Quantum information 2-torial

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1 Overview

The elements of quantum information theory are not so complicated: some linear algebra over the complex numbers and a bit of classical probability theory. But it doesn't take long to come before you run into some intriguing problems that showcase how established experimental results contradict our classical intuition. In this 2-torial we will learn the very basics of quantum theory, see how a Mach–Zehnder interferometer gives a real-world implementation of a classically impossible construction and suggests that we should go all in on complex numbers, and finally how this all applies to the world of theoretical bird watching via the Quantum Zeno effect.

Prerequisites. Complex numbers, (finite) vector spaces and matrices, elementary probability theory.

Key concepts. Bra-ket notation, unitary matrices, probability amplitudes, beam-splitters, Mach–Zehnder interferometers.

Further reading. qubit.guide

2 Exercises

2.1 Some linear algebra

1. Prove that $(|b\rangle\langle a|)^\dagger = |a\rangle\langle b|$.
2. Prove that $\mathrm{tr}(|b\rangle\langle a|) = \langle a \mid b\rangle$.
3. Explain why the eigenvalues of any projection ($P^2 = P$) must be either 0 or 1.

2.2 Quantum theory isn't real

1. Fail to design a classical logic gate that operates on a single bit and such that when it is followed by another, identical, logic gate the output is always the negation of the input. Justify why you could not succeed.
2. Replace probabilities with probability amplitudes, and explain how the double-slit experiment shows that $(a+b)^2 \neq a^2 + b^2$.
3. Combine two symmetric beam-splitters in sequence and describe what happens when following the path of a photon. Relate this to question 1. Explain why the title of this section is a funny joke.

2.3 Quantum bird watching

There is a bird that lives hidden between the leaves of a leafy tree. It is a wonderful bird, and incredibly shy, which means that spotting one and gazing in awe is difficult. In fact, if the bird detects even a single photon that reflects from its shiny feathers into the direction of a potential viewer's eyes, it gets shy and nervous and unhappy. They also get really uncomfortable when people sit directly under their home tree. However, they take delight in people watching, and especially enjoy watching people sat under a leafy tree reading a book.

You would really enjoy sitting under a leafy tree to read a book, but you really don't want to make any birds unhappy. There are lots of trees in the field, but how can you choose one to sit under that doesn't contain a bird if you can't even look to see if there is a bird without making it unhappy if there is one?

Use $\sqrt{\texttt{NOT}}$ to help you.