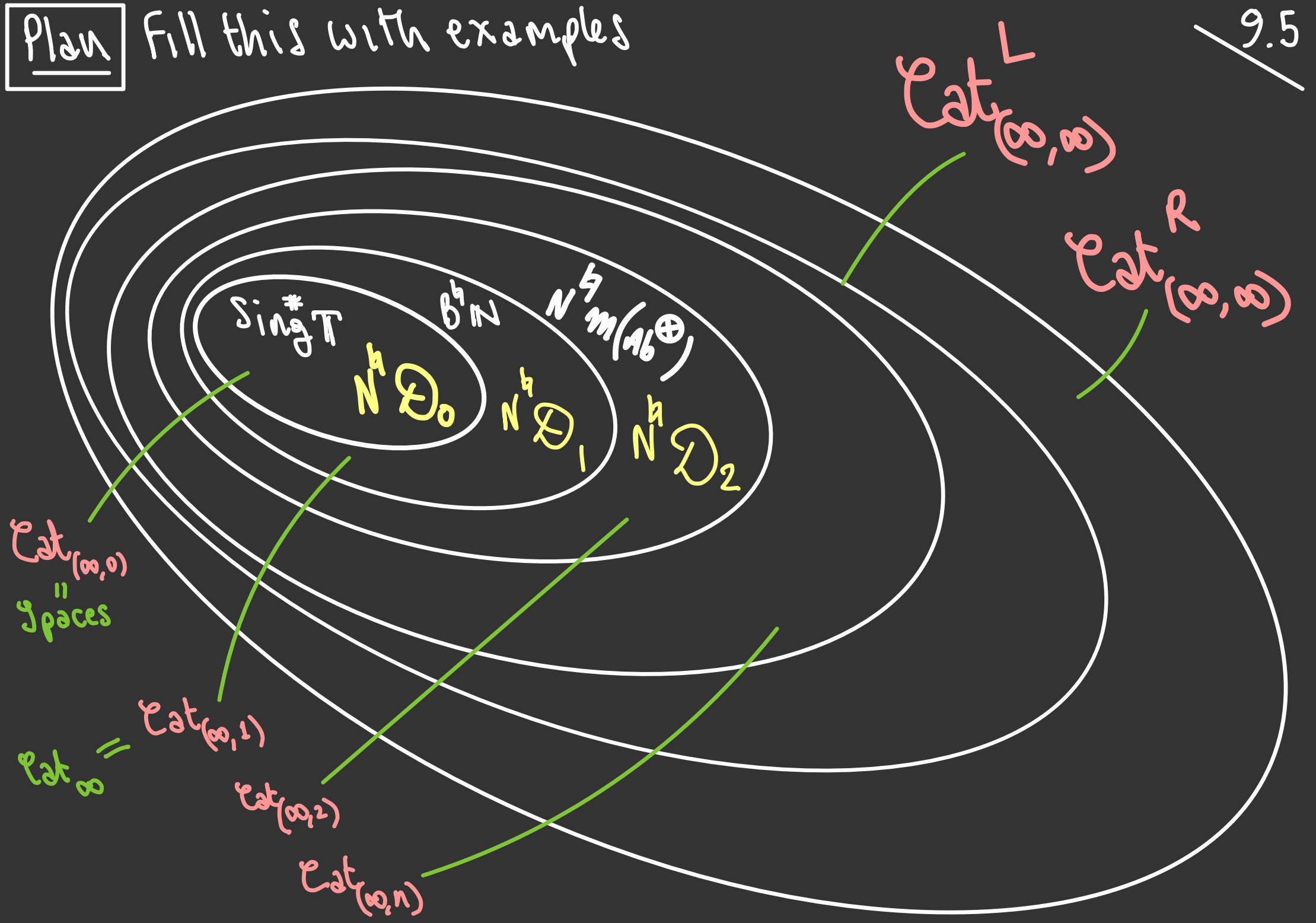
Plan Fill this with examples



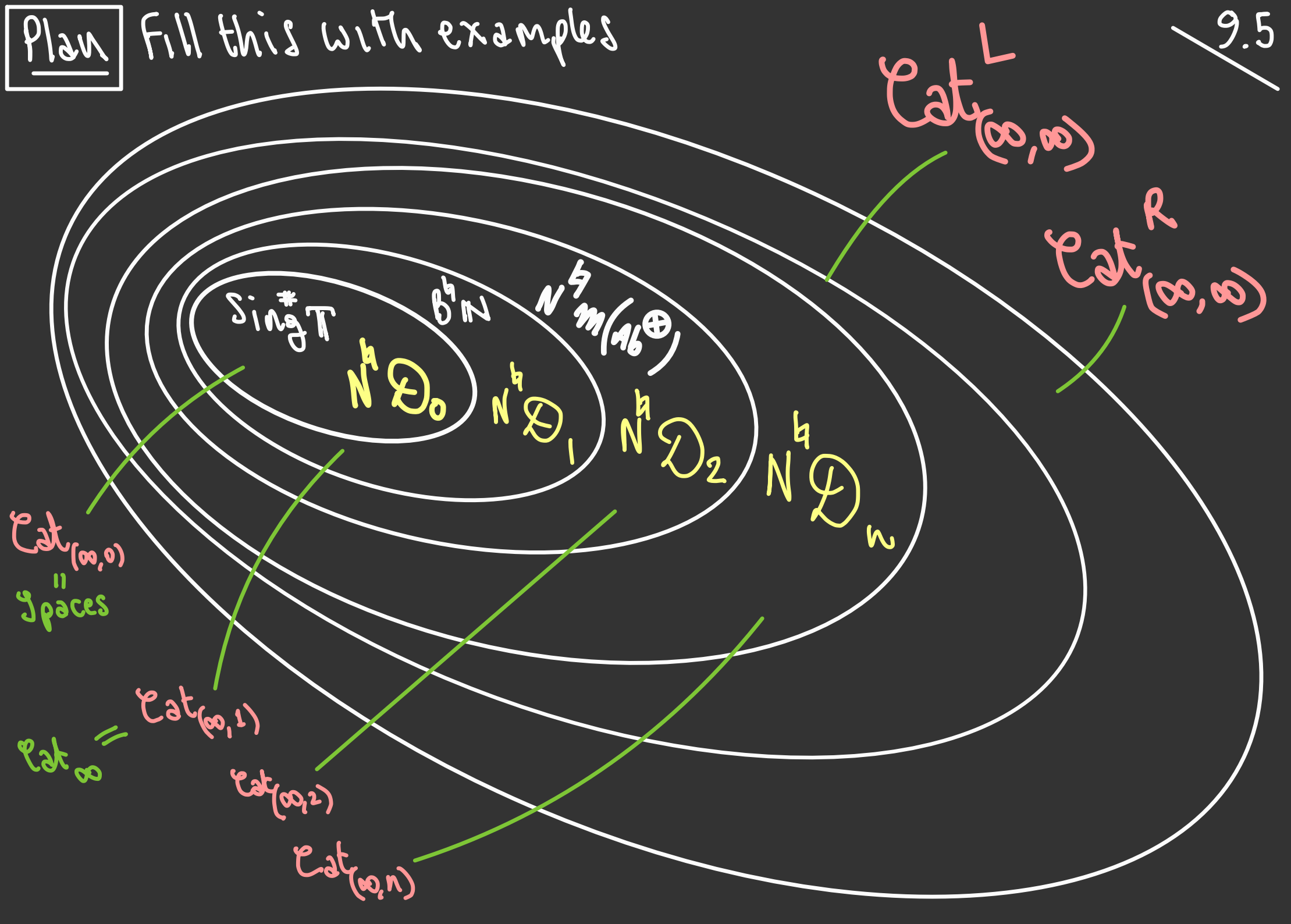
Plan Fill this with examples



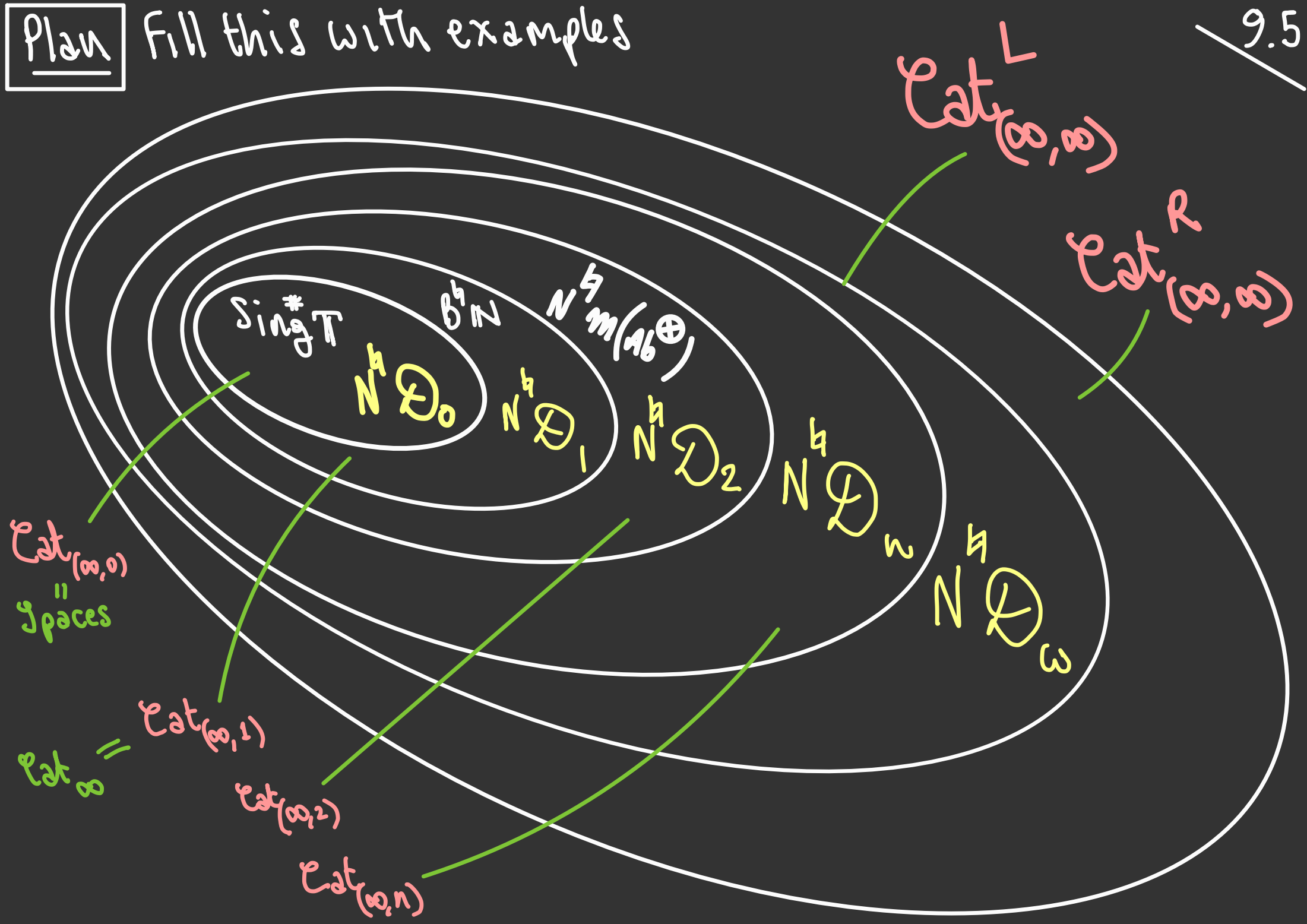
Plan Fill this with examples



Plan Fill this with examples



Plan Fill this with examples



Walking Coinductive Equivalence

Idea :

-
-

Constr

①

②

-
-

Have :

Thm :

Walking Coinductive Equivalence

Idea :

Constr [HLOR] $\mathcal{E} \in \mathcal{W}\mathcal{E}\mathcal{S}\mathcal{t}$

-
-

①

②

-
-

Have :

Thm :

Working Coinductive Equivalence

Idea :

Constr [HLOR] $\mathcal{E} \in \mathcal{W}\mathcal{E}\mathcal{S}\mathcal{t}$
with the properties

①

②

Have :

Thm :

Working Coinductive Equivalence

Idea:

Constr [HLOR] $\mathcal{E} \in \mathbb{W}\mathrm{Eat}$

with the properties

① $\forall \mathcal{D} \in \mathbb{W}\mathrm{Eat}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalence in $\mathcal{D}$ }

②

Have:

Thm:

Working Coinductive Equivalence

Idea:

Constr [HLOR] $\mathcal{E} \in \mathbb{wlst}$

with the properties

① $\forall \mathcal{D} \in \mathbb{wlst}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalence in $\mathcal{D}$ }

② In wlst folk

Have:

Thm:

Working Coinductive Equivalence

Idea:

Constr [HLOR] $\mathcal{E} \in \mathcal{WST}$

with the properties

① $\forall \mathcal{D} \in \mathcal{WST}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalence in $\mathcal{D}$ }

② In $\mathcal{WST}$ folk

$\mathcal{D}_0 \amalg \mathcal{D}_0 \qquad \mathcal{D}_0$

Have:

Thm:

Walking Coinductive Equivalence

Idea:

Constr [HLOR] $\mathcal{E} \in \mathbb{wlat}$

with the properties

① $\forall \mathcal{D} \in \mathbb{wlat}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalence in $\mathcal{D}$ }

② In $\mathbb{wlat}$ folk
$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\cong} \mathcal{D}_0$

Have:

Thm:

Working Coinductive Equivalence

Idea: Build $\mathcal{E}$ as follows \ 10

Constr [HLOR] $\mathcal{E} \in \mathbf{wSet}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wSet}$
$\{\mathcal{E} \to \mathcal{D}\} \cong \{\text{coinductive equivalences in } \mathcal{D}\}$

② In $\mathbf{wSet}$ folk
$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Have:

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wSet}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wSet}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wSet}$ folk
$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows
- start with $f : a \to b$
-
-
-
-

Have:

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wlat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wlat}$ $\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wlat}$ folk

$$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\cong} \mathcal{D}_0$$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
- freely add
  - $f^- : b \to a$
  - $f^+ : b \to a$
- –
- –

10

Have:

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$ with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$ $\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wCat}$ folk

$$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
- freely add

$$\begin{cases} f^- : b \to a + \eta(f) : \mathrm{id}_a \to f^- f \\ f^+ : b \to a + \varepsilon(f) : f f^+ \to \mathrm{id}_b \end{cases}$$

-

-

Have:

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$ with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wCat}$ folk
$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows
- start with $f : a \to b$
- freely add
  - $f^- : b \to a$ + $\eta(f) : \mathrm{id}_a \to f^- f$
  - $f^+ : b \to a$ + $\varepsilon(f) : f f^+ \to \mathrm{id}_b$
- Repeat for η and ε instead of f
-

Have:

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$ $\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wCat}$ folk

$$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$$

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- Repeat for η and ε instead of f
- Keep going

Have:

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wCat}$ folk
$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq\!\!\!\rightarrow} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
- freely add
$$\begin{cases} f^- : b \to a + \eta(f) : \mathrm{id}_a \to f^- f \\ f^+ : b \to a + \varepsilon(f) : f f^+ \to \mathrm{id}_b \end{cases}$$
- Repeat for η and ε instead of f
- Keep going

Have: $N^b \mathcal{E}$ $N^{RS} \mathcal{E}$ $N^{\flat} \mathcal{E}$ $N^{\#} \mathcal{E}$

Thm :

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$ $\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wCat}$ folk

$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
- freely add

$$\begin{cases} f^- : b \to a + \eta(f) : \mathrm{id}_a \to f^- f \\ f^+ : b \to a + \varepsilon(f) : f f^+ \to \mathrm{id}_b \end{cases}$$

- Repeat for η and ε instead of f
- Keep going

Have: $N^b \mathcal{E} \subsetneq N^{RS} \mathcal{E}$ $\qquad N^{\flat} \mathcal{E} \qquad N^{\#} \mathcal{E}$

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$ with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$
$\{\mathcal{E} \to \mathcal{D}\} \cong$ { coinductive equivalences in $\mathcal{D}$ }

② In $\mathbf{wCat}$ folk
$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq\!\!\!\rightarrow} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows
- start with $f : a \to b$
- freely add
$$\begin{cases} f^- : b \to a + \eta(f) : \mathrm{id}_a \to f^- f \\ f^+ : b \to a + \varepsilon(f) : f f^+ \to \mathrm{id}_b \end{cases}$$
- Repeat for η and ε instead of f
- Keep going

Have: $N^b \mathcal{E} \subsetneq N^{RS} \mathcal{E} \subsetneq N^{\natural} \mathcal{E} \qquad N^{\#} \mathcal{E}$

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$

$\{\mathcal{E} \to \mathcal{D}\} \cong \{$ coinductive equivalences in $\mathcal{D}\}$

② In $\mathbf{wCat}$ folk

$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
- freely add

$$\begin{cases} f^- : b \to a + \eta(f) : \mathrm{id}_a \to f^- f \\ f^+ : b \to a + \varepsilon(f) : f f^+ \to \mathrm{id}_b \end{cases}$$

- Repeat for η and ε instead of f
- Keep going

Have: $N^b \mathcal{E} \subsetneq N^{RS} \mathcal{E} \subsetneq N^{\natural} \mathcal{E} = N^{\#} \mathcal{E}$

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$

$\{\mathcal{E} \to \mathcal{D}\} \cong \{$ coinductive equivalences in $\mathcal{D}\}$

② In $\mathbf{wCat}$ folk

$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
- freely add

$$\begin{cases} f^- : b \to a + \eta(f) : \mathrm{id}_a \to f^- f \\ f^+ : b \to a + \varepsilon(f) : f f^+ \to \mathrm{id}_b \end{cases}$$

- Repeat for η and ε instead of f
- Keep going

Have: $N^b \mathcal{E} \subsetneq N^{RS} \mathcal{E} \subsetneq N^{\natural} \mathcal{E} = N^{\#} \mathcal{E}$

complicated?

Thm:

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$

with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$

$\{\mathcal{E} \to \mathcal{D}\} \cong \{$ coinductive equivalences in $\mathcal{D}\}$

② In $\mathbf{wCat}$ folk

$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows

- start with $f : a \to b$
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complicial? ✗

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Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$ **with the properties**

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complicial? ✗ ✓

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Have: $N^b \mathcal{E} \subsetneq N^{RS} \mathcal{E} \subsetneq N^{\natural} \mathcal{E} = N^{\#} \mathcal{E}$

complicat? ✗ ✓ ✓ ✓

Thm :

Walking Coinductive Equivalence

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complicial? ✗ ✓ ✓ ✓

Thm:

Walking Coinductive Equivalence

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Idea: Build $\mathcal{E}$ as follows
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Have: $N^b\mathcal{E} \subsetneq N^{RS}\mathcal{E} \subsetneq N^{\flat}\mathcal{E} = N^{\#}\mathcal{E}$

complicated? ✗ ✓ ✓ ✓

[HLOR] + [Loubaton]

Thm: $N^{\#}\mathcal{E} \in \mathbf{Cat}_{(\infty,0)}$

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$ with the properties

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$\mathcal{D}_0 \amalg \mathcal{D}_0 \hookrightarrow \mathcal{E} \xrightarrow{\simeq} \mathcal{D}_0$

Idea: Build $\mathcal{E}$ as follows
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Have: $N^b\mathcal{E} \subsetneq N^{RS}\mathcal{E} \subsetneq N^{\natural}\mathcal{E} = N^{\#}\mathcal{E}$

complicated? ✗ ✓ ✓ ✓

[HLOR] + [Loubaton] [ORW]

Thm: $N^{\#}\mathcal{E} \in \mathbf{Cat}^{R}_{(\infty,0)}$ + $N^{RS}\mathcal{E} \in \mathbf{Cat}^{R}_{(\infty,\infty)}$

Walking Coinductive Equivalence

Constr [HLOR] $\mathcal{E} \in \mathbf{wCat}$ with the properties

① $\forall \mathcal{D} \in \mathbf{wCat}$ $\{\mathcal{E} \to \mathcal{D}\} \cong \{$coinductive equivalences in $\mathcal{D}\}$

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Idea: Build $\mathcal{E}$ as follows
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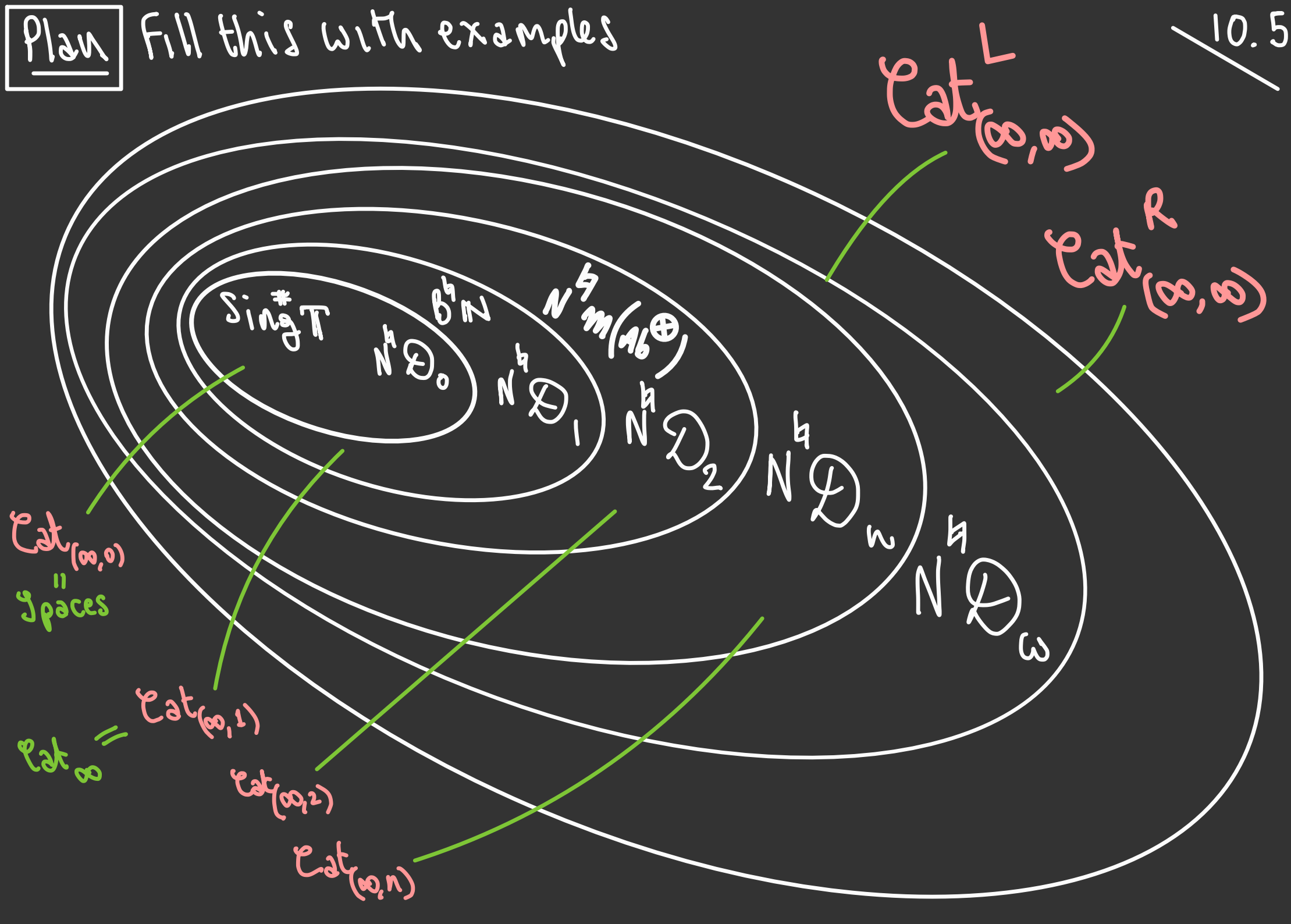
Have: $N^b\mathcal{E} \subsetneq N^{RS}\mathcal{E} \subsetneq N^{\natural}\mathcal{E} = N^{\#}\mathcal{E}$

complicated? ✗ ✓ ✓ ✓

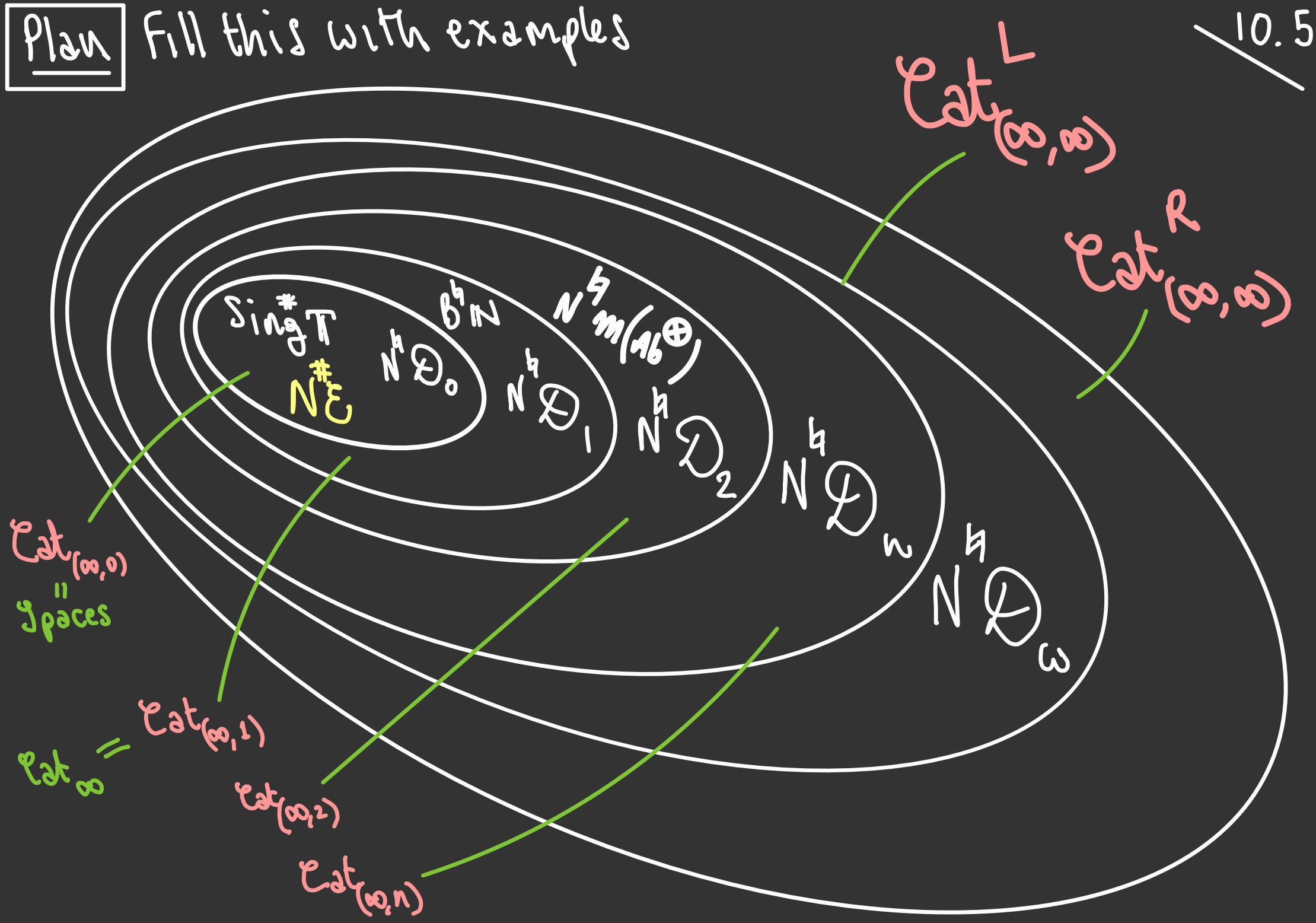
[HLOR] + [Loubaton] [ORV]

Thm: $N^{\#}\mathcal{E} \in \mathbf{Cat}_{(\infty,0)}$ + $N^{RS}\mathcal{E} \in \mathbf{Cat}^{R}_{(\infty,\infty)} \setminus \mathbf{Cat}^{L}_{(\infty,\infty)}$

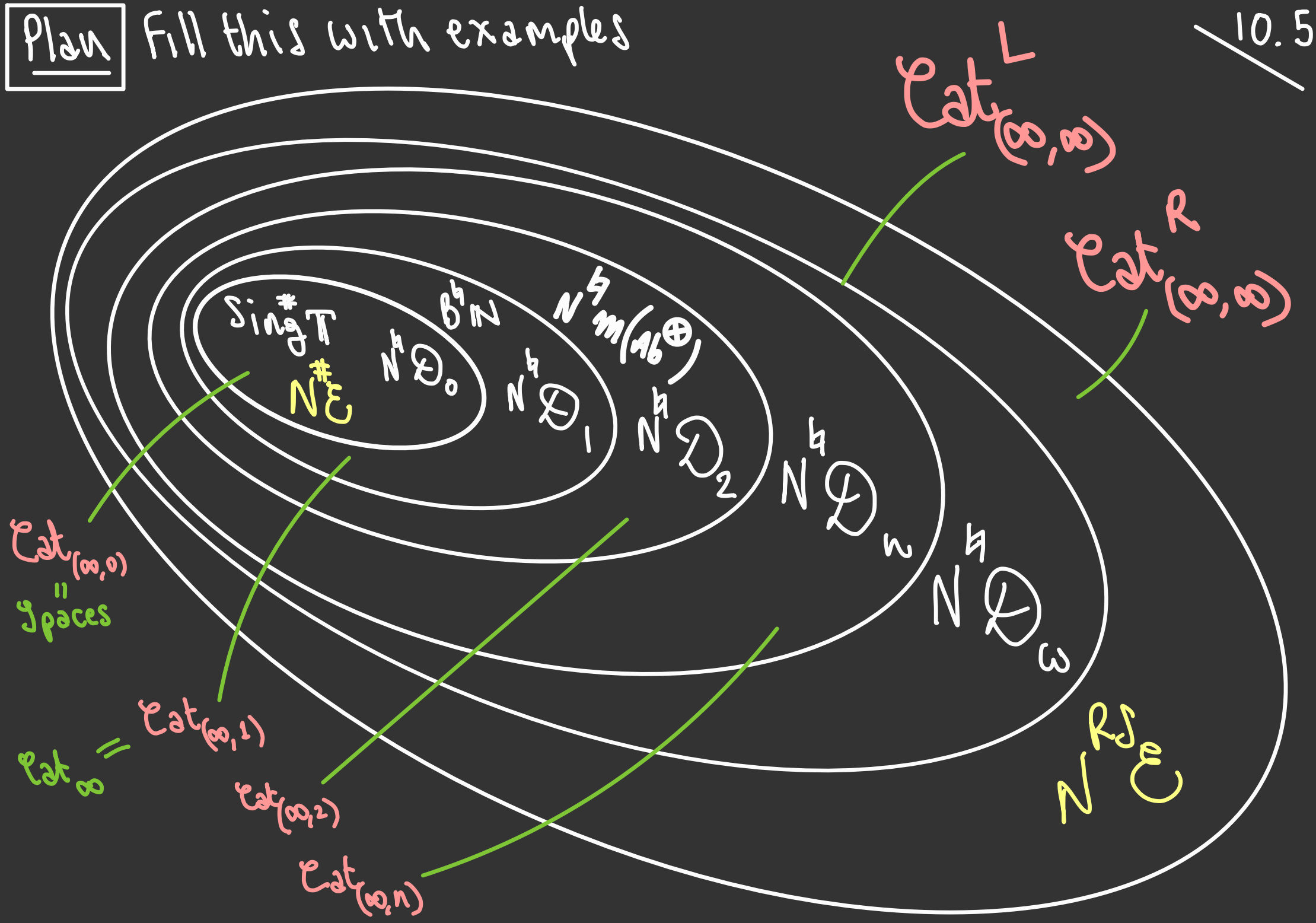
Plan Fill this with examples



Plan Fill this with examples



Plan Fill this with examples



A complicial set of bordisms

In progress

[DLRZ]

A complicial set of bordisms

In progress [DLRZ]

$\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

A complicial set of bordisms

In progress [DLRZ]

$B^{RS} \in Cat_{(\infty,\infty)}^{R}$

$B_0^{RS} = \{$ $\}$

$B_1^{RS} = \{$ $\}$

$B_2^{RS} = \{$ $\}$

A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

$\mathcal{B}^{RS}_0 = \{$ 0-manifolds $\}$

$\mathcal{B}^{RS}_1 = \{$ $\}$

$\mathcal{B}^{RS}_2 = \{$ $\}$

A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

$B_0^{RS} = \{$ 0-manifolds in $[0,1]\}$

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A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

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$B_1^{RS} = \{$ $\}$

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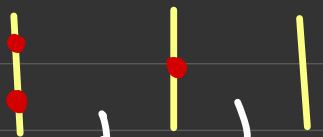
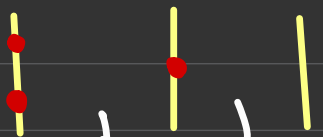
$B_0^{RS} = \{$ 0-manifolds in $[0,1]\} \ni$ 

$B_1^{RS} = \{$ $\}$

$B_2^{RS} = \{$ $\}$

A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

$B_0^{RS} = \{$ 0-manifolds in $\Delta^\circ \times [0,1] \}$ $\ni$ 

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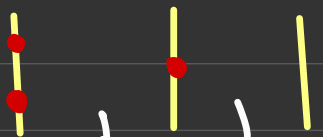
$B_0^{RS} = \{$ 0-manifolds in $\Delta^\circ \times [0,1] \}$ $\ni$ 

$B_1^{RS} = \{$ 1-cobordisms $\}$

$B_2^{RS} = \{$ $\}$

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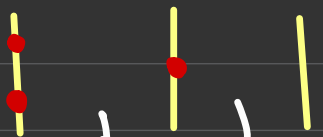
$B_0^{RS} = \{$ 0-manifolds in $\Delta^0 \times [0,1] \}$ $\ni$ 

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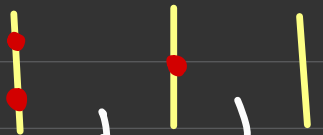
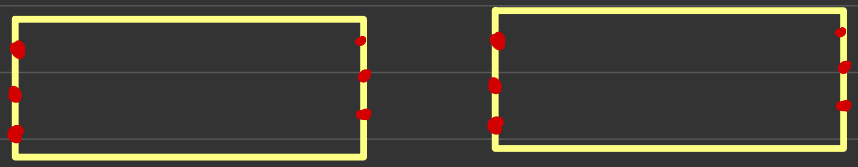
$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ 1\text{-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

$B_2^{RS} = \{ \quad \}$

A complicial set of bordisms

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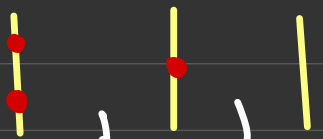
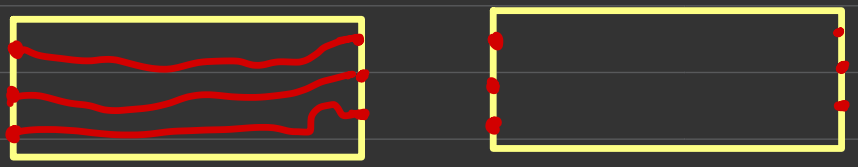
$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ 1\text{-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

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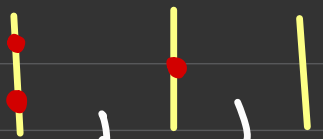
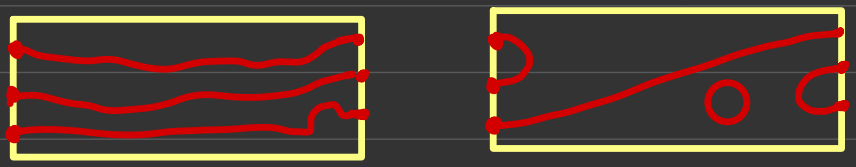
$B_0^{RS} = \{$ 0-manifolds in $\Delta^0 \times [0,1] \}$ $\ni$ 

$B_1^{RS} = \{$ 1-cobordisms in $\Delta^1 \times [0,1] \}$ $\ni$ 

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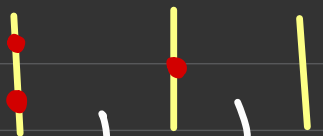
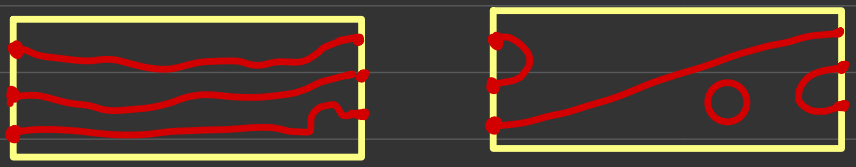
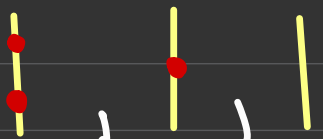
$B_0^{RS} = \{$ 0-manifolds in $\Delta^0 \times [0,1] \}$ $\ni$ 

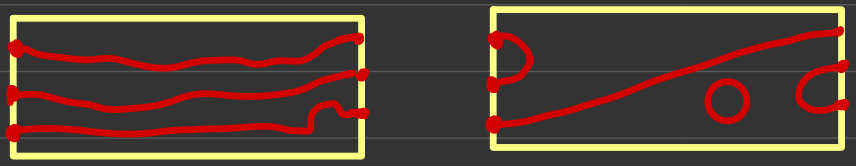
$B_1^{RS} = \{$ 1-cobordisms in $\Delta^1 \times [0,1] \}$ $\ni$ 

$B_2^{RS} = \{$ 2-cobordisms $\}$

A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ 1\text{-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

$B_2^{RS} = \{ 2\text{-cobordisms in } \Delta^2 \times [0,1] \}$

A complicial set of bordisms

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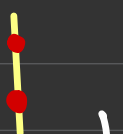
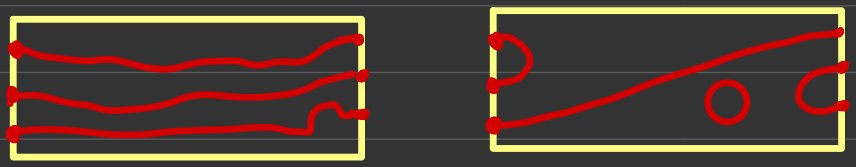
$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ 1\text{-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

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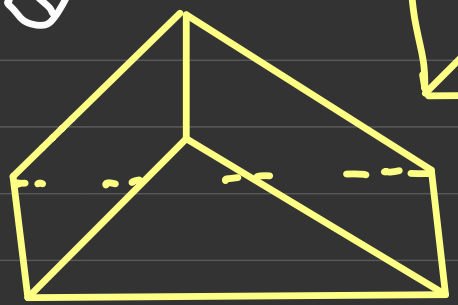
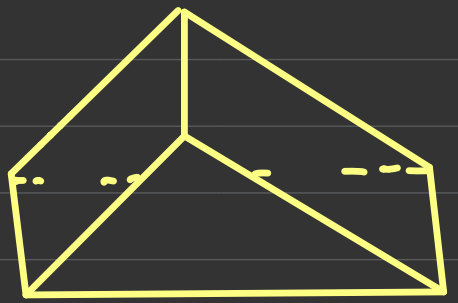
$\ni$ 





A complicial set of bordisms

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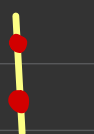
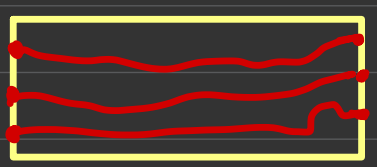
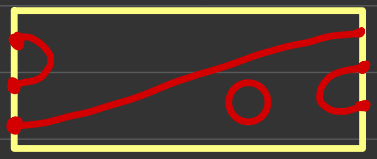
$B_0^{RS} = \{$ 0-manifolds in $\Delta^0 \times [0,1] \}$ $\ni$ 

$B_1^{RS} = \{$ 1-cobordisms in $\Delta^1 \times [0,1] \}$ $\ni$  

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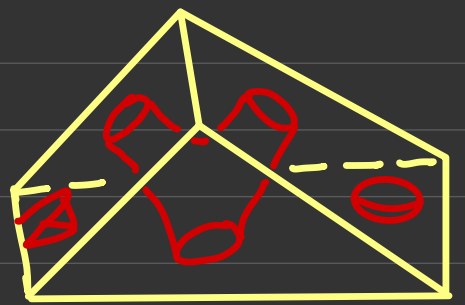
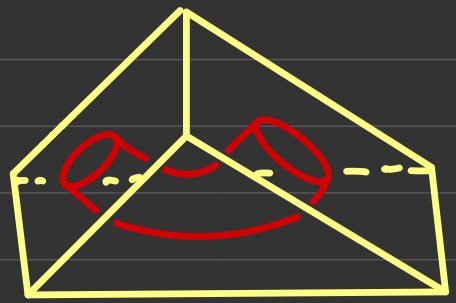
$\ni$   

A complicial set of bordisms

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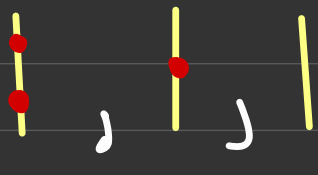
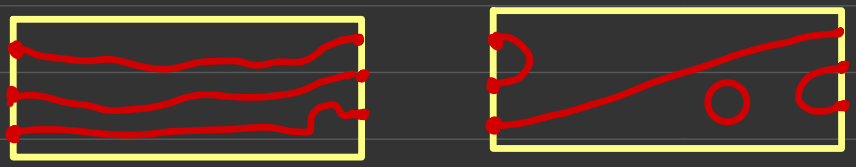
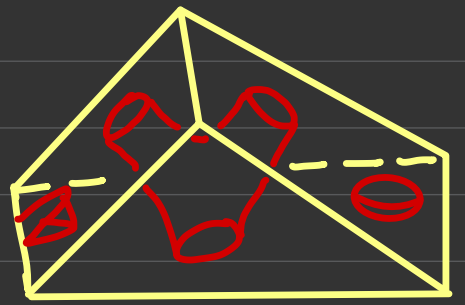
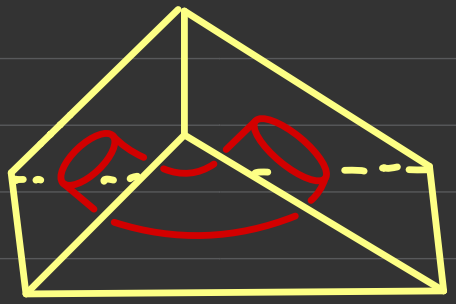
$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ 1\text{-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

$B_2^{RS} = \{ 2\text{-cobordisms in } \Delta^2 \times [0,1] \}$

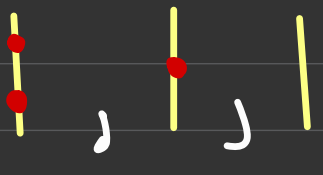


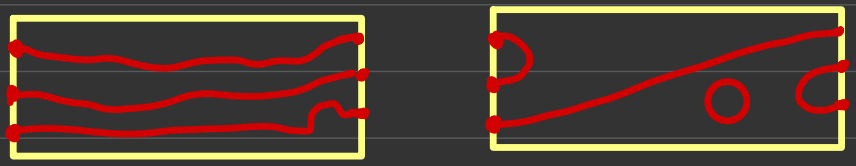




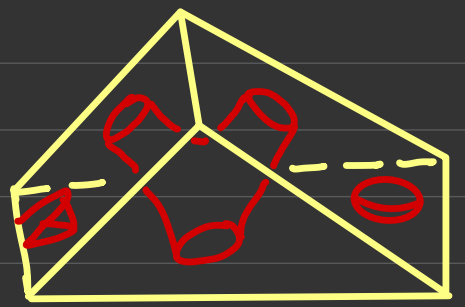
A complicial set of bordisms

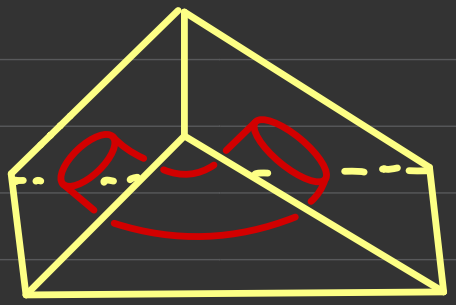
In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{$ "cylinder" 1-cobordisms in $\Delta^1 \times [0,1] \} \ni$ 

$B_2^{RS} = \{$ 2-cobordisms in $\Delta^2 \times [0,1] \}$





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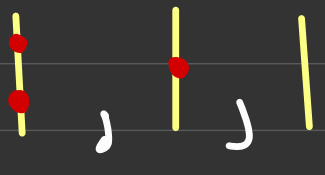
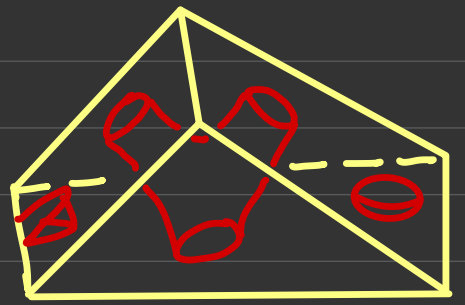
A complicial set of bordisms

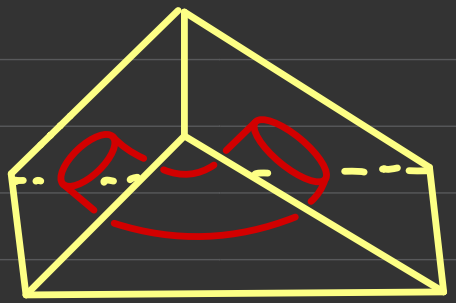
In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

$B_2^{RS} = \{ \text{2-cobordisms in } \Delta^2 \times [0,1] \}$





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A complicial set of bordisms

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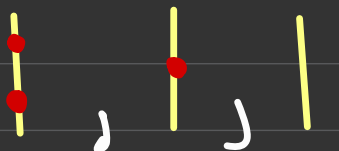
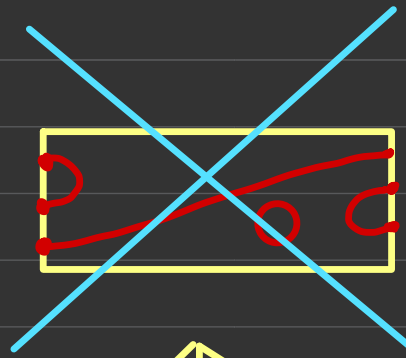
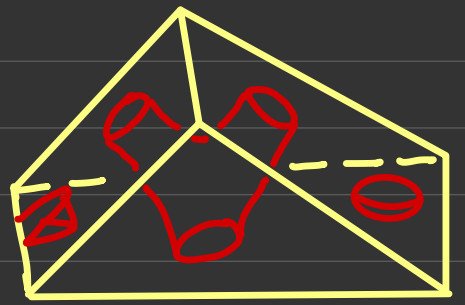
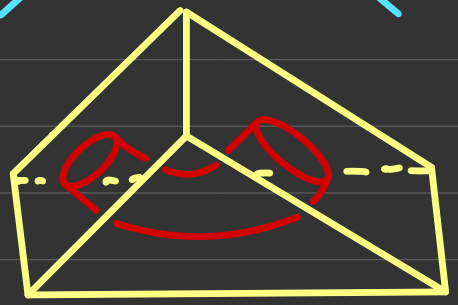
$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 



$B_2^{RS} = \{ \text{2-cobordisms in } \Delta^2 \times [0,1] \}$





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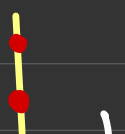
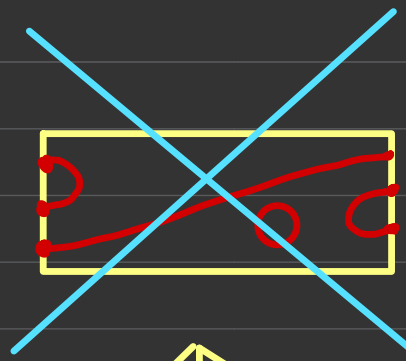
A complicial set of bordisms

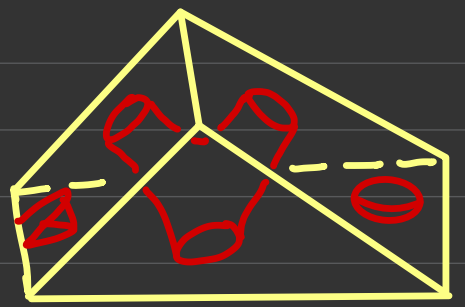
In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)}$

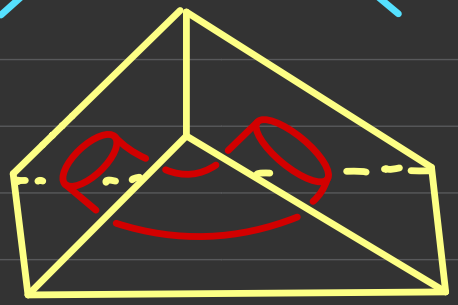
$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

$B_1^{RS} = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 



$B_2^{RS} = \{ \text{2-cobordisms in } \Delta^2 \times [0,1] \}$





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A complicial set of bordisms

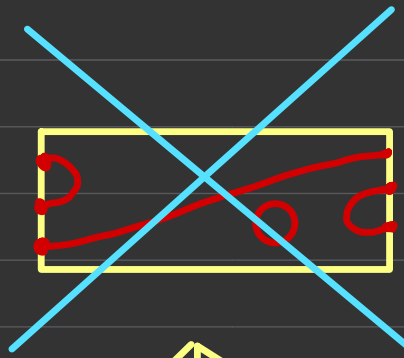
In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{k}_{(\infty,\infty)}$

$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$



$B_1^{RS} = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$





$B_2^{RS} = \{ \text{"cylinder" 2-cobordisms in } \Delta^2 \times [0,1] \}$

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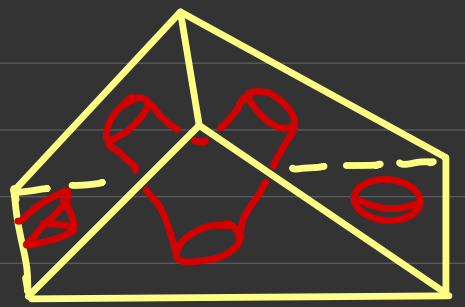
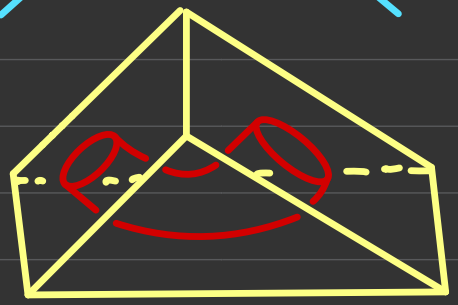
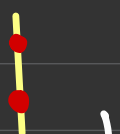
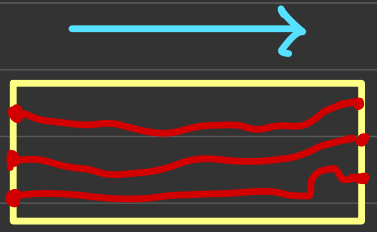


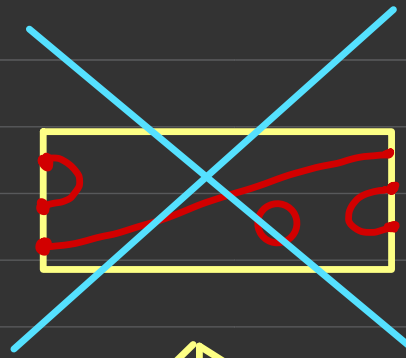


A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{k}_{(\infty,\infty)}$

$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

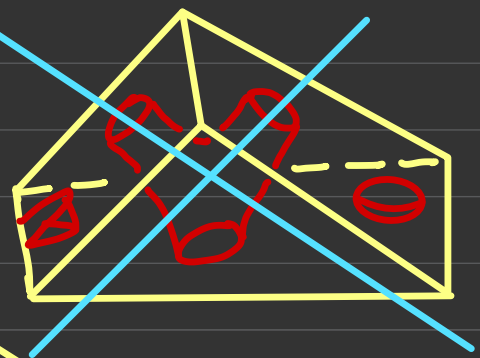
$B_1^{RS} = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

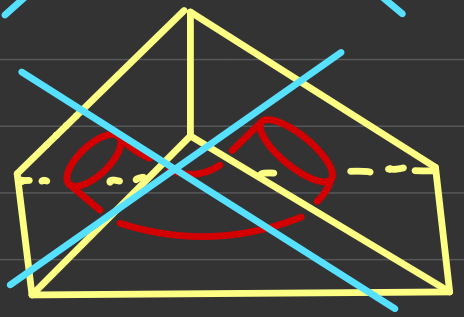


$B_2^{RS} = \{ \text{"cylinder" 2-cobordisms in } \Delta^2 \times [0,1] \}$

$\cup$



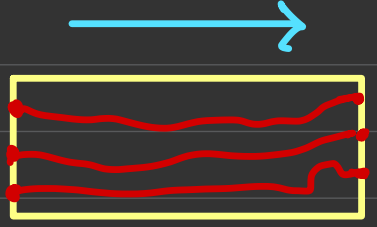


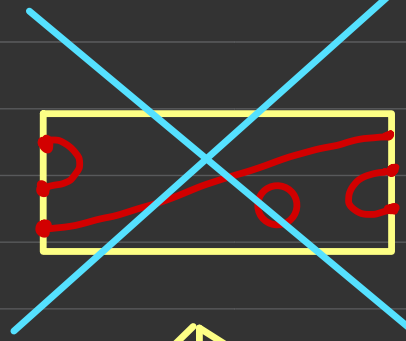


A complicial set of bordisms

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathrm{Cat}^{R}_{(\infty,\infty)} \setminus \mathrm{Cat}^{L}_{(\infty,\infty)}$

$B_0^{RS} = \{ 0\text{-manifolds in } \Delta^\circ \times [0,1] \} \ni$ 

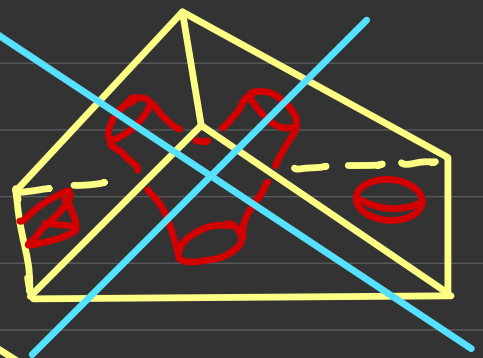
$B_1^{RS} = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

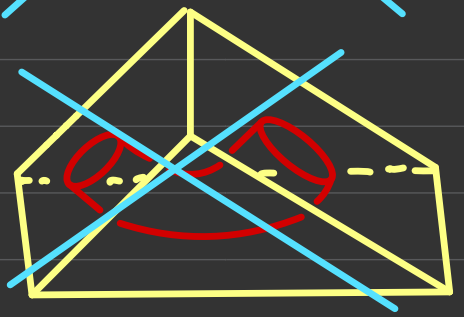


$B_2^{RS} = \{ \text{"cylinder" 2-cobordisms in } \Delta^2 \times [0,1] \}$

$\cup$







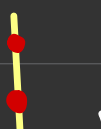
A complicial set of bordisms

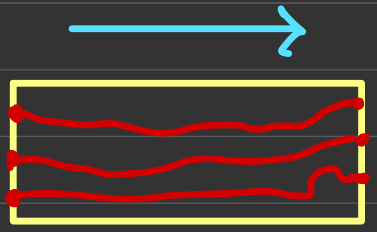
mark all 2

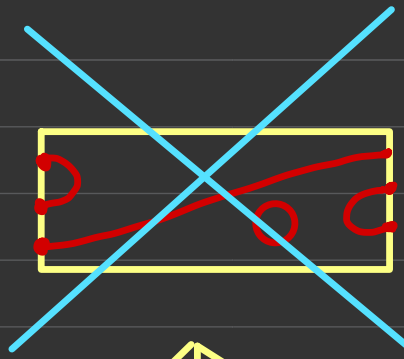
11

In progress [DLRZ]

$B^{RS} \in Cat^{R}_{(\infty,\infty)} \setminus Cat^{L}_{(\infty,\infty)}$ & $B^{\#}$

$B^{RS}_0 = \{ 0\text{-manifolds in } \Delta^0 \times [0,1] \} \ni$ 

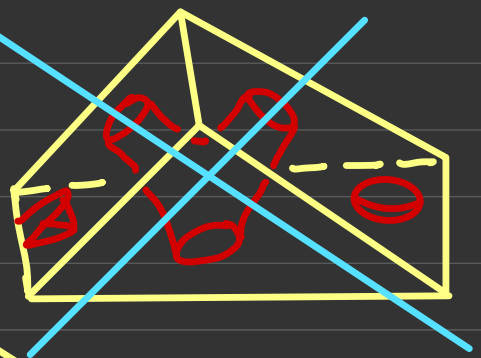
$B^{RS}_1 = \{ \text{"cylinder" 1-cobordisms in } \Delta^1 \times [0,1] \} \ni$ 

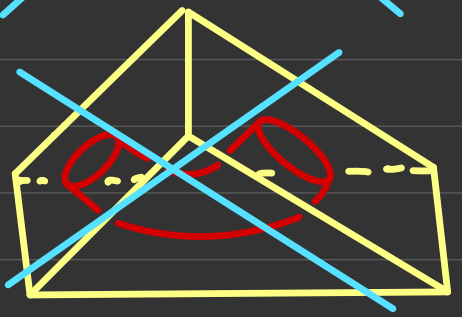


$B^{RS}_2 = \{ \text{"cylinder" 2-cobordisms in } \Delta^2 \times [0,1] \}$

$\cup$







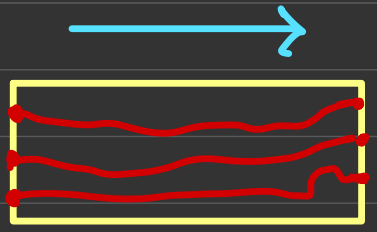
A complicial set of bordisms

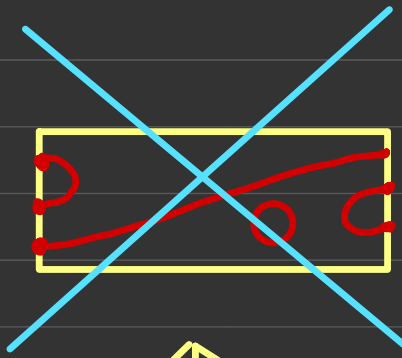
mark all 2

11

In progress [DLRZ] $\mathcal{B}^{RS} \in \mathcal{C}at^{R}_{(\infty,\infty)} \setminus \mathcal{C}at^{L}_{(\infty,\infty)}$ & $\mathcal{B}^{\#} \in \mathcal{C}at_{(\infty,0)}$

$\mathcal{B}^{RS}_0 = \{$ 0-manifolds in $\Delta^0 \times [0,1] \}$ $\ni$ 

$\mathcal{B}^{RS}_1 = \{$ "cylinder" 1-cobordisms in $\Delta^1 \times [0,1] \}$ $\ni$ 



$\mathcal{B}^{RS}_2 = \{$ "cylinder" 2-cobordisms in $\Delta^2 \times [0,1] \}$

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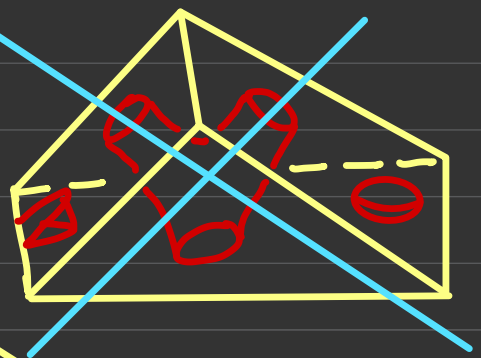
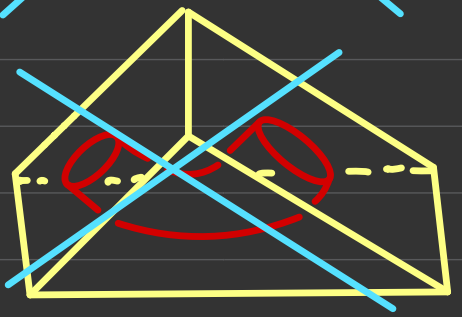
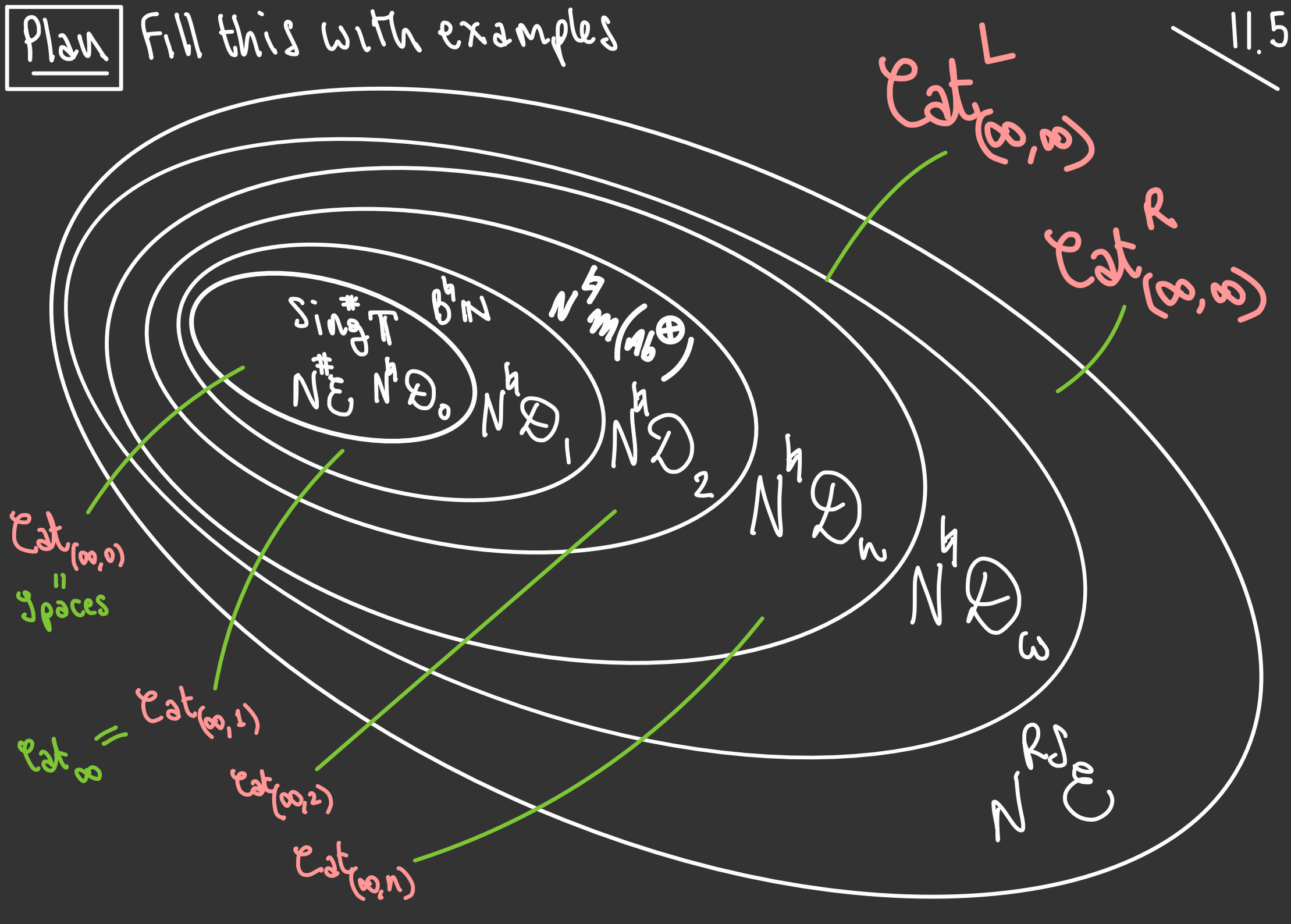
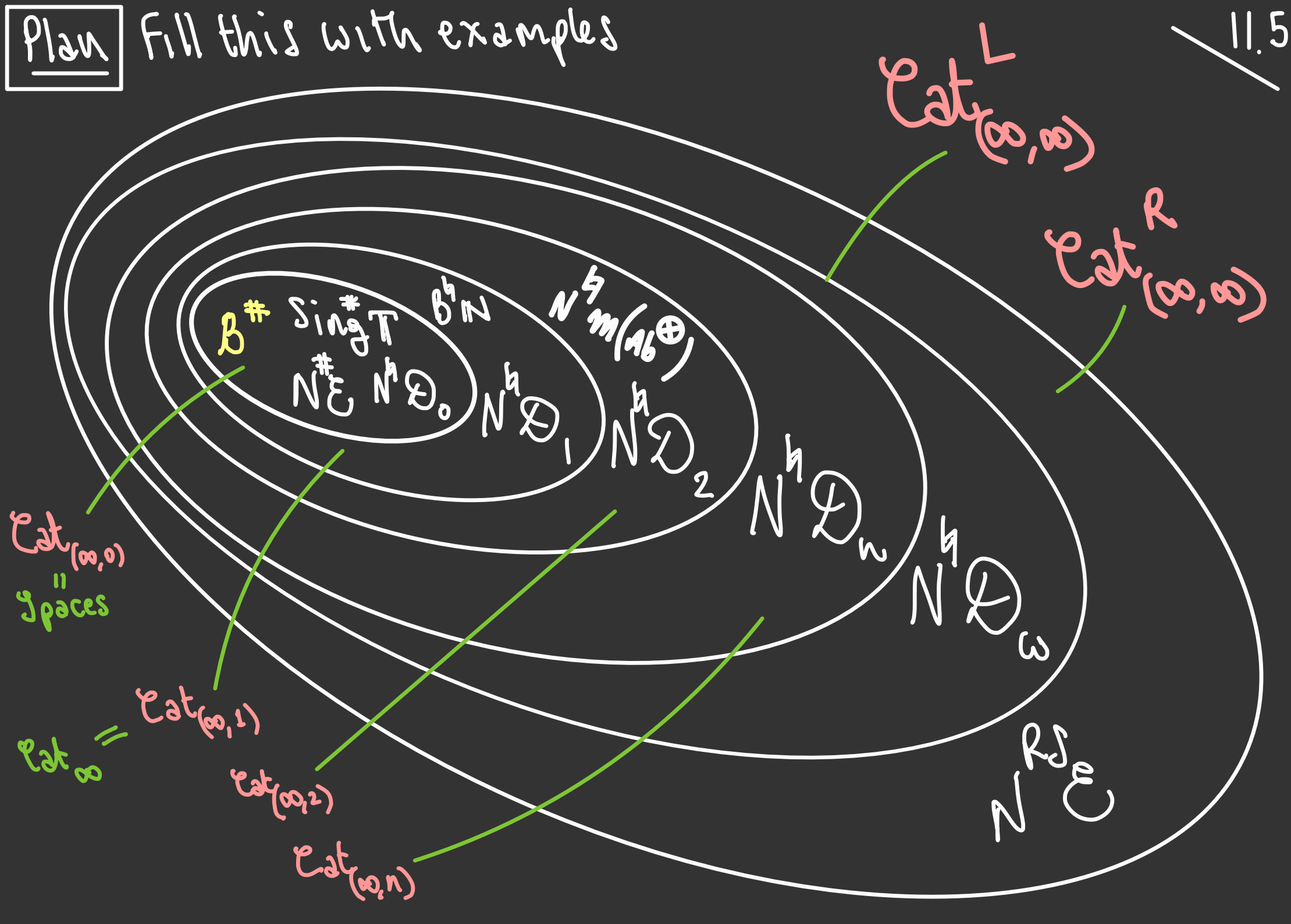




Plan Fill this with examples



Plan Fill this with examples



Plan Fill this with examples

