Applied Category Theory:
Towards a hard science of interdisciplinarity

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Society for Multidisciplinary and Fundamental Research
Interdisciplinary School
2023 July 25

This material was supported by the Air Force Office of Scientific Research under award number FA9550-20-1-0348.

Outline

Why am I here?

In 2007, I read *The Moment of Complexity* by Mark C. Taylor

- It explained that the world was getting increasingly complex.
- More would be different: Anomalies would become the norm.
- It presaged fake news, deep fake videos, difficulty knowing what's true.

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I wanted to do something, to help the world navigate. But how?

- Each person and organization structures its experience of the world.
- My neuron pattern and yours are more different than our fingerprints.
- And these are quite different than the database schemas found in orgs.
- Despite these massive differences, we are able to communicate! How?
- For the world to navigate, we must communicate better.
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Navigation requires coordination. Our efforts need to be more coherent.

- We need to understand interdisciplinarity: how to learn from others.
- I want this to be a hard science, i.e. to be supported by math.

Accounting

We solve big problems together by coordinating our activity.

- When my efforts and yours conflict, it causes friction and loss.
- When we coordinate, we stop stepping on each others' toes.
- To work collectively, our activities must align. How do we align them?
- We give accounts. We explain our activity in *terms* of the collective.

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- Or if your account hides key variables, externalities that I must handle.
- Systematicity increases transparency, communic'n rate, and reliability.
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Note: regularity is different than predictability.

- A chess game is regular (pawns don't move left), not predictable.
- Regulation: "Hey, you can't move a pawn left"; "Oh, oops!".

Mathematical fields as accounting systems

I think of mathematical fields as crystalized accounting systems.

- Arithmetic accounts for the flow of quantities, as in finance.
- Hilbert spaces account for the states of elementary particles, as in QM.
- Probability distributions account for likelihoods, as in game theory.
- Calculus accounts for relative rates of change.
- They're all crystalline in the sense of being structured and lawful.

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- Carefully track the phenomena, articulate the structure, systematize.
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Category theory (CT) is the accounting system for interlocking structures.

- Mathematical definitions are composed of interlocking structures.
- Category theory tracks the layers of structure and their connections.
- This makes analogies—similarities of structure—into formal objects.
- It accounts for the fact that different accounting systems cohere.

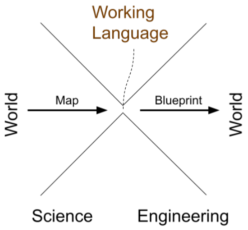
Science, math, and engineering

Roughly: science compresses, engin'ring elaborates.

- Scientists map of the world: lift out patterns.
- Engineers make blueprints to reproduce them.

Betw. science and eng'ring are linguistic artifacts.

- These are compressions of the key ideas.
- More math here is usually considered better.



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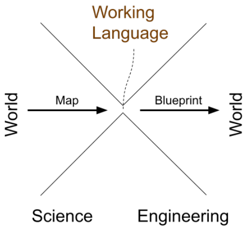

What would a hard science of interdisciplinarity be?

- Hard sciences are ones that are more mathematically accountable.
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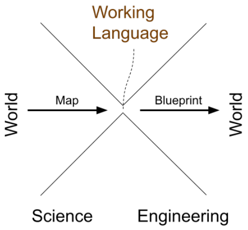

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Category theory is like a conceptual stem cell.

- A stem cell can differentiate into huge variety of forms.
- Coming from a common origin, these forms work together coherently.

Category theory as conceptual stem-cell

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- All forms of pure math... (algebra, topology, logic, number theory, differential equations...)
- Databases and knowledge representation (categories and functors)
- Functional programming languages (cartesian closed categories)
- Dynamical systems and fractals (operad-algebras, co-algebras)
- Shannon Entropy (operad of simplices, internal algebras)
- Taxonomies, metric spaces, and networks (enriched categories)
- Measurements of diversity in populations (magnitude of categories)
- Open economic game theory (Lens categories)
- Collaborative design (enriched categories and profunctors)
- Petri nets and chemical reaction networks (monoidal categories)
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We counter this objection in two ways:

- Couldn't the same objection be made about mathematics?
  - Mathematics is the basis of hard science, used everywhere.
  - CT—like math—explains, models, formalizes many many things.
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  - Conclude that math/CT explains everything and hence nothing?
- Stem cells don't do work until they differentiate.
  - "Adult-level" work requires differentiation and optimization.
  - But the unified origins lead to impressive interoperability.
  - That's what we need for interdisciplinarity.

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- And it's branched out from math in a big way.
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- Operads:

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- Operads: compositional arrangements of things within things.
- They're a branch of category theory that well-represents the spirit.

Plan of the talk

Overarching idea: CT as math to scaffold accounts from many disciplines.

- To pick one, we'll discuss *operads*: compositional arrangements.
- I'll sketch a definition and give a lot of examples.
- I'll explain dynamic operads, e.g. for learning and prediction markets.
- I'll conclude with a summary.

Outline

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Operations take arrangements of many sorts and produce one sort.

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Slightly more formal definition to come.

Operads are everywhere

Operads are used unconsciously in many fields.

- Electrical engineering: “wiring diagrams”
- Design: “set-based design”
- Computer programming: “data flow”
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We want to bring operads to the fore.

- There’s a common theme in the way we think.
- Operads structure this sort of thinking.
- With mathematical structure, we can go much further.

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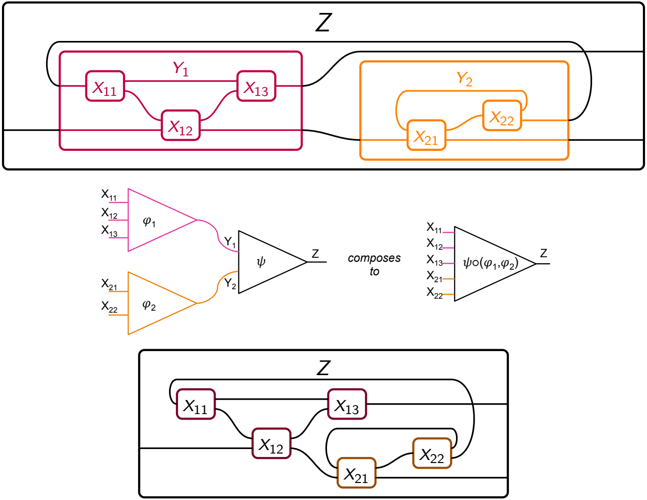
Let's look for sorts, arrangements, and nesting in some examples.

Operad 1: wiring diagrams



Sorts: boxes with ports.

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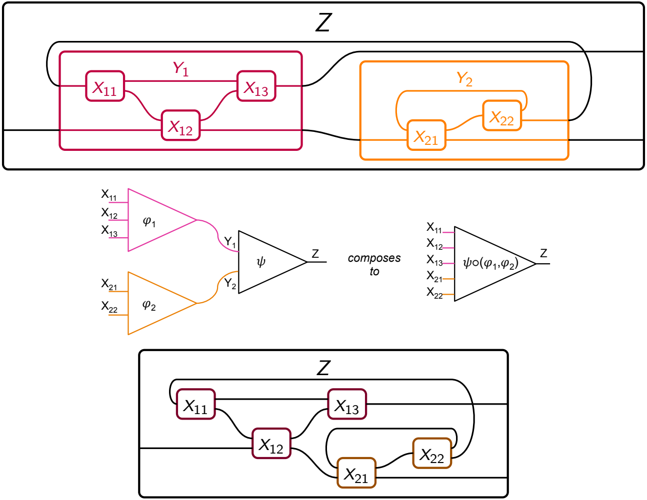

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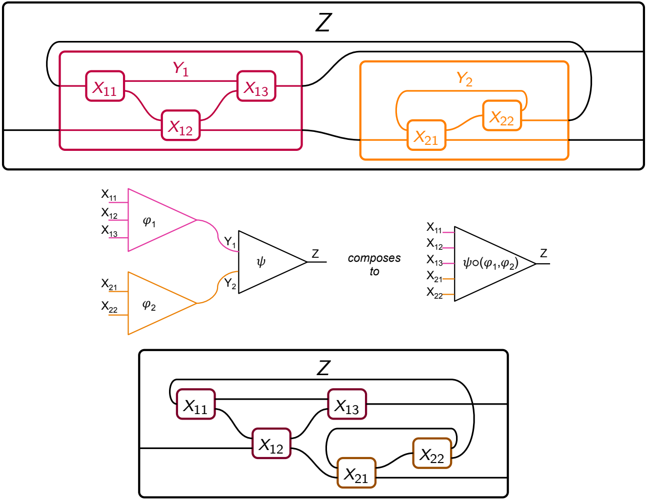


Sorts: boxes with ports. Arrangements: wiring diagrams. Nesting: nesting.

Formal definition of operad

An operad $\mathcal{O}$ consists of

- A set $\mathrm{Ob}(\mathcal{O})$, elements of which are called *sorts*.
- For sorts $X_1, \ldots, X_k, Y \in \mathrm{Ob}(\mathcal{O})$, a set

$$\mathrm{Mor}_{\mathcal{O}}(X_1, \ldots, X_k; Y)$$

  Its elements are called *morphisms* or arrangements of $X_1, \ldots, X_k$ in Y. A k-ary arrangement $\varphi \in \mathrm{Mor}_{\mathcal{O}}(X_1, \ldots, X_k; Y)$ may be denoted

$$\varphi\colon (X_1, \ldots, X_k) \to Y.$$

- For each sort $X \in \mathrm{Ob}(\mathcal{O})$, an identity arrangement $\mathrm{id}_X\colon (X) \to X$.
- A composition, or nesting formula, e.g.,

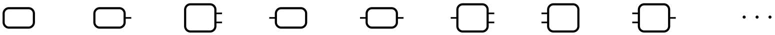
$$\psi \circ (\varphi_1, \ldots, \varphi_k)\colon (X_{i,j}) \xrightarrow{\varphi_i} (Y_i) \xrightarrow{\psi} Z.$$

These are required to satisfy well-known "unital" and "associative" laws.

Operad 1: WDs again

An operad $\mathcal{W}$ for composing wiring diagrams:

- Sort $X \in \mathcal{W}$: any possible box-with-ports.



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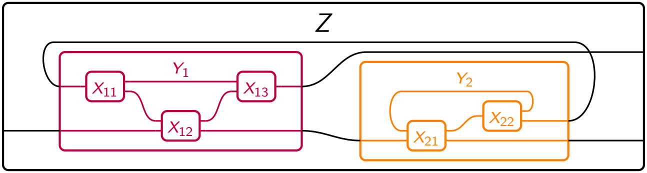
- Arrangement $\varphi\colon X_1, \ldots, X_k \to Y$ in $\mathcal{W}$: any wiring of X's in Y.
- Nesting: the facts about this fractal of wiring possibilities.



- (You could imagine an open dynamical system in each box.)

$\mathcal{W}$ is the decision of what sorts and arrangements you're considering.

Operad 2: hierarchical protein materials

There is an operad $\mathcal{M}$ for composing hierarchical protein materials.

- Why protein materials?
  - Protein materials include your skin: stretchable, breathable, waterproof.
  - Eat hamburgers, make amazing material.
  - Materials scientists would *love* to make materials like this.

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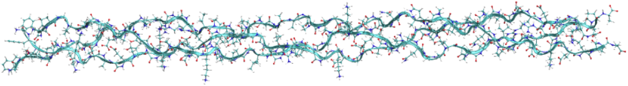
  - Materials scientists would *love* to make materials like this.
- A protein is an arrangement of simpler proteins.
  - There are “atomic” proteins: amino acids.
  - arrange in series or parallel (H-bonds), or
  - arrange in helices, double helices, any conceivable curve, etc.



- Collagen has a nested structure: it is an array, each fiber of which is a triple helix, each strand of which is a helix, each unit of which is an amino acid.[1]

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Operad 3: probabilities

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Arrangement: "In this event, there's a distribution on next events."

- coin flip: $f = (\frac{1}{2}, \frac{1}{2}) \in \mathscr{P}_2$.
  - In the event coin flip, there's a 50-50 distribution on next events.
- die roll: $r = (\frac{1}{6}, \dots, \frac{1}{6}) \in \mathscr{P}_6$.
- card selection: $p = (\frac{1}{52}, \dots, \frac{1}{52}) \in \mathscr{P}_{52}$.

The nesting rule composes distributions by weighted sum:

- Flip a coin: result decides whether to roll a die or pick a card.

$$f \circ (r, p) = \left(\underbrace{\frac{1}{12}, \dots, \frac{1}{12}}_{\text{6 times}}, \underbrace{\frac{1}{104}, \dots, \frac{1}{104}}_{\text{52 times}} \right) \in \mathscr{P}_{58}$$

A zoo of operads: Grammars

Any context-free grammar is an operad.

⟨sentence⟩	::=	⟨noun-phrase⟩ ⟨verb-phrase⟩
⟨noun-phrase⟩	::=	⟨pronoun⟩ | ⟨proper-noun⟩ | ⟨determiner⟩ ⟨nominal⟩
⟨nominal⟩	::=	⟨noun⟩ | ⟨nominal⟩ ⟨noun⟩
⟨verb-phrase⟩	::=	⟨verb⟩ | ⟨verb⟩ ⟨noun-phrase⟩ | ⟨verb⟩ ⟨prep-phrase⟩
⟨prep-phrase⟩	::=	⟨preposition⟩ ⟨noun-phrase⟩

How is this an operad?

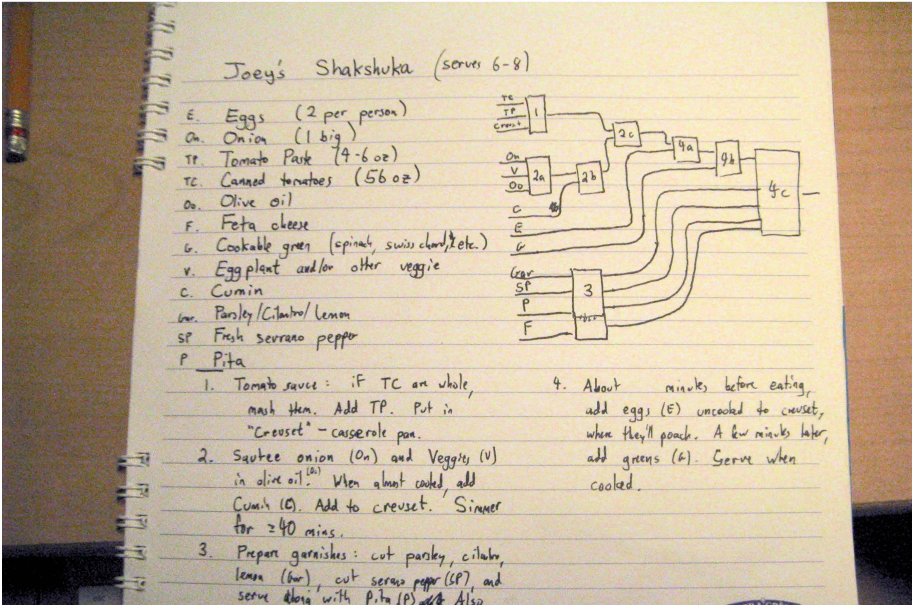
- The sorts are the parts of speech.
- The arrangements are the production rules.
- Nesting is nesting.

An arboretum of operads: Recipes

Each recipe is an operad.

- Combine sub-recipes to make a recipe.
- The outline for this talk is a recipe for getting an idea across.
  - Sorts: points I want to make.
  - Arrangements: putting points together to make a bigger point.
  - Nesting: the well-known outline structure.

An arboretum of operads: Recipes



Summary on operads, and a quick word on algebras

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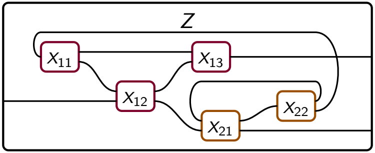

The operad is syntax, the algebra is semantics: something it can be about.

- Imagine putting a little machine into each of these little boxes.
- Any arrangement of machines makes a new machine.
- So wiring diagrams can be "about" machines.

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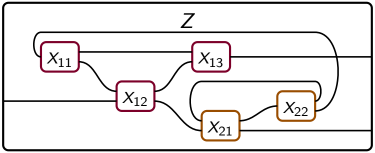

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So far, all of our arrangements have been static. Let's relax that.

Outline

Even machines change over time

People often refer to functions as machines.

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In real life, machines change as you use them.

- An over-used key on your keyboard might have a faded letter.
- Your shoes wear down according to how you walk.
- Similarly for your baseball glove, your brain, your home.
- Automatic, deterministic, total, but they *change based on usage*.

Even machines change over time

People often refer to functions as machines.

- A function $f\colon A \to B$ takes in A's and spits out B's.
- It is automatic, deterministic, total, unchanging through use.

In real life, machines change as you use them.

- An over-used key on your keyboard might have a faded letter.
- Your shoes wear down according to how you walk.
- Similarly for your baseball glove, your brain, your home.
- Automatic, deterministic, total, but they *change based on usage*.

I want to call such a thing a *dynamic function* $A \to B$.

- They're modeled by *Mealy machines*, i.e...
- ...a set S of "states" and a function $f\colon S \times A \to B \times S$.
- For any state $s : S$, you get a machine that takes an $a : A$ and...
- ...not only gives out a B but also updates the state.
- The keys, shoes, glove, brain, and home respond to the input...
- ...and can also be changed by it.

Deep learning, prediction markets, organizations

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We (joint with B.T. Shapiro) have a math model of the first two.

- Also (joint with S. Libkind) a simplified model of Hebbian learning.
- These are all examples of a single mathematical structure.

Dynamic arrangements

Recall that operads were systems for nestable arrangements, e.g.:

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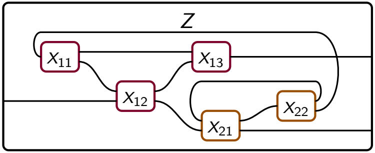
In each case, what would it mean for the arrangements to be dynamic?

- WDs that rewire based on what flows along the wires.



- Protein mat'ls who's configuration changes based on how it's pulled.
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- And so on.

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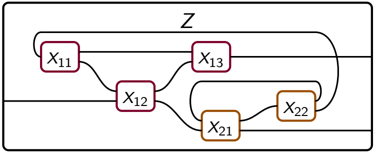

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The state controls the arrangement, and it changes based on what occurs.

Dynamic operads

A *dynamic operad*[2] is a coherent system of dynamic arrangements.

- The coherence means: the changing arrangements nest lawfully.
- Examples: deep learning, Hebbian learning, prediction markets,...

[2] Shapiro, B.T.; Spivak, D.I. "Dynamic operads, dynamic categories: From deep learning to prediction markets".

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- Deep learning: basically the chain rule.
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- ...when they fire together, and it doesn't matter how you chunk it.
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I would love a dynamic operad of wiring diagrams, but I can't think of one.

- How we arrange themselves—who you hang out with—is v. important.
- And same at all levels: certain things like to work together.
- Shouldn't there be some coherent system for this?

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Outline

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- For this we need to integrate multiple perspectives.
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Hopefully this gives a taste of ACT. It's really fun and increasingly useful.

Thanks! Comments and questions welcome...